Week 9 Crane Lab (3D Velocity Kinematics)

Written by: Caiden Bonney

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Modified: 3/12/25

Background:

The motion of rigid bodies in 3D space involves rotations and extensions, which can be complex due to their interactions. Using rotating frames of reference simplifies velocity analysis by breaking motion into manageable components. In this lab, we apply 3D kinematics to study a crane's motion, focusing on velocity equations and rotational transformations.

Required Files:

- ME326_05_Activity6_BonneyCaiden_MatLabFile
- Bar.stl

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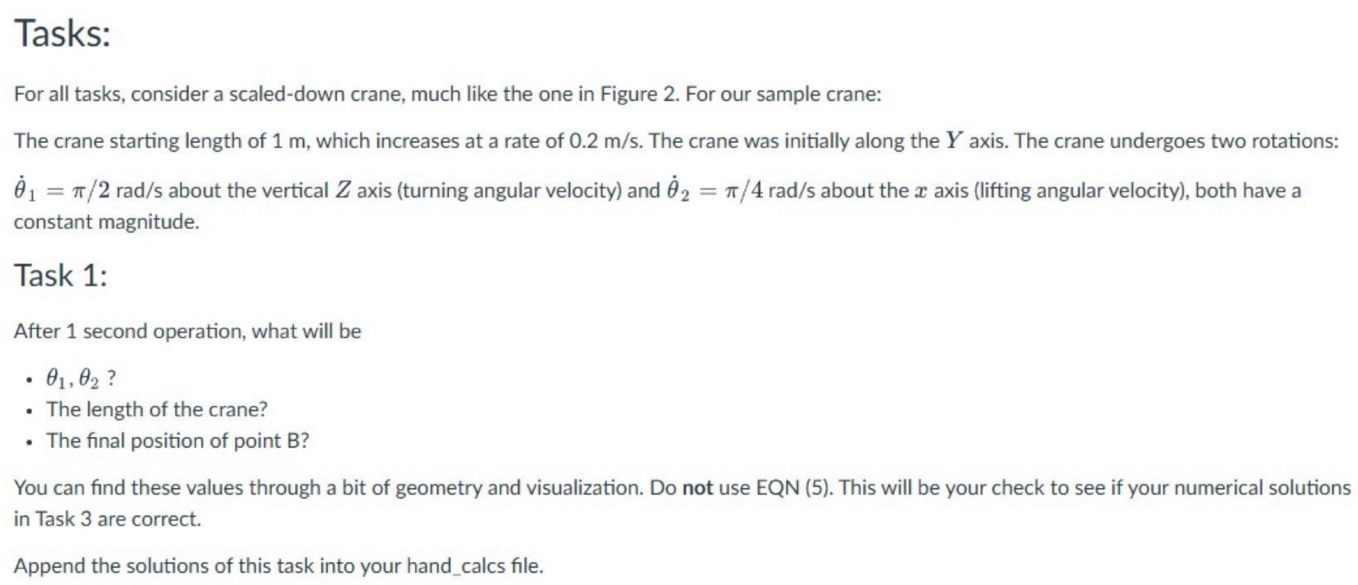
Task 1:



Tasks:

For all tasks, consider a scaled-down crane, much like the one in Figure 2. For our sample crane:

The crane starting length of 1 m, which increases at a rate of 0.2 m/s. The crane was initially along the Y axis. The crane undergoes two rotations:

$\dot{\theta}_1 = \pi/2$ rad/s about the vertical Z axis (turning angular velocity) and $\dot{\theta}_2 = \pi/4$ rad/s about the x axis (lifting angular velocity), both have a constant magnitude.

Task 1:

After 1 second operation, what will be

- θ_1, θ_2 ?
- The length of the crane?
- The final position of point B?

You can find these values through a bit of geometry and visualization. Do **not** use EQN (5). This will be your check to see if your numerical solutions in Task 3 are correct.

Append the solutions of this task into your hand_calcs file.

$r = 1\,\text{m} \qquad \dot{r} = 0.2\,\text{m/s}$

$\theta_1 = 0\,\text{rad} \qquad \dot{\vec{\theta}}_1 = \omega_1 = \frac{\pi}{2}\hat{K}\ \text{rad/s}$

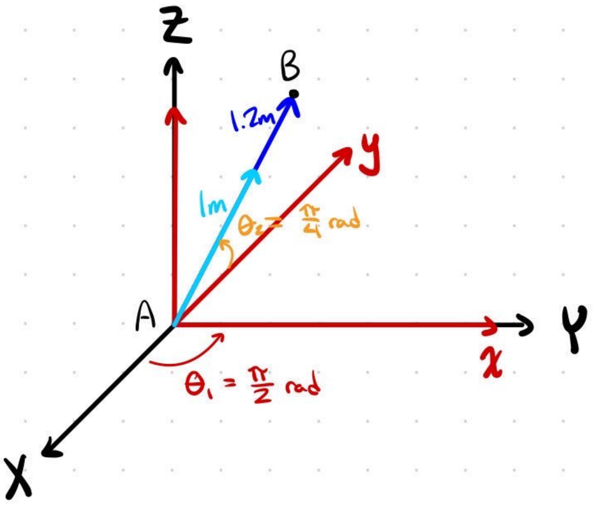
$\theta_2 = 0\,\text{rad} \qquad \dot{\vec{\theta}}_2 = \omega_2 = \frac{\pi}{4}\hat{\imath}\ \text{rad/s}$

