Integrate the Gaussian $y = e^{-x^2}$

$$A = \int e^{-x^2}\, dx = \int e^{-y^2}\, dy$$

$$A^2 = \iint e^{-(x^2+y^2)} dxdy$$

Swap for polar coordinates

$$x = r\cos\theta\,, y = r\sin\theta$$

Jacobian

	dr	$d\theta$
dx	$\cos\theta$	$-\mathrm{r}\sin\theta$
dy	$\sin\theta$	$r\cos\theta$

Determinant of Jacobian

$$|J| = r\cos^2\theta - (-\mathrm{r}\sin^2\theta) = r(\cos^2\theta + \sin^2\theta) = r$$

$$A^2 = \int_0^{r=\infty}\int_0^{\theta=2\pi} e^{-r^2} r\, drd\theta = 2\pi\int_0^{r=\infty} e^{-r^2} r\, dr$$

Substitute

$$s = -r^2, ds = -2rdr$$

$$A^2 = 2\pi\int_0^{s=-\infty} e^s \frac{ds}{-2} = -\pi\int_0^{s=-\infty} e^s\, ds = -\pi[e^{-\infty} - e^0] = \pi$$

$$\int e^{-x^2}\, dx = \sqrt{\pi}$$