Appendix E

The Vector-Loop

Vector-loop analysis can be applied to any planar mechanism, however the following example will focus on the crank-slider with a rotating collar shown below in Figure E.1. As the crank rotates at constant velocity ω, the slider reciprocates within the collar.

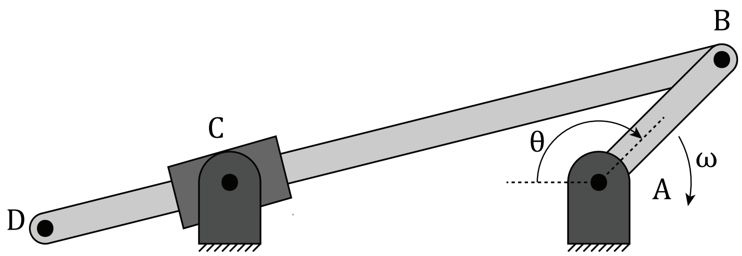

Figure E.1: Link $\overline{\mathrm{BD}}$ slides through the pivoted collar at C, driven by crank $\overline{\mathrm{AB}}$ rotating clockwise at constant angular velocity ω.

E.1 Forming the vector-loop

The vector-loop is formed by representing each link in the mechanism with a vector $\boldsymbol{r}_n$. A valid loop is formed if a set of vectors $\boldsymbol{r}_1 ... \boldsymbol{r}_n$, each representing a link in the mechanism, forms a *closed* loop.

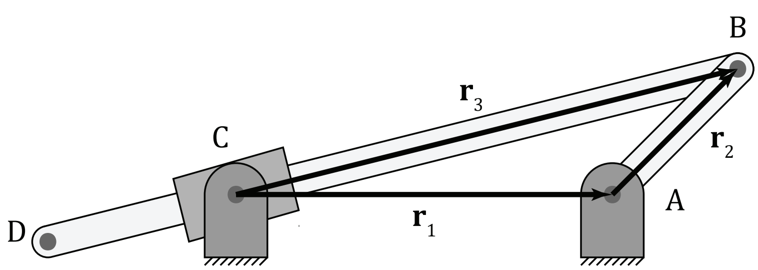

Figure E.2: Vectors $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, and $\boldsymbol{r}_3$ are overlaid on the crank-slider mechanism to form a closed vector-loop.

For the crank-slider mechanism shown in Figure E.2:

$$\boldsymbol{r}_1 + \boldsymbol{r}_2 = \boldsymbol{r}_3 \tag{E.1}$$

This equation is the fundamental relationship that will be used to derive the kinematic relationships for the crank-slider mechanism. Note that different signs can be assumed for the vectors representing each link, and that the resulting equation can be written in several equivalent forms[1]; however, the kinematic relationships derived will be the same.

A vector can be thought of as a directed line segment, implying the notion of length and direction. Figure E.3 shows the vector loop freed from the mechanism in order to clearly illustrate the length and direction of each vector $\boldsymbol{r}_1$ through $\boldsymbol{r}_3$.

Note: Even though the crank angle is defined clockwise in the original problem as θ, a new angle θ_2 must be defined. This is to enforce the convention that all angles are defined with respect to the positive extension of the x-axis.

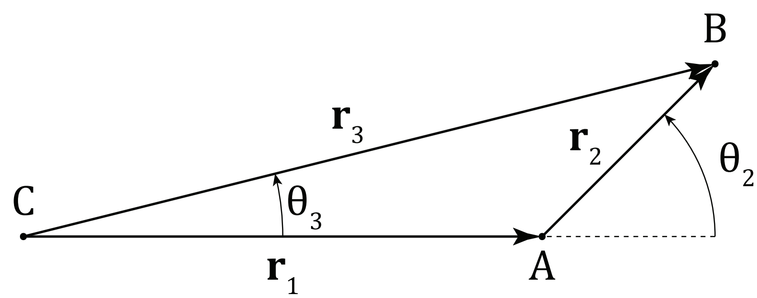

Figure E.3: Vectors $\boldsymbol{r}_2$ and $\boldsymbol{r}_3$ both change angle and vector $\boldsymbol{r}_3$ changes length.