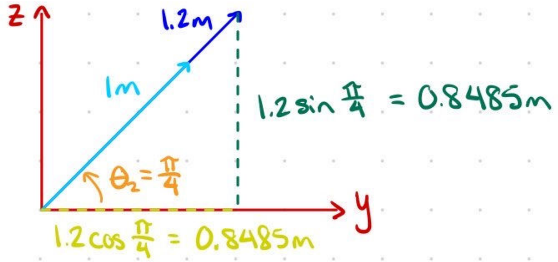
After 1 second of operation

$\theta_1 = 0 + \omega_1 \cdot 1\text{s} = \frac{\pi}{2}\ \text{rad}$

$\theta_2 = 0 + \omega_2 \cdot 1\text{s} = \frac{\pi}{4}\ \text{rad}$

$r = 1 + \dot{r} \cdot 1 = 1.2\,\text{m}$

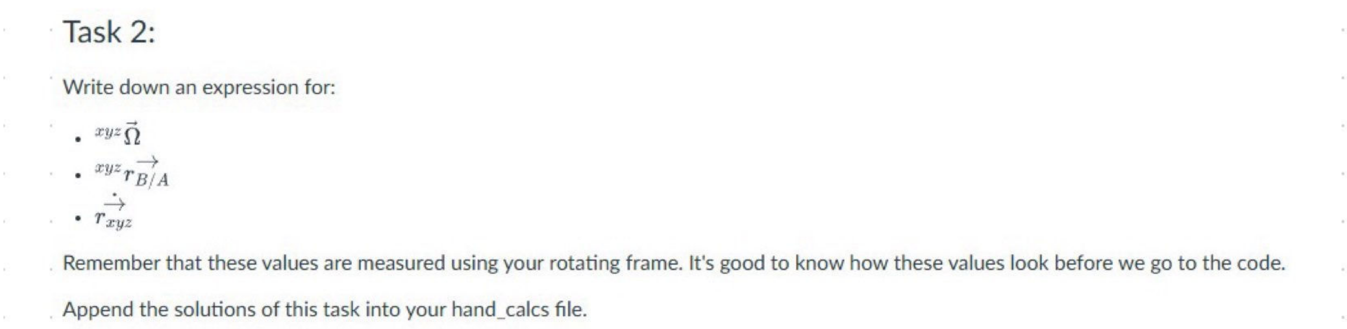

Z
B
1.2m
y
1m
$\theta_2 = \frac{\pi}{4}$ rad
A
Y
x
$\theta_1 = \frac{\pi}{2}$ rad
X


z
1.2m
1m
$1.2 \sin\frac{\pi}{4} = 0.8485\text{m}$
$\theta_2 = \frac{\pi}{4}$
y
$1.2 \cos\frac{\pi}{4} = 0.8485\text{m}$

${}^{xyz}\vec{r}_{B/A} = 0.8485\hat{\jmath} + 0.8485\hat{k}$

${}^{XYZ}\vec{r}_{B/A} = -0.8485\hat{I} + 0.8485\hat{K}$

Task 2:



Task 2:

Write down an expression for:

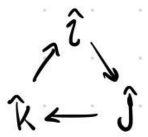
- ${}^{xyz}\vec{\Omega}$
- ${}^{xyz}\overrightarrow{r_{B/A}}$
- $\dot{\overrightarrow{r_{xyz}}}$

Remember that these values are measured using your rotating frame. It's good to know how these values look before we go to the code.

Append the solutions of this task into your hand_calcs file.

$${}^{xyz}\vec{\Omega} = \dot{\theta}_1 = \omega_1 \hat{k} \ \frac{rad}{s}$$

$${}^{xyz}\vec{r}_{B/A} = \left(r_{mag} + (\dot{r}\,\Delta t)\right)\left(\cos\theta_2\,\hat{j} + \sin\theta_2\,\hat{k}\right)$$



