

Report

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1 In Experiment 1, estimate the probability of winning \$80 within 1000 sequential bets. Explain your reasoning.

The probability of winning \$80 is 100% (or to be more accurate, very close to 100%). As shown in Fig.1, all 10 simulations get to \$80 at about 200 bets, long before the end. And also in the Fig.2, all the means is \$80 and standard deviations are 0 after about 200. Because the strategy that double the bet amount after losing one spin will turn it around once hit one win. For the worst case scenario except losing, if keep losing for 999 times, the episode winnings will be $\sum_{i=0}^{998} 2^i$. Then the next bet amount is 2^{999} . If win, the last winning is greater than $0(2^{999} - \sum_{i=0}^{998} 2^i > 0)$. The only case of losing is 1000 sequential losing. The probability of losing 1000 sequential bets is $(1 - \text{win_prob})^{1000} = (1 - 18/38)^{1000} \approx 1.76 \times 10^{-279}$. So the estimate the probability of winning \$80 within 1000 sequential bets is $1 - 1.76 \times 10^{-279} \approx 1$.

2 In Experiment 1, what is the expected value of our winnings after 1000 sequential bets? Explain your reasoning. Go here to learn about expected value: https://en.wikipedia.org/wiki/Expected_value

According to the reason from above, the probability of winning \$80 is $1 - 1.76 \times 10^{-279}$. And the other case is losing $-\sum_{i=0}^{999} 2^i$ with the probability of 1.76×10^{-279} . Therefore the expected value of our winnings after 1000 sequential bets is $80 \times (1 - 1.76 \times 10^{-279}) + (-\sum_{i=0}^{999} 2^i) * (1.76 \times 10^{-279}) \approx 80$.

3 In Experiment 1, does the standard deviation reach a maximum value then converge or stabilize as the number of sequential bets increases? Explain why it does (or does not).

Yes. The standard deviation reach a maximum value and the converage to 0. At the beginning of the simulation, the episode winnings in each simulation is very random. And then almost 100% the winning will become \$80, which means the standard deviation will be 0, as shown in [?].

../ex1SD.png

- 4 In Experiment 2, estimate the probability of winning \$80 within 1000 sequential bets. Explain your reasoning.**

In the experiment 2, I got 640 simulations with winning \$80 out of 1000 simulations. So my estimated the probability of winning \$80 within 1000 sequential bets is 64%.

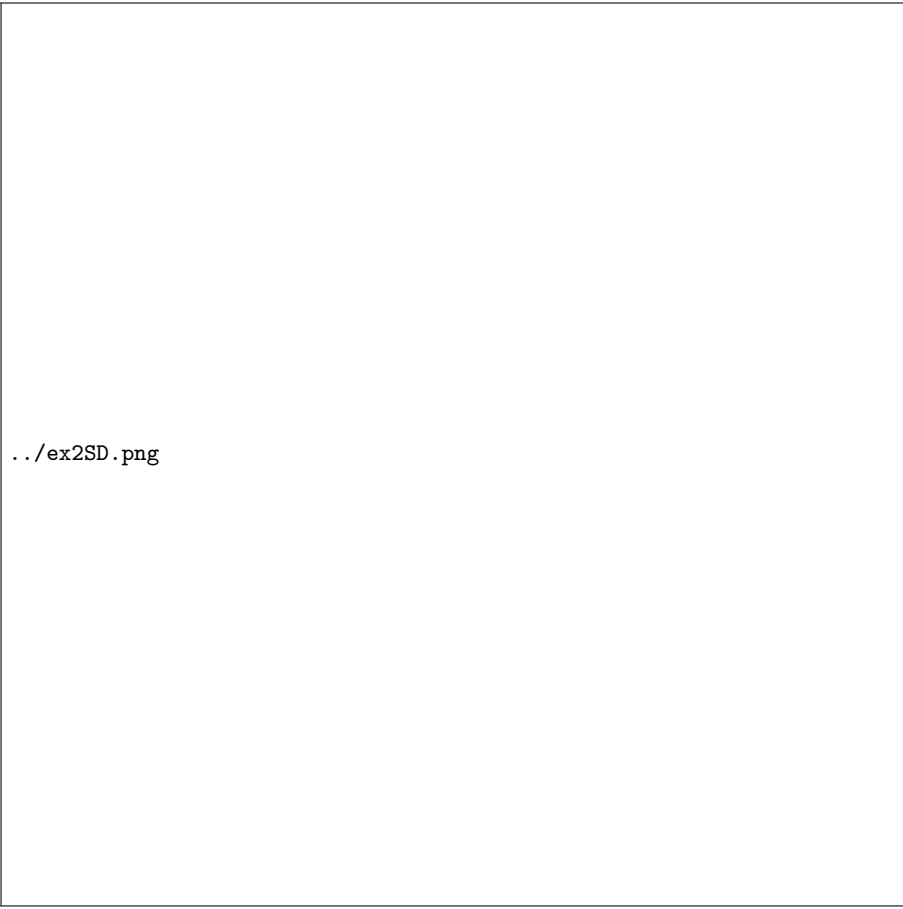
- 5 In Experiment 2, what is the expected value of our winnings after 1000 sequential bets? Explain your reasoning.**