Equation E.1 can be rewritten using the idea of length and direction to help bring further insight into the vector loop. Analysis of each vector in the loop shows which variables will be unknown, which will be known, and which will be inputs to the system.

$$\boldsymbol{r}_1(\overset{\checkmark}{r_1}, \overset{\checkmark}{\theta_1}) + \boldsymbol{r}_2(\overset{\checkmark}{r_2}, \overset{I}{\theta_2}) = \boldsymbol{r}_3(\overset{?}{r_3}, \overset{?}{\theta_3}) \tag{E.2}$$

Here, a check mark (✓) indicates a known variable, a question mark (?) indicates an unknown variable, and the input is indicated by an (I). Both the length and angle of $\boldsymbol{r}_1$ are constant. The length of $\boldsymbol{r}_2$ is constant but its angle is changing in a known manner so it is indicated as an input. Neither the length nor the angle of $\boldsymbol{r}_3$ is known, so both the length, r_3, and angle, θ_3, will be considered unknown.

[1]For the vector signs assumed in Figure , the following equations are several of the equivalent ways of writing the corresponding vector loop equation: $\boldsymbol{r}_1 + \boldsymbol{r}_2 = \boldsymbol{r}_3$; $\boldsymbol{r}_1 + \boldsymbol{r}_2 - \boldsymbol{r}_3 = 0$; $\boldsymbol{r}_1 = \boldsymbol{r}_3 - \boldsymbol{r}_2$; $\boldsymbol{r}_2 = \boldsymbol{r}_3 - \boldsymbol{r}_1$.

The two lower order terms will be grouped together to compose the state vector $\boldsymbol{x}$. For properly constrained mechanisms[2] the number of lower order unknowns should always be two, satisfied by the two vector loop component equations.

$$\boldsymbol{x} = \begin{bmatrix} r_3 \\ \theta_3 \end{bmatrix}$$

Similarly, the input to the system along with its derivatives will be grouped together to compose the input vector $\boldsymbol{u}$.

$$\boldsymbol{u} = \begin{bmatrix} \theta_2 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \end{bmatrix}$$

Note: It is critical to properly identify which variables are part of the state vector, which are constants, and which are inputs to the system in order to properly simulate the behavior of the mechanism.

E.2 Formulating differential equations

In order to find a differential equation, the system first needs to be split into horizontal and vertical components. If a Cartesian coordinate system is overlaid at the origin of each link, then the length and direction of each associated vector give

$$x_n = r_n \cos\theta_n \text{ and } y_n = r_n \sin\theta_n$$

where r_n is the length of the vector and θ_n is the angle measured CCW between the positive extension of the x-axis and the vector. For simplicity and compactness of notation, a shorthand can be defined as

$$c_n = \cos\theta_n \text{ and } s_n = \sin\theta_n$$

so that

$$x_n = r_n c_n \text{ and } y_n = r_n s_n$$

Again consider Figure E.3, which depicts the vectors $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, and $\boldsymbol{r}_3$ forming a valid vector loop. To derive the kinematic equations from the vector loop, the x-components and y-components of the vector-loop equation must be considered separately.

[2] That is, for properly constrained *single degree of freedom* mechanisms, the number of unknowns should be two.

Note: Getting the signs correct in this process is facilitated by distorting the vector loop so that each vector is in the first quadrant[3]. Of course any distortion of the mechanism must preserve the relationship $\boldsymbol{r}_1 + \boldsymbol{r}_2 = \boldsymbol{r}_3$.

Crank-slider Position Equations

Horizontal Components:

$$r_1 + r_2 c_2 = r_3 c_3 \tag{E.3}$$

Vertical Components:

$$r_2 s_2 = r_3 s_3 \tag{E.4}$$

Differentiation of equations E.3 and E.4 with respect to time[4] yields two coupled first order differential equations. The two coupled equations can be put into a single matrix equation of the form $A\dot{\boldsymbol{x}} = \boldsymbol{b}$, where $A = A(\boldsymbol{x})$ is a matrix that is a function of lower order terms and $\boldsymbol{b} = \boldsymbol{b}(\boldsymbol{x}, \boldsymbol{u})$ is a vector that is a function of the lower order terms and the input to the system.

[3] The first quadrant is helpful in determining signedness because both the x- and y-components are positive.