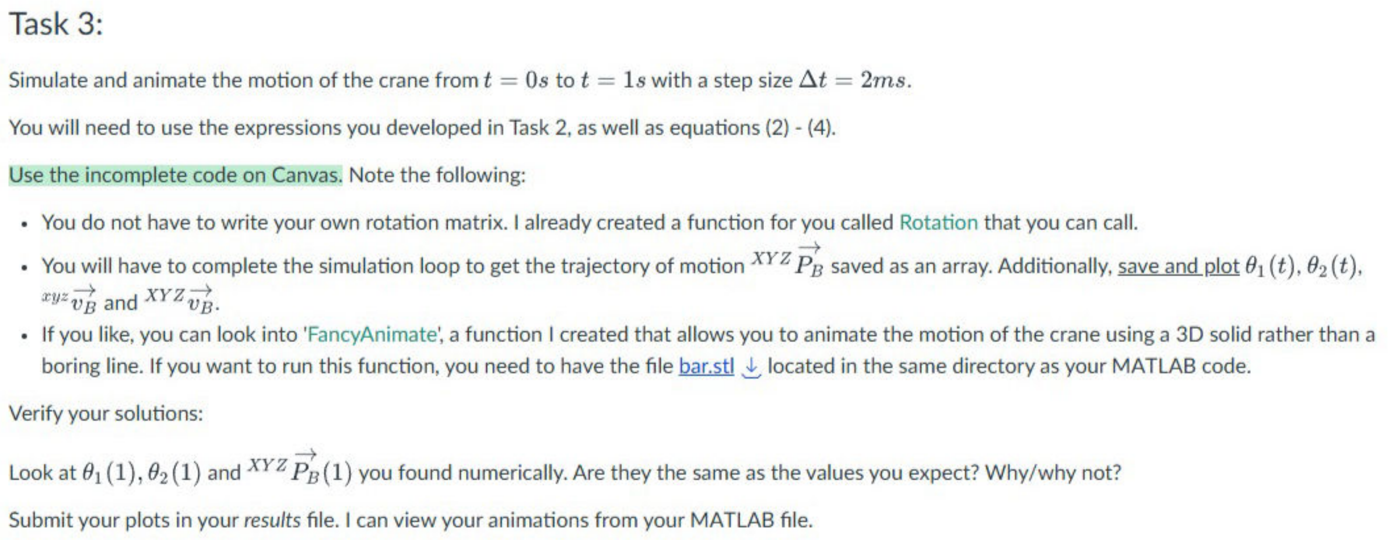
$\hat{j} \times \hat{j} = 0$

$${}^{xyz}\dot{\vec{r}}_{xyz} = \dot{r} \cdot {}^{xyz}\vec{r}_{A/B} + {}^{xyz}\vec{\omega}_2 \times {}^{xyz}\vec{r}_{B/A}$$

$$= \dot{r}\cos\theta_2\,\hat{j} + \dot{r}\sin\theta_2\,\hat{k} + \dot{\theta}_2\hat{j} \times \left(r_{mag} + (\dot{r}\,\Delta t)\right)\left(\cancelto{0}{\cos\theta_2\,\hat{j}} + \sin\theta_2\,\hat{k}\right)$$

$${}^{xyz}\dot{\vec{r}}_{xyz} = \dot{r}\cos\theta_2\,\hat{j} + \dot{r}\sin\theta_2\,\hat{k} + \dot{\theta}_2\left(r_{mag} + (\dot{r}\,\Delta t)\right)\sin\theta_2\,\hat{i}$$

Task 3:



Task 3:

Simulate and animate the motion of the crane from $t = 0s$ to $t = 1s$ with a step size $\Delta t = 2ms$.

You will need to use the expressions you developed in Task 2, as well as equations (2) - (4).

Use the incomplete code on Canvas. Note the following:

- You do not have to write your own rotation matrix. I already created a function for you called Rotation that you can call.
- You will have to complete the simulation loop to get the trajectory of motion ${}^{XYZ}\overrightarrow{P_B}$ saved as an array. Additionally, save and plot $\theta_1(t)$, $\theta_2(t)$, ${}^{xyz}\overrightarrow{v_B}$ and ${}^{XYZ}\overrightarrow{v_B}$.
- If you like, you can look into 'FancyAnimate', a function I created that allows you to animate the motion of the crane using a 3D solid rather than a boring line. If you want to run this function, you need to have the file bar.stl ↓ located in the same directory as your MATLAB code.

Verify your solutions:

Look at $\theta_1(1)$, $\theta_2(1)$ and ${}^{XYZ}\overrightarrow{P_B}(1)$ you found numerically. Are they the same as the values you expect? Why/why not?

Submit your plots in your *results* file. I can view your animations from your MATLAB file.

The goal of this code is to simulate and animate the motion of a scaled-down crane. The crane can undergo 2 rotations (that intersect at A) and the length of the crane changes as well.

```
% sample code (3D velocity kinematics)
% Note: here, I set up a rotating frame that rotates at w1 (about Z)
clear
clc

dt = 0.002; % simulation step size
t = 0:dt:1; % time vector

w1 = pi/2; % [rad/s] angular velocity magnitude about the Z axis (primary)
w2 = pi/4; % [rad/s] angular velocity magnitude about the x axis (secondary)
r_dot = 0.2; % [m/s] magnitude of velocity of crane extension outwards
w1_vec = [0; 0; w1]; % [rad/s] angular velocity theta1_dot as a vector
xyzw2_vec = [w2; 0; 0]; % [rad/s] angular velocity theta2_dot as a vector

theta1(1) = 0; % [rad] initial angle
theta2(1) = 0; % [rad] initial angle

r_mag(1) = 1; % [m] length of the crane initially

XYZr(:,1) = r_mag*[0;1;0]; % [m] Initially, the position vector r_B/A is
% along the +Y direction
xyzr(:,1) = XYZr(:,1); % Initially the rotationald and global coordinate
% systems align

% Simulation loop:
for i=1:length(t)

    R_theta2 = Rotation('x',theta2(i)); % represents the rotation due to
    % theta2 when left-multiplied to a vector
    XYZR_xyz = Rotation('z',theta1(i)); % also could be called R_theta1
    % because it represents the rotation due to theta1 when left-multiplied
    % to a vector

    xyzr(:,i) = (r_mag(i)*R_theta2*[0;1;0]); % extension position vector of
    % the end of the arm expressed in local xyz frame. (same as the
    % position of B in xyz frame)

    xyzrdot_xyz(:,i) = r_dot*xyzr(:,i) + cross(xyzw2_vec, xyzr(:,i));
    % velocity of the end of the rod due to theta2 in the xyz coordinate
    % system

    xyzv(:,i) = xyzrdot_xyz(:,i);
    % velocity of the end of the rod in the xyz coordinate system

    XYZr(:,i) = XYZR_xyz*xyzr(:,i); % rotating the xyz coordinate
    % system rod position to the XYZ coordinates

    XYZv(:,i) = XYZR_xyz*xyzv(:,i); % rotating the xyz coordinate
    % system rod velocity to the XYZ coordinates

    % Find the next angle
    theta1(i+1) = theta1(i) + w1*dt;
    theta2(i+1) = theta2(i) + w2*dt;

    r_mag(i+1) = r_mag(i) + r_dot*dt; % calculated the length of the arm
    % over time
end
```

```
% Animate the motion (using lines)
figure(1)
animate(XYZr, theta1);
```

