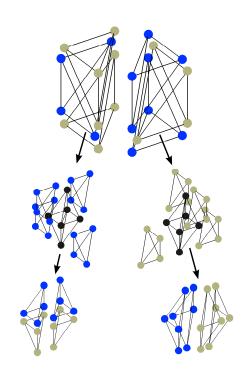
Woptic User's Guide

Optical conductivity with Wannier functions.

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for version 1.0.0- α



Introduction

To compute the optical conductivity σ in the basis of maximally-localized Wannier orbitals the package wortic is provided. It uses the Green function formalism, which yields

$$\sigma^{\alpha\beta}(\Omega) = \frac{e^2}{(2\pi)^2} \int \! \mathrm{d}^3k \int \! \mathrm{d}\omega \,\, \frac{f(\omega) - f(\omega + \Omega)}{\Omega} \, \mathrm{tr} \left[A(k,\omega) \, V^\alpha(k) \, A(k,\omega + \Omega) \, V^\beta(k) \right], \quad \ (*)$$

where $\sigma^{\alpha\beta}$ is the (α,β) element of the optical conductivity tensor $(\alpha,\beta\in\{x,y,z\})$, $V_{\rm uc}$ the unit cell volume, f the Fermi function, $A=\frac{\mathrm{i}}{2\pi}(G-G^\dagger)$ the generalized spectral function [1, 2, 3], and V^α the group velocity in direction α . The numerical bottleneck in evaluating (*) is the k-summation, since usually many k-points are required to obtain converged results. For a speed-up in k-mesh convergence, worth therefore employs an adaptively refined tetrahedral tiling of k-space.

WOPTIC consists of two main programs: woptic_main, which calculates the optical conductivity, and refine_tetra, where the k-mesh is refined; as well as several smaller support programs. The individual programs are normally called by means of the driver script woptic. This guide provides technical documentation for WOPTIC. For details on the underlying formalism and algorithm, see Refs. [4, 5, 6].

Acknowledgement Development of this software was supported by Vienna University of Technology, Graz University of Technology, and the European Research Council through grant agreement no. 306447.

Caution Following the many recent changes in WIEN2WANNIER and WOPTIC, some parts of the older version of WOPTIC have not yet been adapted. As such, they must be considered experimental. We keep such untested (or even known to be broken) features in the code and in this guide, where they are marked with a "dangerous bend" sign, \(\bigsige \).

Citation In any scientific publications arising from the use of woptic, we ask that you cite Ref. [6],

E. Assmann, P. Wissgott, J. Kuneš, A. Toschi, P. Blaha, and K. Held, Comput. Phys. Commun. 202, 1 (2016) arXiv:1507.04881,

to acknowledge your use of our code. This is in addition to the appropriate citations to acknowledge other codes used (such as Wienzk [7], wannier90 [8], and wienzwannier [9]).

Common options and other resources This guide attempts to document the features most relevant to the WOPTIC user; it will not list every option or every file used by every command. Most commands honor the option --help, which should provide a definitive list of options for that command. The WOPTIC distribution also includes a terse instruction sheet as doc/WOPTICHEAT.

Contributing Woptic is Free Software. You can contribute to it through GitHub. Bug reports, feature requests, etc. can be submitted there under Issues.

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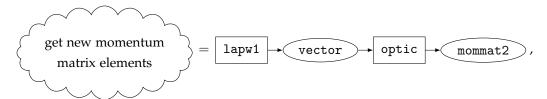
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1 The driver script woptic

This is the main user-callable program. It runs the other programs as necessary until a set number of iterations is completed (or an error occurs) — convergence has to be checked manually. If you include an outer window in your interp calculation, you should check the localization of $W^{\alpha\beta}(R,\omega)$ and/or the interpolation errors in the optical conductivity.