[4] Note on differentiation of implicit functions: c_n and s_n are functions of θ_n, which is a function of time. This functional relationship makes c_n and s_n implicit functions of time, and the chain rule must be utilized, e.g., $\frac{dc_n}{dt} = \frac{dc_n}{d\theta_n}\frac{d\theta_n}{dt}$. Therefore,

$$\dot{c}_2 = -\dot{\theta}_2 s_2 \qquad \dot{s}_2 = \dot{\theta}_2 c_2$$
$$\dot{c}_3 = -\dot{\theta}_3 s_3 \qquad \dot{s}_3 = \dot{\theta}_3 c_3$$

Crank-slider Velocity Equations

Horizontal Components:

$$\begin{aligned}
\frac{d}{dt}\big(r_1 + r_2c_2 &= r_3c_3\big) \\
\dot{r}_1 + \dot{r}_2c_2 + r_2\dot{c}_2 &= \dot{r}_3c_3 + r_3\dot{c}_3 \\
0 + 0 - r_2s_2\dot{\theta}_2 &= c_3\dot{r}_3 - r_3s_3\dot{\theta}_3 \\
c_3\dot{r}_3 - r_3s_3\dot{\theta}_3 &= -r_2s_2\dot{\theta}_2
\end{aligned} \tag{E.5}$$

Vertical Components:

$$\begin{aligned}
\frac{d}{dt}\big(r_2s_2 &= r_3s_3\big) \\
\dot{r}_2s_2 + r_2\dot{s}_2 &= \dot{r}_3s_3 + r_3\dot{s}_3 \\
0 + r_2c_2\dot{\theta}_2 &= s_3\dot{r}_3 + r_3c_3\dot{\theta}_3 \\
s_3\dot{r}_3 + r_3c_3\dot{\theta}_3 &= r_2c_2\dot{\theta}_2
\end{aligned} \tag{E.6}$$

Matrix Form:

$$\overset{A}{\begin{bmatrix} c_3 & -r_3s_3 \\ s_3 & r_3c_3 \end{bmatrix}} \overset{\dot{\boldsymbol{x}}}{\begin{bmatrix} \dot{r}_3 \\ \dot{\theta}_3 \end{bmatrix}} = \overset{\boldsymbol{b}}{\begin{bmatrix} -r_2s_2\dot{\theta}_2 \\ r_2c_2\dot{\theta}_2 \end{bmatrix}} \begin{array}{l} (From\ Eq.\ E.5) \\ (From\ Eq.\ E.6) \end{array} \tag{E.7}$$

Now Equation E.7 can be solved to find the velocities from the lower order terms and the input.

$$\dot{\boldsymbol{x}} = A(\boldsymbol{x})^{-1}\boldsymbol{b}(\boldsymbol{x}, \boldsymbol{u}) \tag{E.8}$$

Or more simply,

$$\dot{\boldsymbol{x}} = A^{-1}\boldsymbol{b} \tag{E.9}$$

Further differentiation of equations E.5 and E.6 with respect to time results in acceleration equations. The acceleration equations can also be put into a matrix equation of the form $A\ddot{\boldsymbol{x}} = \boldsymbol{b}$ where $A = A(\boldsymbol{x})$ and $\boldsymbol{b} = \boldsymbol{b}(\boldsymbol{x}, \dot{\boldsymbol{x}}, \boldsymbol{u})$.

Crank-slider Acceleration Equations

Horizontal Components:

$$\frac{d}{dt}\left(c_3\dot{r}_3 - r_3 s_3\dot{\theta}_3 = -r_2 s_2\dot{\theta}_2\right)$$

$$\dot{c}_3\dot{r}_3 + c_3\ddot{r}_3 - \dot{r}_3 s_3\dot{\theta}_3 - r_3\dot{s}_3\dot{\theta}_3 - r_3 s_3\ddot{\theta}_3 = -\dot{r}_2 s_2\dot{\theta}_2 - r_2\dot{s}_2\dot{\theta}_2 - r_2 s_2\ddot{\theta}_2$$

$$-s_3\dot{r}_3\dot{\theta}_3 + c_3\ddot{r}_3 - s_3\dot{r}_3\dot{\theta}_3 - r_3 c_3\dot{\theta}_3^2 - r_3 s_3\ddot{\theta}_3 = -0 - r_2 c_2\dot{\theta}_2^2 - 0$$

$$c_3\ddot{r}_3 - r_3 s_3\ddot{\theta}_3 = -r_2 c_2\dot{\theta}_2^2 + 2s_3\dot{r}_3\dot{\theta}_3 + r_3 c_3\dot{\theta}_3^2 \tag{E.10}$$