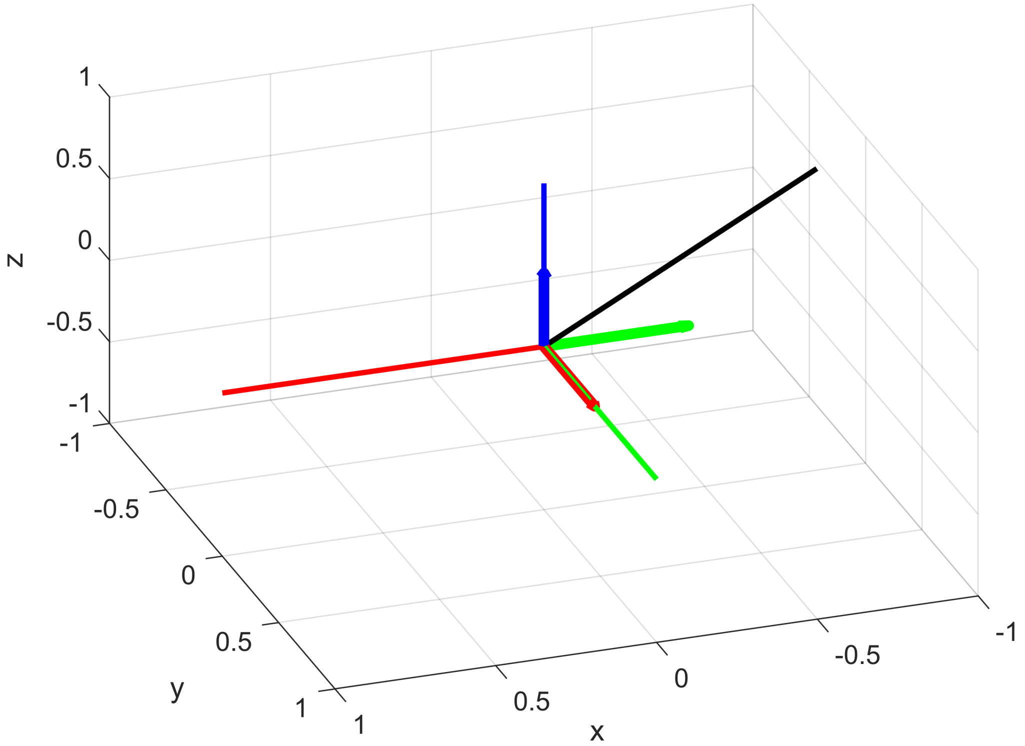


Figure 1 depicts the animated crane as it rotates about the Z axis whilst extending the beam and rotating the beam up in the j axis.

```
% Fancier animation using some STL files to represent the structure
% rather than lines
figure(2)
FancyAnimate(XYZr, theta1,theta2)
```