Since the procedure is a little involved, we provide Fig. 1.1 to give an overview of the files and programs involved in one iteration (but note that not all files that might be involved are shown). The computation of the group velocities $V^{\alpha}(k)$ for the new k-points varies according to the option matelmode. Wortic implements two modes using the full momentum matrix elements $V^{\alpha}_{ab}(k) = \langle \psi \ ak | \ \widehat{p}_{\alpha} \ | \psi \ bk \rangle$ as group velocities:

optic mode takes the matrix elements from Wienzk's optic module,

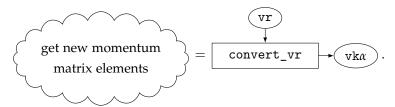


and transforms them to the Wannier basis using the matrices U(k) which diagonalize the Wannier-interpolated Hamiltonian, $U_{nu}(k)H_{uv}^{w}(k)U_{vm}^{\dagger}(k)=\delta_{nm}\epsilon_{n}(k)$. But the diagonalization fixes the eigenvectors only up to a phase, which leads to a *random-phase problem* in (*) and associated uncertainties in the optical conductivity. The problem is absent whenever the self-energy is orbital-independent (by symmetry, or in a noninteracting model). In such a case, optic mode should be dependable. Otherwise, the results should be checked for the influence of the random-phase problem.

interp mode applies Wannier interpolation to the matrix elements directly in order to overcome the random-phase problem. compute_vr calculates the Wannier momentum matrix elements in direct space

$$V_{uv}^{w,\alpha}(R) = \frac{1}{N_k} \sum_k e^{-ik \cdot R} U_{un}^{\dagger}(k) V_{nm}^{\alpha}(k) U_{mv}(k) = \langle w \ u0 | \widehat{p}_{\alpha} | w \ vR \rangle,$$

and convert_vr interpolates them to the new k-points,1



The interpolation works well for the Wannier-Wannier transitions ($V^{W,\alpha}$), but interpolation errors may become large for the mixed transitions governed by $W^{\alpha\beta}(R,\omega)$, where

$$W_{uv}^{\alpha\beta}(k,\omega) = \sum_{i} V_{ui}^{\alpha}(k) A_{ii}(\omega) V_{iv}^{\beta}(k)$$

with the index *i* running over the non-Wannier states (i.e. the outer window) and the matrix elements are transformed into the Wannier basis on one side only. Note that the interpolation errors typically only affect the interband optical conductivity; as long as the low-energy degrees of freedom are described by the Wannier functions, the static quantities (dc conductivity and thermopower) should be reliable.



In addition, Peierls mode uses the Peierls approximation $V(k) \approx i \nabla_k H(k)$ instead of the momentum matrix elements. It is currently unsupported.

Further reading. See [4] for the original description of wortic in the optic and Peierls modes. See [5] for a detailed description of interp mode and a numerical comparison to optic mode including an analysis of the errors committed in each of them. Ref. [1] tests the Peierls approximation against the full momentum matrix elements.



Disentanglement is supported only in interp mode in the case where only Wannier-Wannier transitions are included. This may be useful when the Wannier model is expected to describe all the salient features of a system, but disentanglement is necessary, e.g., to remove extraneous states at the band edges.

¹In the interest of full disclosure, the diagram is accurate in the case where only Wannier-Wannier transitions are taken into account. With mixed transitions, vr is supplemented by vvr and vk α by vvk $\alpha\beta$, and optic is also called for the mixed [Wannier-Bloch] matrix elements.

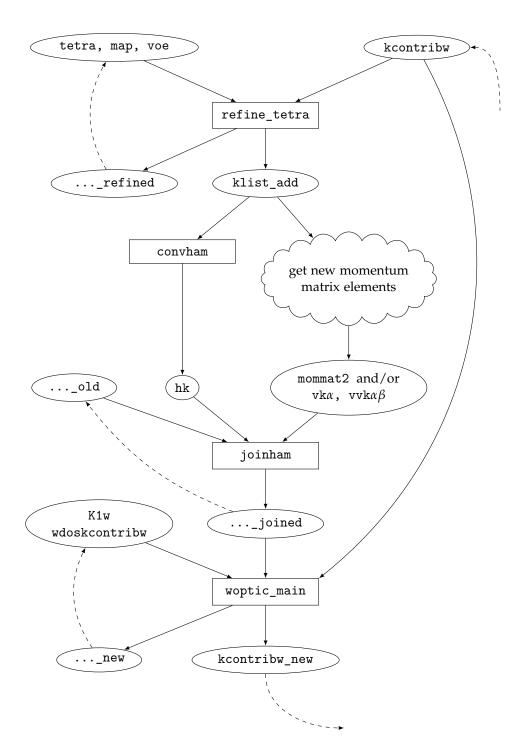


Figure 1.1: Flow of control and information in the main woptic loop: programs (rectangles) and selected files (ellipses). Dashed lines indicate a file is taken from the previous iteration.