Vertical Components:

$$\frac{d}{dt}\left(s_3\dot{r}_3 + r_3 c_3\dot{\theta}_3 = r_2 c_2\dot{\theta}_2\right)$$

$$\dot{s}_3\dot{r}_3 + s_3\ddot{r}_3 + \dot{r}_3 c_3\dot{\theta}_3 + r_3\dot{c}_3\dot{\theta}_3 + r_3 c_3\ddot{\theta}_3 = \dot{r}_2 c_2\dot{\theta}_2 + r_2\dot{c}_2\dot{\theta}_2 + r_2 c_2\ddot{\theta}_2$$

$$c_3\dot{r}_3\dot{\theta}_3 + s_3\ddot{r}_3 + c_3\dot{r}_3\dot{\theta}_3 - r_3 s_3\dot{\theta}_3^2 + r_3 c_3\ddot{\theta}_3 = 0 - r_2 s_2\dot{\theta}_2^2 + 0$$

$$s_3\ddot{r}_3 + r_3 c_3\ddot{\theta}_3 = -r_2 s_2\dot{\theta}_2^2 - 2c_3\dot{r}_3\dot{\theta}_3 + r_3 s_3\dot{\theta}_3^2 \tag{E.11}$$

Matrix Form:

$$\begin{array}{ccccc} A & \ddot{\boldsymbol{x}} & & \boldsymbol{b} & \\ \begin{bmatrix} c_3 & -r_3 s_3 \\ s_3 & r_3 c_3 \end{bmatrix} & \begin{bmatrix} \ddot{r}_3 \\ \ddot{\theta}_3 \end{bmatrix} & = & \begin{bmatrix} -r_2 c_2\dot{\theta}_2^2 + 2s_3\dot{r}_3\dot{\theta}_3 + r_3 c_3\dot{\theta}_3^2 \\ -r_2 s_2\dot{\theta}_2^2 - 2c_3\dot{r}_3\dot{\theta}_3 + r_3 s_3\dot{\theta}_3^2 \end{bmatrix} & \begin{array}{l} (From\ Eq.\ E.10) \\ (From\ Eq.\ E.11) \end{array} \end{array} \tag{E.12}$$

Note: While the velocity equation, $A\dot{\boldsymbol{x}} = \boldsymbol{b}$, and the acceleration equation, $A\ddot{\boldsymbol{x}} = \boldsymbol{b}$, have the same form, the A and $\boldsymbol{b}$ matrices *are not* generally the same for both the velocity and acceleration equations.

Now that the acceleration equations have been represented as a single matrix equation, it is possible to find the acceleration as a function of the lower order terms, $\boldsymbol{x}$ and $\dot{\boldsymbol{x}}$, and the input $\boldsymbol{u}$.

$$\ddot{\boldsymbol{x}} = A^{-1}\boldsymbol{b} \tag{E.13}$$

The accelerations in $\ddot{\boldsymbol{x}}$ can be integrated to find the lower order terms, $\dot{\boldsymbol{x}}$ and $\boldsymbol{x}$, in a simulation or can be used to extend the analysis to include mechanism kinetics in addition to kinematics.

E.3 Initial Conditions

It can be especially challenging to select initial conditions for a mechanism like the crank-slider because the state variables are constrained; that is, the variables need to be consistent with each other for the equation to solve properly.

E.3.1 Initial Positions

For the crank-slider example, the initial values of r_3 and θ_3 must be consistent with the input θ_2 such that both the horizontal and vertical vector loop equations are valid. In other words, Equations E.3 and E.4 must both be satisfied by the selected initial conditions. If the initial values for r_3 and θ_3 are not consistent with the selected θ_2 then there will be a residual "gap" in the mechanism.

A simple procedure can be used to select valid initial positions. Consider Figure E.4 which depicts the crank-slider mechanism with improper initial conditions. Point B and B* should be coincident, but because of the incorrect length r_3 and angle θ_3 there is a gap $\boldsymbol{e}$.

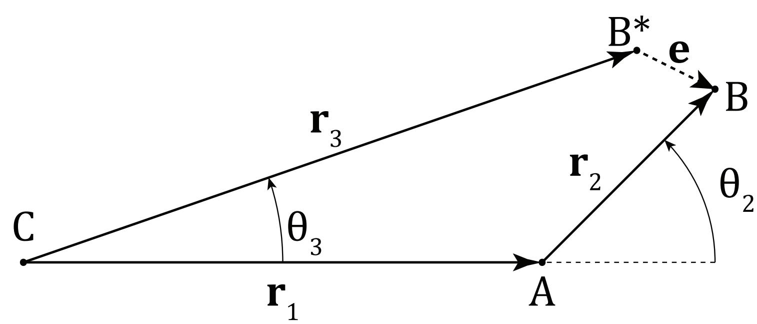

Figure E.4: When the initial conditions are not consistent a gap will be present in the vector-loop.