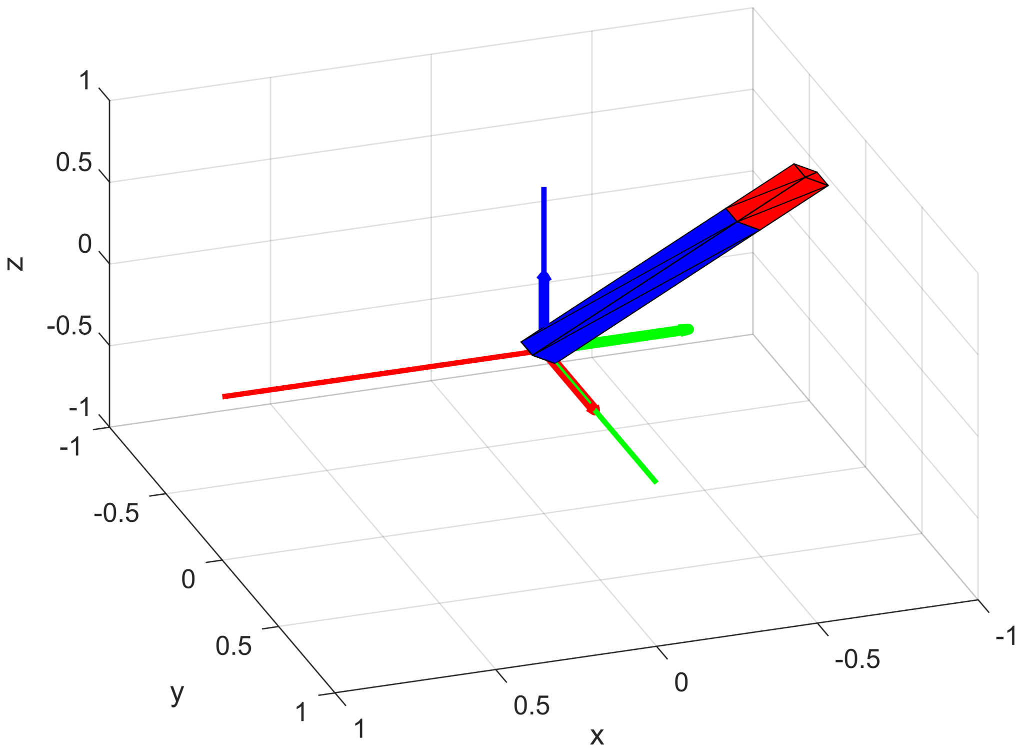


Figure 2 depicts the Fancy Animate version of the previously described Figure 1.

```
figure(3)
plot(t, theta1(1:end-1))
title('Theta1')
xlabel('Time [s]')
ylabel('Theta1 [rad]')
```

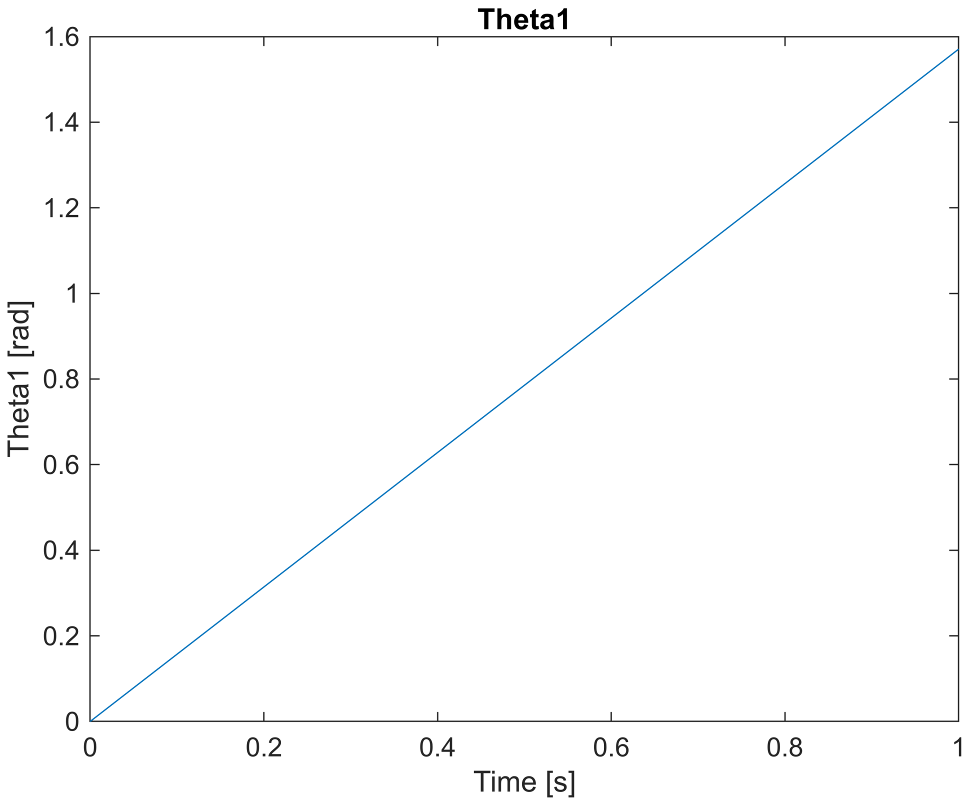


Figure 3 depicts the angle theta1 in radians over the time period at which the simulation is run. Theta1 changes at a constant positive rate since w1 is constant and positive.

```
figure(4)
plot(t, theta2(1:end-1))
title('Theta1')
xlabel('Time [s]')
ylabel('Theta2 [rad]')
```