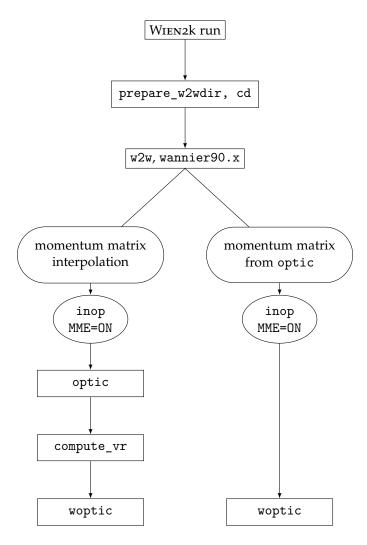


Figure 1.2: Flow of control and information in woptic initialization.

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Synopsis

```
woptic [-i N_{tot} ] [--restart I] [more options]
```

Options 1.2

```
-i N_{tot} stop after iteration N_{tot} (default: 5)
```

- --restart *I* restart from *case* .wophist.zip at the beginning of iteration *I* (default: 1)
- --restore I restore iteration I from case .wophist.zip without continuing
- **--theta** Θ refinement harshness ($\Theta = 0$: uniform mesh, $\Theta = 1$: most adaptive; default: 0.5)
- --inter focus refinement on larger Ω
- --init N_i initial uniform refinement steps (default: 3)



--band compute optical conductivity contributions along case.klist_band to be processed by kanalysis

-p call the parallelized versions of lapw1, lapwso, optic (wortic itself is not parallelized)

The iterations in -i, --restart, and --restore are "absolute" in the sense that iteration 1 always corresponds to the initial k-mesh. Thus, woptic --restart 3 -i 5 does three iterations: nos. 3, 4, and 5. Iteration no. 1 starts with a uniform k-mesh whose density is determined by N_i . The starting mesh corresponds to $(2^{N_i+1})^3$ k-points in the full BZ.

Let T be a tetrahedron and $\epsilon(T)$ its associated integration error estimate. The precise meaning of the "harshness" $\Theta \in [0,1]$ is: T will be refined if

$$\epsilon(T) \ge \Theta \max_T \epsilon(T)$$
 (--theta)

(but may also be refined due to other rules) [4, 6]. The --inter option scales the contributions to the error estimates by the frequency,

$$\epsilon(T;\Omega) \leftarrow \Omega \cdot \epsilon(T,\Omega),$$
 (--inter)

for the purposes of refinement.

The input file case.inwop

The main input file for WOPTIC is

```
case.inwop -
                        # mode; matelmode; intra-uc hop? (Peierls)
OPT
      interp
               3 1e-5 # Emax/O, dE, delta (eV); omega-div, tol-cutoff
10 0.05 0.05
                       # inner window (= WFs); [outer window]
# inv. temp. (eV^-1); chemical potential (eV)
21 23 12 25
40.0
       0.00
                        # Drude-sumrule sep. [eV]; compute orb.resolv.?
1.0
       F
                       # read self-energy?
                                             #bands
                        # bands with self-energy (if #b=0: inner window)
  21 22 23
F F 1.23 0
                        # rotate WFs?; scissors?, shift (eV), #bands
  24 25
                        # bands to shift
```

- end of ${\it case.}$ inwop -

This file is read using Fortran list-oriented reads, i.e., items are separated by white space.

line 1 mode, matelmode, intrahop — modes of operation

```
mode
                           compute the optical conductivity
                           compute the joint density of states (noninteracting only)
            JOINT
            1 | Peierls
                           use dH^{w}/dk as momentum matrix elements
matelmode
            2 | interp
                           Wannier-interpolated momentum matrix elements
            3 | optic
                           matrix elements computed by Wienzk's optic
            4 | Bloch
                           for testing (noninteracting only)
                           for testing (noninteracting only; should be similar to
            5 | LDA
                           optic-joint-kram)
                           whether to use intra-uc hopping in Peierls mode [2, 3]
intrahop
             logical
                           (needs case.intrahop)
```

Only modes interp and optic are thoroughly tested; Peierls, Bloch, and LDA must be considered experimental.