Equations E.3 and E.4 are valid only if the vector-loop is a valid closed loop, which will only be the case if the angle and length of $\boldsymbol{r}_3$ are consistent with the crank angle. The equations can be adjusted to include extra terms, e_x and e_y, representing the gap in the mechanism. Valid initial conditions are found when the vector magnitude of the gap is zero.

Crank-slider Position Equations With Gap

Horizontal Components:

$$\begin{aligned} r_1 + r_2 c_2 &= r_3 c_3 + e_x \\ e_x &= r_1 + r_2 c_2 - r_3 c_3 \end{aligned} \tag{E.14}$$

Vertical Components:

$$\begin{aligned} r_2 s_2 &= r_3 s_3 + e_y \\ e_y &= r_2 s_2 - r_3 s_3 \end{aligned} \tag{E.15}$$

Magnitude:

$$\begin{aligned} e &= \sqrt{e_x^2 + e_y^2} \\ e &= \sqrt{(r_1 + r_2 c_2 - r_3 c_3)^2 + (r_2 s_2 - r_3 s_3)^2} \end{aligned} \tag{E.16}$$

The following procedure can be used to select valid initial conditions by minimizing the magnitude of the gap, e.

1. Represent the horizontal and vertical gap in terms of the variables in the state vector and the input vector. For the crank slider, that is r_3, θ_3, and θ_2.
2. Represent the residual gap as a scalar by combining the horizontal and vertical components of the gap.
3. Select an initial value for the input. For the crank slider, select the crank angle[5], $\theta_{2,i}$.
4. Estimate initial conditions that result in a small starting gap given the selected initial input. For the crank-slider, guess $r_{3,g}$ and $\theta_{3,g}$ based on the selected value for $\theta_{2,i}$.
5. Attempt to reduce the residual gap to zero by minimization. For the crank-slider, minimize e starting with $r_3 = r_{3,g}$ and $\theta_3 = \theta_{3,g}$. The values of $r_{3,i}$ and $\theta_{3,i}$ that minimize e will be valid initial conditions.

Many different algorithms can be used to minimize the residual function described in the steps above but most work by minimizing a scalar output, like e, based off of a vector input, like $\boldsymbol{x}$. One such method is called the simplex method and is implemented in MATLAB® by the function `fminsearch`. Consider the contour plot shown in Figure E.5 which shows the value e as a function of r_3 and θ_3.

[5] It is necessary to ensure that valid initial conditions exist for a selected input. In the case of the crank-slider it is not an issue because every angle of θ_2 results in a valid configuration. Some mechanisms do not have complete range of motion so the initial input must be selected carefully.

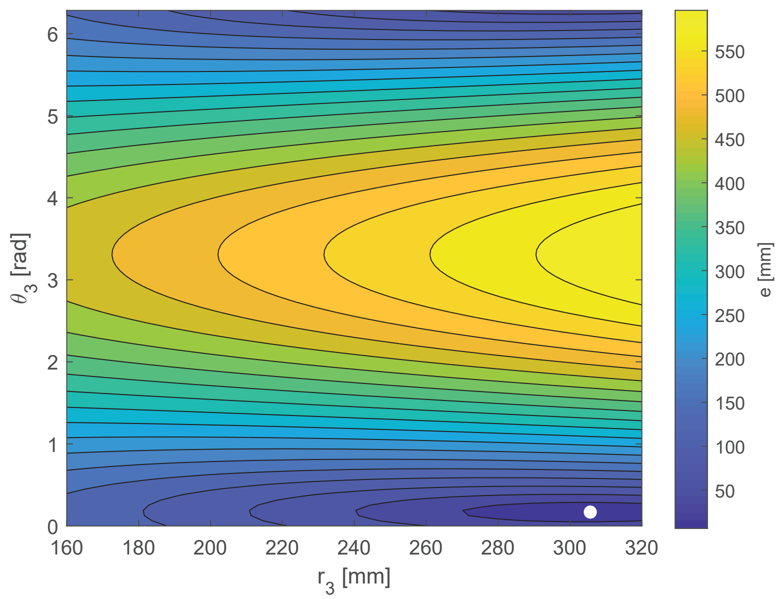

Figure E.5: The gap size e is minimum when r_3 and θ_3 form valid initial conditions shown by the white dot on the plot.

E.3.2 Initial Velocities

The initial velocities can be calculated explicitly once the initial positions are known. The initial velocities can be found by solving Equation E.7 using the initial positions computed by minimization.

