好题分享：深刻理解动态规划

用动态规划方法手工求解以下问题

有8万元的投资可以投给3个过目，每个项目在不同筒子数额下（以万元为单位）的利润如下表

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 投资额  盈利  项目 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 项目1 | 0 | 5 | 15 | 40 | 80 | 90 | 95 | 98 | 100 |
| 项目2 | 0 | 5 | 15 | 40 | 60 | 70 | 73 | 74 | 75 |
| 项目3 | 0 | 4 | 26 | 40 | 45 | 50 | 51 | 52 | 53 |

请安排投资计划，使总的利润最大。

写出你所设的状态变量、决策变量、状态转移方程与递推关系式和手工求解的详细步骤及结构。

2.

K：项目的编号，k=1,2,3；

Xk：投资的数目，显然xk=0,1,2,3,4,5,6,7,8；

gk(xk)：把数目为xk钱投资到项目k得到的收益；

fk(xk)：把数目为xk的钱投资得到的最大收益（可以投资的范围为项目k~n）；

状态变量xk: 表示投资的数目；

决策变量uk: 表示投给项目k的投资额；

状态转移方程：f(k)(xk)=max{gk(uk)+f(k+1)(xk-uk)}, uk=0,1,……,xk;

fn(xn)= gn(xn)

项目个数n=3，xk最大取8，问题划分为三个阶段：

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| k=3 | | 只选项目三 | |  | |  | |  | |  | |  |  |  | |  | |
|  | | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | 6 | 7 | | 8 | |
| 1 | | 0 | | 4 | | 26 | | 40 | | 45 | | 50 | 51 | 52 | | 53 | |
| k=2，3 | |  | |  | |  | |  | |  | |  | |  | |  | |  | |
| 下K=2 uk/右k=3 uk | | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | |
| 0 | | **0** | | 4 | | **26** | | **40** | | 45 | | 50 | | 51 | | 52 | | 53 | |
| 1 | | **5** | | 9 | | 31 | | 45 | | 50 | | 55 | | 56 | | 57 | |  | |
| 2 | | 15 | | 19 | | 41 | | 55 | | 60 | | 65 | | 66 | |  | |  | |
| 3 | | **40** | | 44 | | 66 | | 80 | | 85 | | 90 | |  | |  | |  | |
| 4 | | **60** | | 64 | | **86** | | **100** | | 105 | |  | |  | |  | |  | |
| 5 | | **70** | | 74 | | 96 | | **110** | |  | |  | |  | |  | |  | |
| 6 | | 73 | | 77 | | 99 | |  | |  | |  | |  | |  | |  | |
| 7 | | 74 | | 78 | |  | |  | |  | |  | |  | |  | |  | |
| 8 | | 75 | |  | |  | |  | |  | |  | |  | |  | |  | |

这一步实际上就是暴力法枚举了所有情形。

把k=3.k=2的结果斜着看，每条斜线上取最大值即为k=1的右边的uk值、就是在只选2,3项目情况下，uk=0~8的最大情况。因为斜线就是uk=0~8的所有不同组合下计算出来的利润。

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| k=1，2,3 |  |  |  |  |  |  |  |  |  |
| 下k=1的uk/右边2,3下的uk | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|  | 0 | 5 | 26 | 40 | 60 | 70 | 86 | 100 | 110 |
| 1 | 5 |  |  |  |  |  |  | 105 |  |
| 2 | 15 |  |  |  |  |  | 101 |  |  |
| 3 | 40 |  |  |  |  | 110 |  |  |  |
| 4 | 80 |  |  |  | **140** |  |  |  |  |
| 5 | 90 |  |  | 130 |  |  |  |  |  |
| 6 | 95 |  | 121 |  |  |  |  |  |  |
| 7 | 98 | 103 |  |  |  |  |  |  |  |
| 8 | 100 |  |  |  |  |  |  |  |  |

由上表可知：最大获益为140万元，其中对项目1投资4万，项目2投资4万，项目3投资0元。