line 2 Ω_{max} , $\Delta\Omega$, δ , $N_{\omega/\Omega}$, ϵ — frequency grids and broadening

```
\begin{array}{lll} \Omega_{\max} & \mathrm{eV} & \mathrm{maximum} \ \mathrm{external} \ \mathrm{frequency} \ \mathrm{for} \ \mathrm{which} \ \sigma(\Omega=\Omega_{\mathrm{max}}) \ \mathrm{is} \ \mathrm{computed} \\ \Delta\Omega & \mathrm{eV} & \Omega \ \mathrm{grid} \ \mathrm{spacing} \\ \delta & \mathrm{eV} & \mathrm{broadening} \ \mathrm{parameter} \ \mathrm{for} \ \mathrm{noninteracting} \ \mathrm{bands} \ (\mathrm{where} \ \Sigma \leftarrow \mathrm{i} \delta) \\ N_{\omega/\Omega} & \mathit{int} & \mathrm{internal} \ \mathrm{frequency} \ \mathrm{density} \ (N_{\omega/\Omega} \ \mathrm{internal} \ \omega \ \mathrm{per} \ \mathrm{external} \ \Omega) \\ \epsilon & \mathit{real} & \mathrm{tolerance} \ \mathrm{for} \ \omega\text{-integration} \ \mathrm{limits} \ (-\Omega_{\mathrm{max}} \lesssim \omega \lesssim 0) \end{array}
```

Note: Set $\Omega_{\text{max}} = 0$ to compute only the dc quantities.

line 3 Wlo, Whi, [Blo, Bhi] — band windows

```
Wlo, Whi int band indices corresponding to WFs
Blo, Bhi int the outer ("Bloch") window (optional, default: Wlo, Whi)
```

line 4 β , μ — grand-canonical ensemble parameters

```
\beta eV<sup>-1</sup> the inverse temperature (To convert to temperature in Kelvin: T = 11604/\beta.) \mu eV the chemical potential (applied only to the interacting bands [see iself], use the DMFT value; for noninteracting calculations, set \mu = 0)
```

line 5 Drudesep, orbresolv

```
Drudesep eV cutoff for Drude sumrule integration orbresolv logical whether to compute observables per-orbital (currently deactivated)
```

line 6 selfE, Nself — self-energy specification

```
 \begin{array}{ll} \texttt{selfE} & \textit{logical} & \texttt{whether to read self-energy } \Sigma_i(\omega) \; \texttt{from } \textit{case} \, . \, \texttt{selfE} \\ \texttt{Nself} & \textit{int} & \texttt{number of bands with self-energy, or 0} \; \textit{(in this case, Whi-Wlo+1)} \\ \end{array}
```

```
line 7 iself — interacting bands (ignored if selfE=.false. or Nself=0)
iself    int(Nself) indices of interacting bands (if Nself=0: inner window)
```

```
line 8 wfrot, shift, Eshift, Nshift — wF rotation and scissors operator
    wfrot logical whether to apply unitary matrix from case.wfrot to wF basis
    shift logical whether to apply rigid "scissors" shift
    Eshift eV shift value
    Nshift int number of bands to shift

line 9 ishift — scissor bands
    ishift int(Nshift) indices of bands to shift
```

1.4 Environment variable \$SCRATCH

woptic and the programs it calls support Wienzk-style \$SCRATCH for temporary files. This means that lapw1 and optic will place <code>case.vector</code> and <code>case.mommat2</code> in the directory \$SCRATCH; likewise, woptic_main and refine_tetra will place certain temporary files in that directory.

However, note that a true local SCRATCH on a cluster is not supported. In this case, the individual case .mommat2_i would have to be transferred to the "master" node.

1.5 Output files

Over the course of the iterations, woptic writes diagnostic information to standard output and lists the executed commands in :log. In particular, the current values of the quantities thermopower, dc conductivity, and sumrules are extracted from woptic_main's output file <code>case.outputwop</code>, as well as the integration error estimator from refine_tetra's <code>case.outputref</code>. The latter is given in arbitrary units and should decrease over the iterations.