E.4 Adding kinetics

Once the initial vector loop has been used to develop acceleration equations in matrix form, the system can be augmented to also compute reaction forces and moments. Consider Figures E.6 and E.7 which depict free-body and kinetic diagrams for links 2 and 3 respectively. Summation of forces and moments for each link result in a new set of equations coupled to the acceleration equations found in the kinematic anlaysis.

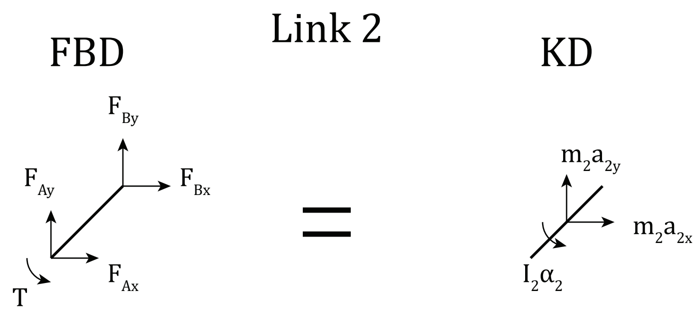

Figure E.6: Free-body and kinetic diagrams for link 2. The crank is driven by an input torque T.

Link 2 Kinetics:

$\Sigma F(\rightarrow)$:

$$\begin{aligned} F_{Ax} + F_{Bx} &= m_2 a_{x2} \\ F_{Ax} + F_{Bx} &= -\frac{1}{2} m_2 r_2 c_2 \dot{\theta}_2^{\,2} \end{aligned} \tag{E.17}$$

$\Sigma F(\uparrow)$:

$$\begin{aligned} F_{Ay} + F_{By} &= m_2 a_{y2} \\ F_{Ay} + F_{By} &= -\frac{1}{2} m_2 r_2 s_2 \dot{\theta}_2^{\,2} \end{aligned} \tag{E.18}$$

$\Sigma M_g(\circlearrowleft)$:

$$\begin{aligned} T + \frac{1}{2} r_2 s_2 F_{Ax} - \frac{1}{2} r_2 s_2 F_{Bx} - \frac{1}{2} r_2 c_2 F_{Ay} + \frac{1}{2} r_2 c_2 F_{By} &= I_2 \alpha_2 \\ T + \frac{1}{2} r_2 s_2 F_{Ax} - \frac{1}{2} r_2 s_2 F_{Bx} - \frac{1}{2} r_2 c_2 F_{Ay} + \frac{1}{2} r_2 c_2 F_{By} &= 0 \end{aligned} \tag{E.19}$$

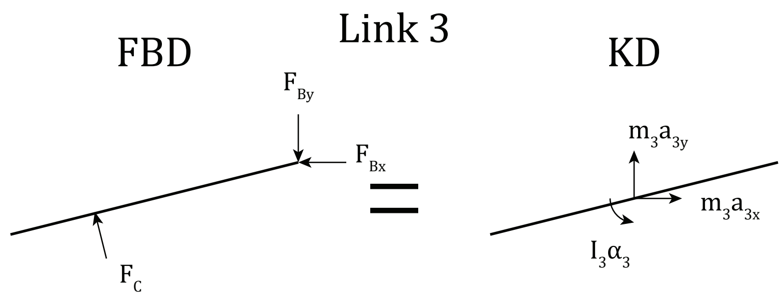

Figure E.7: Free-body and kinetic diagrams for link 3.

Link 3 Kinetics:

$\Sigma F(\rightarrow)$:

$$-s_3F_c - F_{Bx} = m_3a_{x3}$$
$$-s_3F_c - F_{Bx} = m_3\Big(c_3\ddot{r}_3 - 2s_3\dot{r}_3\dot{\theta}_3 - (r_3 - \frac{1}{2}L_3)c_3\dot{\theta}_3^2 - (r_3 - \frac{1}{2}L_3)s_3\ddot{\theta}_3\Big)$$
$$-s_3F_c - F_{Bx} - m_3c_3\ddot{r}_3 + m_3(r_3 - \frac{1}{2}L_3)s_3\ddot{\theta}_3 = -2m_3s_3\dot{r}_3\dot{\theta}_3 - m_3(r_3 - \frac{1}{2}L_3)c_3\dot{\theta}_3^2 \tag{E.20}$$