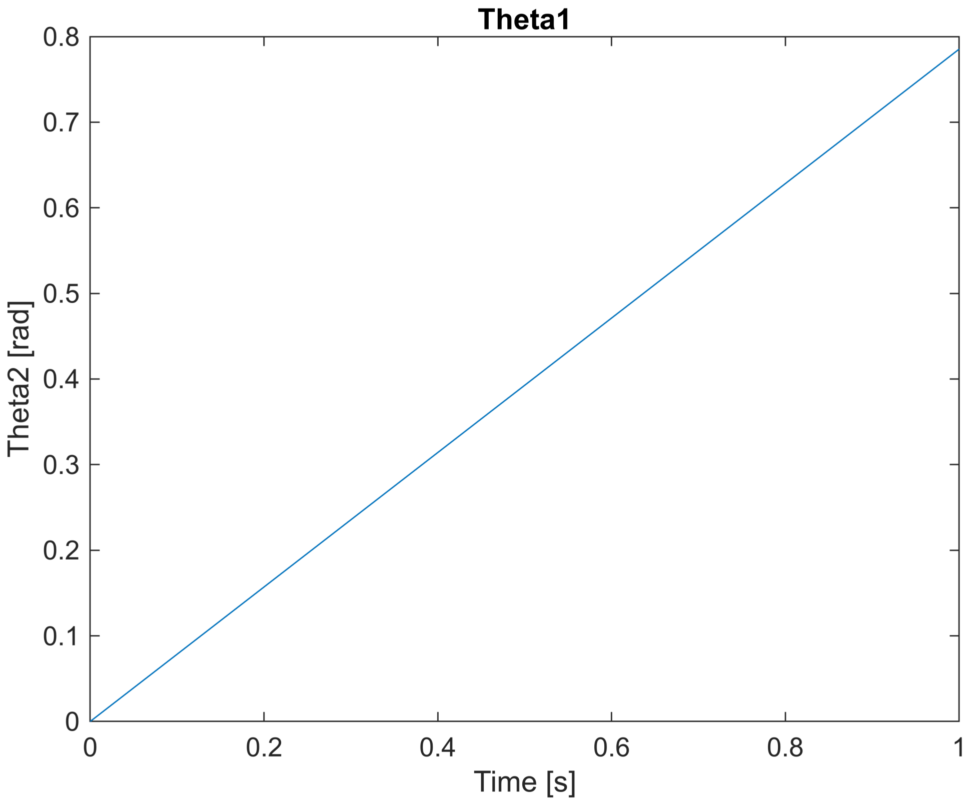


Figure 4 depicts the angle theta2 in radians over the time period at which the simulation is run. Theta2 changes at a constant positive rate since w2 is constant and positive.

```
figure(5)
plot(t, xyzv(1,:), 'DisplayName', 'x') % Adding labels for legend
hold on
plot(t, xyzv(2,:), 'DisplayName', 'y') % Adding labels for legend
plot(t, xyzv(3,:), 'DisplayName', 'z') % Adding labels for legend
title('Velocity (xyz Coordinates)')
xlabel('Time [s]')
ylabel('Velocity [m/s]')
hold off
legend('Location','best');
```

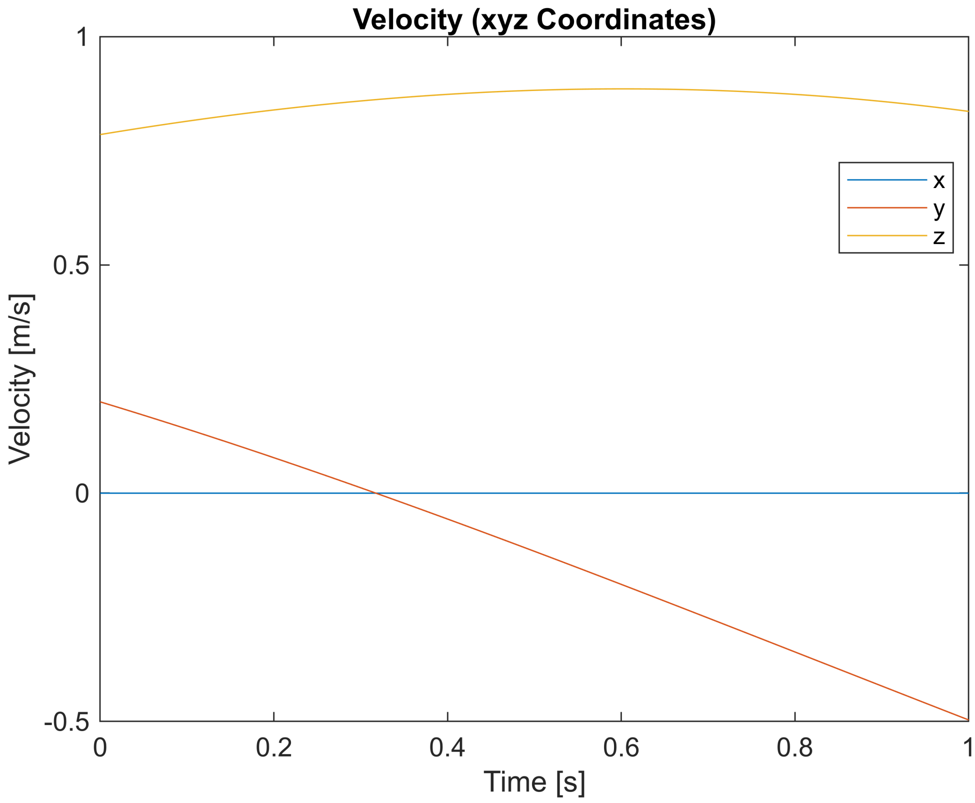


Figure 5 depicts the velocity displayed in the xyz coordinates, this does not account for the rotation due to w1 since the coordinate system itself is rotating. This is why the velocity in the x direction is zero throughout the course of the simulation.

```
figure(6)
plot(t, XYZv(1,:), 'DisplayName', 'X') % Adding labels for legend
hold on
plot(t, XYZv(2,:), 'DisplayName', 'Y') % Adding labels for legend
plot(t, XYZv(3,:), 'DisplayName', 'Z') % Adding labels for legend
title('Velocity (XYZ Coordinates)')
xlabel('Time [s]')
ylabel('Velocity [m/s]')
hold off
legend('Location','best');
```