The optical conductivity is written to <code>case.optcondw</code>. For comparison with Wienzk's standard optic module, note that there is a factor ≈ 1112.65 between optic's output (given in Gaussian ccs units of $10^{15}\,\mathrm{s^{-1}}$ in <code>case.sigmak</code>) and wortic's (in si units of S cm $^{-1}$). Expressed in the si, the conversion is optic = wortic \cdot $4\pi\,\varepsilon_0\,10^{15}\,\mathrm{Hz}\,\Omega$ cm.

The density of states is written to case .wdos. The files case .optcondw and case .wdos always correspond to the latest iteration. Together with certain other files, they are archived in case .wophist.zip with a suffix .I for iteration I.

2 Support Programs

In this section, the sub-programs called by woptic are documented, roughly in order of decreasing user-callability.

2.1 wopticlean — remove left-over files from woptic runs

wopticlean preserves files which serve as input to woptic and its sub-programs, as well as the archive file <code>case</code>.wophist.zip. The number of files considered for deletion is substantial; to check which ones are, use the <code>--recon</code> option or the source.

Synopsis

```
wopticlean [--recursive] [--mrproper] [--recon] [directory ...]
```

Options and arguments

- -r|--recursive Operate recursively on all directories below
- -A|--mrproper delete also files whose basename does not match the containing directory
- -n|--recon dry-run; print file names that would be deleted

The arguments specify *directories* to operate on (*default: .*). Before a big cleanup (especially when using ¬A or ¬r), you are advised to do a dry-run.

2.2 compute_vr — compute $V^{\mathrm{w}}(R)$

compute_vr computes the dipole matrix elements $V^{W,\alpha}(R)$ and $W^{\alpha\beta}(R,\omega)$ in direct space by applying the matrices U(k) and a Fourier transform for use with the interp mode. You should check these matrix elements (especially $W^{\alpha\beta}$) for decay in R when using this mode.

Synopsis

```
compute_vr [--text] case
```

Option

```
--text output case.vvr in plain text
```

Files read

```
case.inwop input file
case.chk WANNIER90 checkpoint file
case_hr.dat Hamiltonian in direct space
case.mommat2 momentum matrix elements
case.struct WIEN2k master input file (mixed transitions)
case.energy energies from lapw1 (mixed transitions)
case.fermi Fermi energy (mixed transitions)
case.inwf w2w input file (disentanglement)
```

Files written

```
case.outputvr \log file case.vr V^{W,\alpha}(R) case.vvr W^{\alpha\beta}(R,\omega) (mixed transitions)
```

riangleq 2.3 kanalysis — $\sigma(k,\omega)$ on a BZ path

This program generates files that can be used for analysis of the contributions to the optical conductivity in woptic. Required is a run of woptic with the --band option, such that it computes the contributions to the optical conductivity along <code>case.klist_band</code> and stores them in <code>case.kcontribw_band</code>. kanalysis reads this file and generates 2D data in ω - and k-space readable e.g. by gnuplot.

Synopsis

```
kanalysis n_{\Omega}^{\min} case [mode]
```

Arguments

 n_{O}^{\min} minimum frequency index for output

mode (optional) by default, the output includes extra newlines for convenient plotting with
gnuplot (splot "case.optanalysis_band" with pm3d); if mode = 1, these newlines
are omitted

🕏 2.4 obtain_dist — intra-UC hopping for Peierls

In wortic, for the generalized Peierls approximation [2], the distances between the Wannier centers are required. This program reads <code>case_centres.xyz</code> which is produced by <code>wanniergo</code> and <code>generates case.intrahop</code> which can then be used by <code>wortic</code>.

Synopsis

```
obtain_dist case
```

2.5 inwopcheck — parse inwop file

A helper program for woptic. Reads an inwop file and outputs information suitable for reading in a shell script or for inspection.

Synopsis

```
inwopcheck case.inwop
```

2.6 woptic_main — k-integration

woptic_main computes the optical conductivity contributions $\sigma(k,\omega)$ on the k-mesh constructed by refine_tetra and performs the k- and ω -integration. It is normally called by woptic, but it may be useful to call it manually after a woptic run.

Synopsis

```
woptic_main [--band] case
```

Option



--band compute optical conductivity contributions along case.klist_band to be processed by kanalysis

Files used by woptic_main are listed below. *Updated* files are written with a suffix _new. Which files precisely are used depends on the options in effect, this dependence is partially indicated below.