$\Sigma F(\uparrow)$:

$$c_3F_c - F_{By} = m_3a_{y3}$$
$$c_3F_c - F_{By} = m_3\Big(s_3\ddot{r}_3 + 2c_3\dot{r}_3\dot{\theta}_3 - (r_3 - \frac{1}{2}L_3)s_3\dot{\theta}_3^2 + (r_3 - \frac{1}{2}L_3)c_3\ddot{\theta}_3\Big)$$
$$c_3F_c - F_{By} - m_3s_3\ddot{r}_3 - m_3(r_3 - \frac{1}{2}L_3)c_3\ddot{\theta}_3 = 2m_3c_3\dot{r}_3\dot{\theta}_3 - m_3(r_3 - \frac{1}{2}L_3)s_3\dot{\theta}_3^2 \tag{E.21}$$

$\Sigma M_g(\circlearrowleft)$:

$$-F_c(r_3 - \frac{1}{2}L_3) - \frac{1}{2}L_3c_3F_{By} + \frac{1}{2}L_3s_3F_{Bx} = I_3\alpha_3$$
$$-F_c(r_3 - \frac{1}{2}L_3) - \frac{1}{2}L_3c_3F_{By} + \frac{1}{2}L_3s_3F_{Bx} - I_3\ddot{\theta}_3 = 0 \tag{E.22}$$

Equations E.17 through E.22 can also be packed into a matrix equation. First a new output vector, $\boldsymbol{y}$, can be composed from all of the unknown forces as

$$\boldsymbol{y} = \begin{bmatrix} F_{Ax} \\ F_{Ay} \\ F_{Bx} \\ F_{By} \\ F_C \\ T \end{bmatrix}$$

A new, quite large, matrix can be composed that will solve the kinetics and kinematics simultaneously.

$$\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{x}} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \end{bmatrix} \tag{E.23}$$

Where A_{11} and $\boldsymbol{b}_1$ represent the A matrix and $\boldsymbol{b}$ vector for the kinematics only and A_{21}, A_{22}, and $\boldsymbol{b}_2$ solve the newly added kinetics. A_{21}, A_{22}, and $\boldsymbol{b}_2$ can be determined from Equations E.17 through E.22.

$$A_{21} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -m_3 c_3 & m_3(r_3 - \frac{1}{2}L_3)s_3 \\ -m_3 s_3 & -m_3(r_3 - \frac{1}{2}L_3)c_3 \\ 0 & -I_3 \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ \frac{1}{2}r_2 s_2 & -\frac{1}{2}r_2 c_2 & -\frac{1}{2}r_2 s_2 & \frac{1}{2}r_2 c_2 & 0 & 1 \\ 0 & 0 & -1 & 0 & -s_3 & 0 \\ 0 & 0 & 0 & -1 & c_3 & 0 \\ 0 & 0 & \frac{1}{2}L_3 s_3 & -\frac{1}{2}L_3 c_3 & -(r_3 - \frac{1}{2}L_3) & 0 \end{bmatrix}$$

$$\boldsymbol{b}_2 = \begin{bmatrix} -\frac{1}{2}m_2 r_2 c_2 \dot{\theta}^2 \\ -\frac{1}{2}m_2 r_2 s_2 \dot{\theta}^2 \\ 0 \\ -2m_3 s_3 \dot{r}_3 \dot{\theta}_3 - m_3(r_3 - \frac{1}{2}L_3)c_3\dot{\theta}_3^2 \\ 2m_3 c_3 \dot{r}_3 \dot{\theta}_3 - m_3(r_3 - \frac{1}{2}L_3)s_3\dot{\theta}_3^2 \\ 0 \end{bmatrix}$$

The equation $A_{21}\ddot{\boldsymbol{x}} + A_{22}\boldsymbol{y} = \boldsymbol{b}_2$ is too large to display in its fully expanded form. The output forces, $\boldsymbol{y}$, can be solved for by inverting the square matrix A_{22}.

$$\boldsymbol{y} = A_{22}^{-1}\left(\boldsymbol{b}_2 - A_{21}\ddot{\boldsymbol{x}}\right) \tag{E.24}$$

E.5 Simulating the mechanism

The kinematics and kinetics of the mechanism can be simulated in a variety of ways. One effective method is to use a Simulink® model to route the information and perform the numerical integration but to use a MATLAB® function to implement the matrix operations necessary for the equations derived above. Figures E.8 and E.9 show to possible implementations in Simulink®, one for kinematics only and one for both kinematics and kinetics.