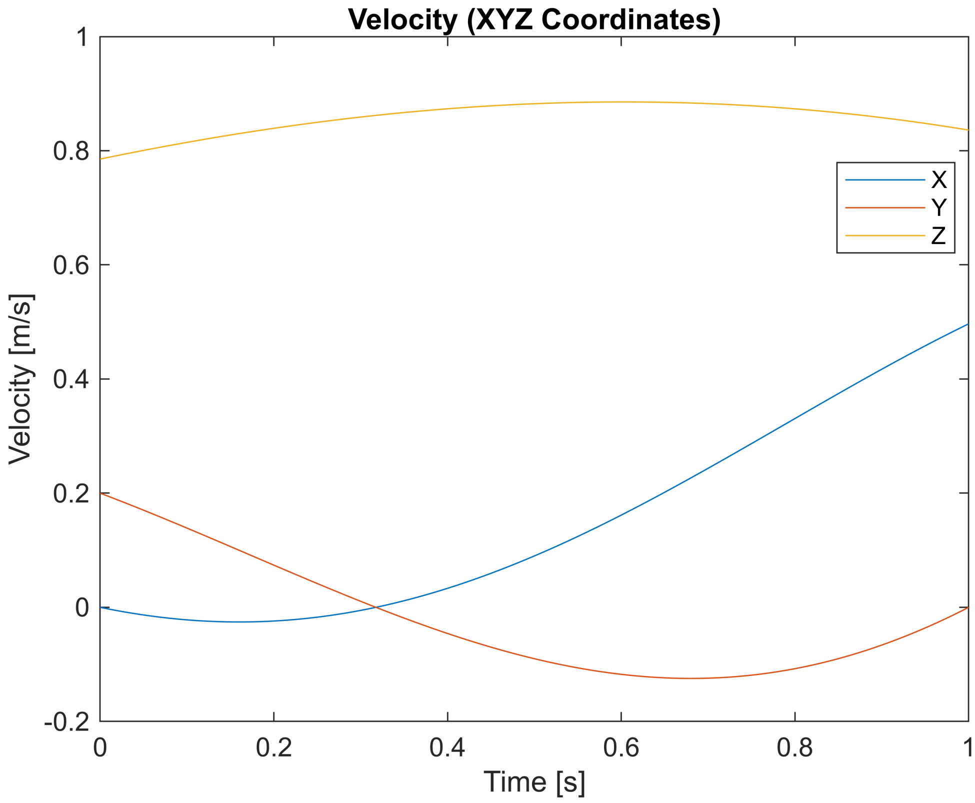


Figure 6 depicts velocity displayed in the XYZ coordinates over the course of the simulation.

```
theta1(length(t))
```

```
ans =
1.5708
```

The expected value found in the Task 1 is:

Theta1 = pi/2 = 1.57079

It can be noted that the values do match the precalculated values after rounding to the amount of digits given by matlab.

```
theta2(length(t))
```

```
ans =
0.7854
```

The expected value found in the Task 1 is:

Theta2 = pi/4 = 0.78539

It can be noted that the values do match the precalculated values after rounding to the amount of digits given by matlab.

```
xyzr(:,length(t))
```

```
ans = 3×1
```

```
     0
0.8485
0.8485
```

The expected value found in the Task 1 is:

xyzr = 0 i_hat + 0.8485 j_hat + 0.8485 k_hat

It can be noted that the values do match the precalculated values after rounding to the amount of digits given by matlab.

```
XYZr(:,length(t))
```

```
ans = 3×1
   -0.8485
   -0.0000
    0.8485
```

The expected value found in the Task 1 is:

xyzr = -0.8485 I_hat + 0 J_hat + 0.8485 K_hat

It can be noted that the values do match the precalculated values after rounding to the amount of digits given by matlab.

(Note there are especially rounding errors for the second term as the value is -9.36859738872830e-15 which is equivalent to 0 in this scope of this simulation).

Functions

Below are some functions I used for convenience:

Rotation: Produces a rotation matrix about a chosen axis and a chosen angle.

Inputs:

- axis: a string that specifies the axis of rotation (string).
- theta: the angle of rotation about the specified axis (in rad).

Animate: An animation loop that produces the line plots.

Inputs:

- Trajectory :a 3xn matrix representing the XYZ coordinates of the end of the arm.
- theta1: a 1xn matrix representing the angle theta1 as a function of time. (used to find the orientation of the rotating frame at each instance in time).

FancyAnimate: Produces a fancier animation using a 3D solid rather than lines.