Files read

```
case.inwop woptic main input file (always)
case.struct Wien2k master input file (always)
case.symop symmetry operations from optic (always)
case.klist symmetrized k-points (always)
case.tetra symmetrized tetrahedra (always)
case.energy energies from lapw1
case.fermi Fermi energy
case.mommat2 matrix elements from optic
case.chk wannier-interpolated matrix elements (interp)
```

```
case.vvk\alpha\beta Wannier-interpolated mixed matrix elements (interp) case.hk Wannier Hamiltonian H^w(R) case.selfE self-energy \Sigma(\omega) (selfE) case.wfrot Wannier function rotation matrix (wfrot) case.klist_full unsymmetrized k-points (Peierls) case.tetra_full unsymmetrized tetrahedra (Peierls) case.map mapping of klist_full to klist (Peierls) case.intrahop wf center distance matrix (Peierls & intrahop)
```

Files written

```
case.outputwop diagnostic output (always)
case.optcondw optical conductivity (always)
case.wdos (joint) density of states (always)

case.optcondw_orbαβ orbitally resolved optical conductivity (orbresolv) (currently deactivated)
```

Files updated

```
case.kcontribw optical conductivity contributions
case.K1w thermopower contributions
case.wdoskcontribw dos contributions
```

2.7 refine_tetra — k-mesh refinement

refine_tetra uses the optical conductivity contributions $\sigma(k,\omega)$ to compute integration error estimates and refine the k-mesh. It is normally called by woptic.

Synopsis

```
refine_tetra [options] case
```

Options

- --theta Θ $0 \le \Theta \le 1$ defines the "harshness" of refinement (see corresponding option of woptic)
- --init N_i initial refinement with N_i steps (see corresponding option of woptic)
- --inter give larger weight to higher-energy contributions (see corresponding option of
 woptic)

Files used by refine_tetra are listed below. *Updated* files are written with a suffix _refined.

Files read

```
case.inwop woptic main input file
case.struct Wien2k master input file
```

Files written

```
case.kcontribw function values for estimator on case.klist
case.outputref log file
```

Files updated

```
case.klist symmetrized k-points
case.klist_full unsymmetrized k-points
case.tetra symmetrized tetrahedra
case.tetra_full unsymmetrized tetrahedra
case.voe list of k-points on tetrahedral edges
case.map internal mapping of case.klist_full to case.klist
```

2.8 joinham — combine hk / mommat2 / vk α / vvk $\alpha\beta$ files

joinham combines _old files from the previous iteration with new files corresponding to added k-points. It is normally called by woptic.

Synopsis

```
joinham case |
joinham hk mommat |
joinham hk1 hk2 hkout |
joinham hk1 hk2 hkout mom1 mom2 momout
```

Arguments

```
hk* a file of type case.hk, case.vk\alpha, or case.vvk\alpha\beta mom* a file of type case.mommat2
```

In the first form, case .hk_old is joined with case .hk, and, if they exist, case .mommat2_old with case .mommat. In the second form, hk_old is joined with hk and mom_old with mom. In both cases, output file names are suffixed with _joined. Unformatted mom files are handled automatically and result in unformatted output.

2.9 convert_vr — Fourier-interpolate $V^{w}(k)$

convert_vr computes $case.vk\alpha$ and $case.vvk\alpha\beta$ from case.vr and case.vvr by Fourier transform. The procedure is analogous to convham, case.hk, and $case_hr.dat$.

Synopsis

```
convert_vr [--text] case
```

Option

-t, --text write $case.vvk\alpha\beta$ in plain text instead of unformatted ($case.vk\alpha$ is always plain text)

Files read

```
case.struct Wienzk master input file case.klist target k-points case.inwop woptic input file case.vr direct-space matrix elements V^{W,\alpha}(R) case.vvr direct-space mixed matrix elements W^{\alpha\beta}(R,\omega) (mixed transitions)
```

Files written

```
case.outputvk log file  \begin{aligned} &\textbf{case.vk\{x,y,z}\} & \text{k-space matrix elements } V^{W,\alpha}(k) \\ &\textbf{case.vvk\{xx,xy,xz,yy,yz,zz}\} & \text{k-space mixed matrix elements } W^{\alpha\beta}(R,\omega) \text{ (mixed transitions)} \end{aligned}
```

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