From my experiment 2, I got 640 simulations with \$80 winning and 359 simulations with \$-256 winning and one time with winning \$42. So that the expected

value of our winnings after 1000 sequential bets can be $0.64 \times 80 + 0.359 \times (-256) + 0.001 \times 42 = -40.662$

6 In Experiment 2, does the standard deviation reach a maximum value then converge or stabilize as the number of sequential bets increases? Explain why it does (or does not).

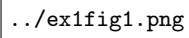
No. Unlike the case in experiment 1, the standard deviation has no obvious maximum. The standard deviation keeps increasing and converges to a value close to \$160.



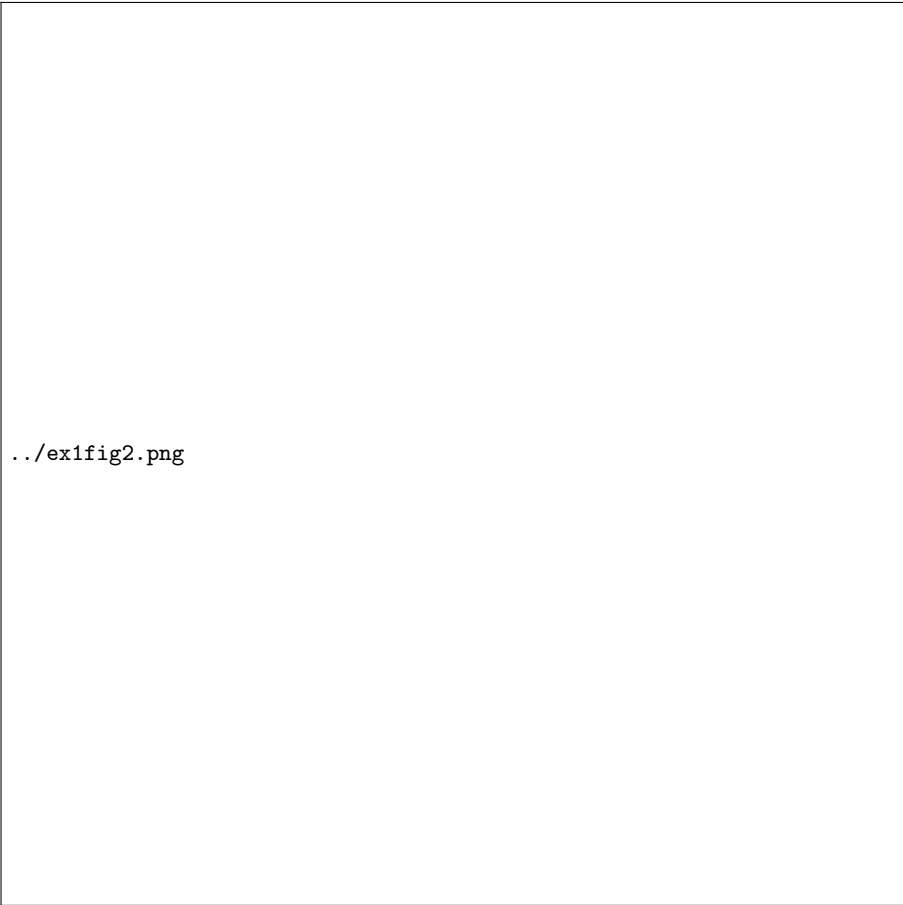
../ex2SD.png

7 Include figures 1 through 5.

The figures 1 through 5 are attach as Fig.3 to 7 as below.



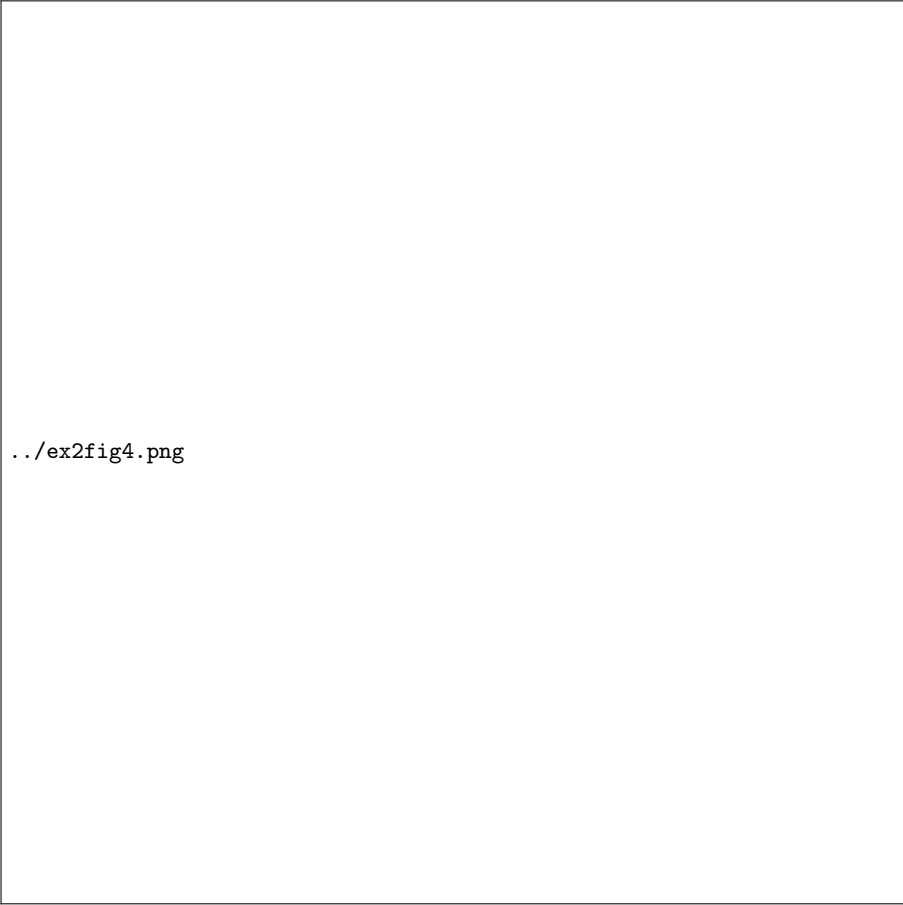
`../ex1fig1.png`



`../ex1fig2.png`

More info about this, go to: <http://quantsoftware.gatech.edu/Martingale>

../ex1fig3.png



../ex2fig4.png

../ex2fig5.png