Note that FancyAnimate requires 'bar.stl' to be in the same directory as this matlab file.

Inputs:

- Trajectory :a 3xn matrix representing the XYZ coordinates of the end of the arm.

- theta1: a 1xn vector representing the angle theta1 as a function of time. (used to find the orientation of the rotating frame at each instance in time).
- theta2: a 1xn vector representing the angle theta2 as a function of time.

```
function R = Rotation(axis, theta)
    if axis == 'x' %your rotation is about the x axis
        R = [1 0 0;
             0 cos(theta) -sin(theta);
             0 sin(theta) cos(theta)];
    elseif axis == 'y' %your rotation is about the y axis
        R = [cos(theta) 0 sin(theta);
             0 1 0;
             -sin(theta) 0 cos(theta)];
    else %your rotation is about the z axis
        R = [cos(theta) -sin(theta) 0;
             sin(theta) cos(theta) 0;
             0 0 1];
    end
end

function animate(Trajectory, theta1)
    %here, trajectory is a 3xn vector
    %each row is the [X,Y,Z] position of the end of the crane
    for i=1:length(Trajectory(1,:))
        figure(1)
        plot3([0;1],[0;0],[0;0],'r',LineWidth=2)
        hold on
        plot3([0;0],[0;1],[0;0],'g',LineWidth=2)
        plot3([0;0],[0;0],[0;1],'b',LineWidth=2)
        XYZ_R_xyz = Rotation('z',theta1(i));
        %Find an expression for the x, y, z axes at this instance
        x = XYZ_R_xyz*[0.5;0;0];
        y = XYZ_R_xyz*[0;0.5;0];
        z = XYZ_R_xyz*[0;0;0.5];

        %plot the x y z axes using quiver:
        quiver3(0, 0, 0,  x(1), x(2), x(3), 'Color', 'red', 'LineWidth', 4);
        quiver3(0, 0, 0,  y(1), y(2), y(3), 'Color', 'green', 'LineWidth', 4);
        quiver3(0, 0, 0,  z(1), z(2), z(3), 'Color', 'blue', 'LineWidth', 4);

        %plot the crane as a line
        plot3([0;Trajectory(1,i)],[0;Trajectory(2,i)],
[0,Trajectory(3,i)],'k',LineWidth=2)
        hold off
        xlabel 'x'
        ylabel 'y'
        zlabel 'z'

        view([160.69 40.76]);
        grid
        xlim ([-1,1])
        ylim ([-1,1])
        zlim ([-1,1])
        pause(0.01)
    end
end

function FancyAnimate(Trajectory, theta1,theta2)
    %here, trajectory is a 3xn vector
    %each row is the [X,Y,Z] position of the end of the crane

    Bar = stlread('bar.stl');
    BarPoints1 = [0.001,0.001,0.001].*Bar.Points;
    BarPoints1Starting = (Rotation('z',pi/2)*BarPoints1')';
    BarConnectivity1 = Bar.ConnectivityList;
    BarPoints2 = [0.001/9,0.001,0.001].*Bar.Points+[0.9,0,0];
    BarPoints2Starting = (Rotation('z',pi/2)*BarPoints2')';
    BarConnectivity2 = Bar.ConnectivityList;

    for i=1:length(Trajectory(1,:))
        figure(2)
        plot3([0;1],[0;0],[0;0],'r',LineWidth=2)
        hold on
        plot3([0;0],[0;1],[0;0],'g',LineWidth=2)
        plot3([0;0],[0;0],[0;1],'b',LineWidth=2)
        XYZ_R_xyz = Rotation('z',theta1(i));
        x = XYZ_R_xyz*[0.5;0;0];
        y = XYZ_R_xyz*[0;0.5;0];
        z = XYZ_R_xyz*[0;0;0.5];
        quiver3(0, 0, 0,  x(1), x(2), x(3), 'Color', 'red', 'LineWidth', 4);
        quiver3(0, 0, 0,  y(1), y(2), y(3), 'Color', 'green', 'LineWidth', 4);
        quiver3(0, 0, 0,  z(1), z(2), z(3), 'Color', 'blue', 'LineWidth', 4);

        R = Rotation('z',theta1(i))*Rotation('x',theta2(i));
        BarPoints1 = (R*BarPoints1Starting')';
        BarPoints2 = [0.001/9*(norm(Trajectory(:,i))-0.9)/
0.1,0.001,0.001].*Bar.Points+[0.9,0,0];
        BarPoints2Starting = (Rotation('z',pi/2)*BarPoints2')';
        BarPoints2 = (R*BarPoints2Starting')';

trimesh(BarConnectivity1,BarPoints1(:,1),BarPoints1(:,2),BarPoints1(:,3),'edgecolor'
,'k','facecolor','b')

trimesh(BarConnectivity2,BarPoints2(:,1),BarPoints2(:,2),BarPoints2(:,3),'edgecolor'
,'k','facecolor','r')

        hold off
        xlabel 'x'
        ylabel 'y'
        zlabel 'z'
        %view([150,30]);
        view([160.69 40.76]);
        grid
        xlim ([-1,1])
        ylim ([-1,1])
        zlim ([-1,1])
        pause(0.1)
    end
end
```