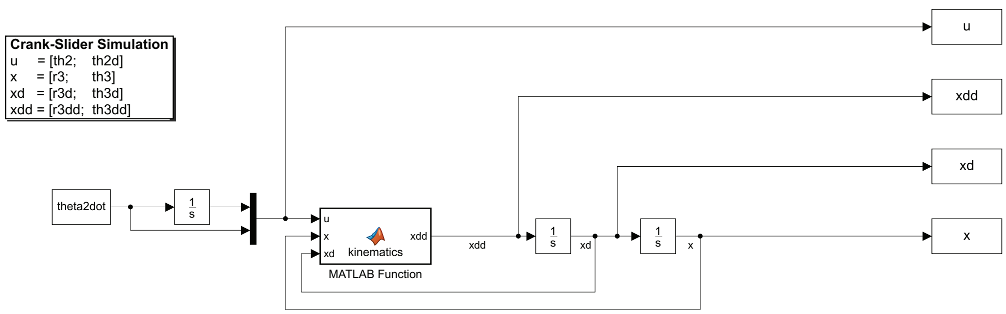

Figure E.8: This model shows a method for simulating only the kinematics of the crank-slider mechanism. The MATLAB® function block is the core of the Simulink® diagram.

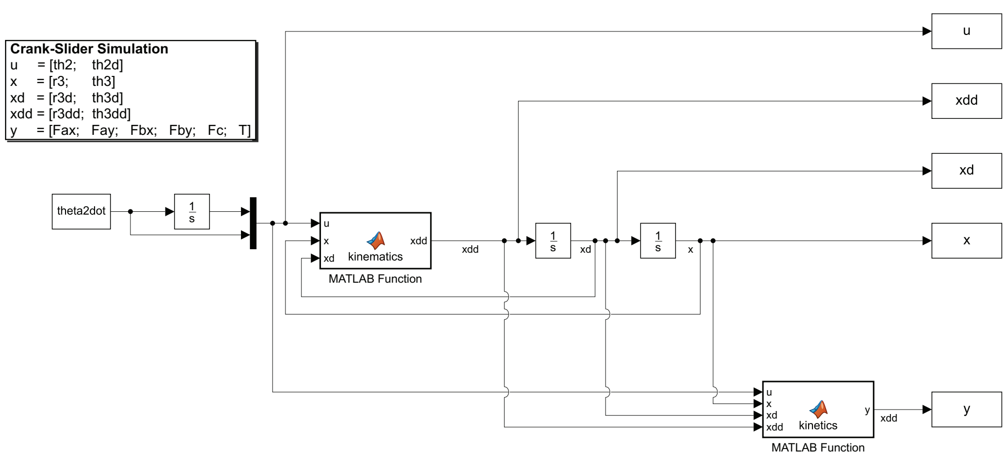

Figure E.9: This model shows a method for simulating both the kinematics and kinetics of the crank-slider mechanism. Here two MATLAB® function blocks make up the core of the Simulink® diagram.

E.6 Visualizing the mechanism

The code below can be used to generate an animation of the crank-slider mechanism using simulation output data.

AnimationTemplate.m

```
%% Crank Slider Geometry
r1 = 240;                                       % Length of vector r1 (ground)
r2 = 80;                                        % Length of vector r2 (crank)
L  = 350;                                       % Length of slider

%% Input
th2_i  = 0;                                     % Initial crank angle
th2d_i = 0;                                     % Initial crank angle

%% Simulation
% The code in this section may eventually be replaced by simulation data.
u = [th2_i th2d_i];                             % input vector
x = [r1+r2 0];                                  % state vector

%% Animation
N = size(x,1);                                  % Number of rows of data

c2 = cos(u(:,1));                               % cos(theta2)
s2 = sin(u(:,1));                               % sin(theta2)

c3 = cos(x(:,2));                               % cos(theta3)
s3 = sin(x(:,2));                               % sin(theta3)

% X and Y components for each point on the crank-slider mechanism.
%            Pt. C        Pt. A            Pt. B        Pt. D
x_coord = [ zeros(N,1)  r1*ones(N,1)    r1+r2*c2     r1+r2*c2-L*c3 ];
y_coord = [ zeros(N,1)  zeros(N,1)         r2*s2        r2*s2-L*s3 ];

% Loop through the frames and plot them all.
for n = 1:N
    figure(1);                                  % Make sure the animation stays put
    plot(x_coord(n,:),y_coord(n,:));            % Plot the whole mechanism at once
    axis('equal');                              % Makes x- and y- scalle equal
    drawnow('limitrate');                       % Forces MATLAB to update plot
end
```

If desired, the animation can be saved as a GIF using the `imwrite` command.