Exercise 5

Mechanical Yaw-rate Sensor Design

5.1 Background

The goal of this minilab is to design and tune a mechanical sensor to accurately measure the yaw-rate of an aircraft in a level turn. Consider the apparatus shown below in Figure 5.1 which consists of a gimballed flywheel spinning at a constant velocity $\Omega_x = 10000$ [RPM]. The flywheel has a diameter, D, of 40 [mm] and a mass, m, of 200 [g]. The flywheel is supported on each side by a strut consisting of a spring and a damper in parallel. Neglect the additional contribution to the moment of inertia due to the gimbal assembly. Assume that the two struts are identical; that is, $k_1 = k_2$ and $b_1 = b_2$. Each strut is a distance $r = 50$ [mm] from the clevis joint. Also assume that the aircraft is performing a level turn at a yaw-rate of Ω_z. The estimated yaw-rate, $\hat{\Omega}_z$, can be determined by considering the angle of deflection of the flywheel.

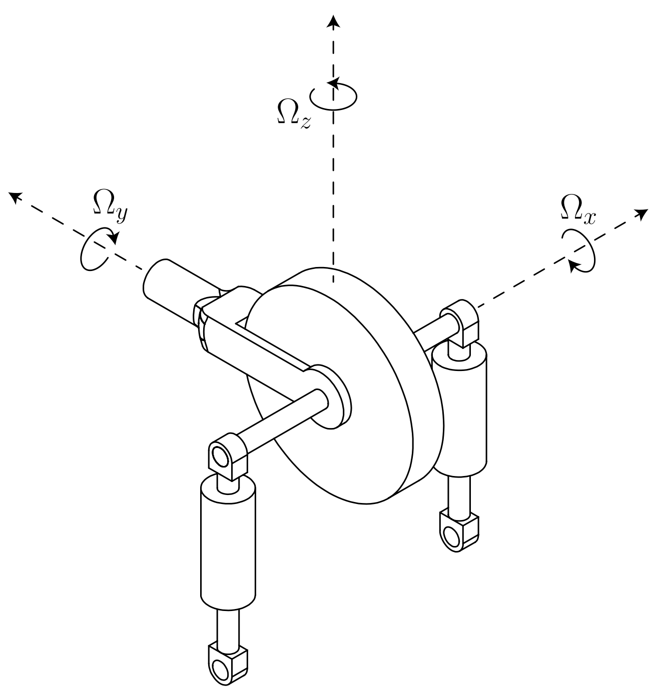

Figure 5.1: Mechanical yaw-rate sensing apparatus.

Gyrators

A gyrator is a type of transducer originally proposed by Bernard Tellegen in 1948. The name gyrator is a portmanteau of gyroscope and the suffix -tor typically used to name electrical circuit elements. This naming convention arose due to the nature of gyroscopic motion, specifically the torque and angular velocity relationship. Consider the gyroscope shown below in Figure 5.2.

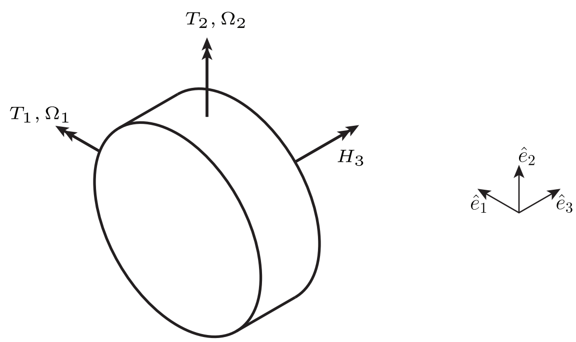

Figure 5.2: Gyroscope spinning about its $\hat{e}_1$ axis.

A gyroscope spinning about an axis will induce a torque when the spin axis changes. Conversely, the spin axis, indicated as $\hat{e}_1$ in the figure, will change when an external torque, T_1 or T_2, is applied about any axis other than the spin axis. If the angular momentum, $\boldsymbol{H} = H_3\hat{e}_3$, is constant and due solely to spin, then this relationship can be described using a single cross product.

$$\boldsymbol{T} = \boldsymbol{\Omega} \times \boldsymbol{H} \tag{5.1}$$

Where $\boldsymbol{\Omega}$ is the angular velocity of the spin axis and $\boldsymbol{H}$ is the angular momentum due to the spin velocity of the gyroscope. Expanding the cross product in Equation 5.1 produce the familiar gyrator equations.

These simplifications assume that the angular momentum is only in the direction of the $\hat{e}_3$ axis; that is, $\boldsymbol{H} = H_3\hat{e}_3$ and that the angular velocity of the spin axis is in the $\hat{e}_1\hat{e}_2$ plane; that is, $\boldsymbol{\Omega} = \Omega_1\hat{e}_1 + \Omega_2\hat{e}_2$.

$$\boldsymbol{T} = \boldsymbol{\Omega} \times \boldsymbol{H} \tag{5.2}$$

$$\boldsymbol{T} = (\Omega_1\hat{e}_1 + \Omega_2\hat{e}_2) \times H_3\hat{e}_3 \tag{5.3}$$

$$\boldsymbol{T} = H_3\Omega_2\hat{e}_1 - H_3\Omega_1\hat{e}_2 \tag{5.4}$$

After splitting Equation 5.4 into components and rearranging terms, a standard [2×2] transducer matrix can be formed.

$$\Omega_1 = -\frac{1}{H_3} T_2 \tag{5.5}$$

$$T_1 = H_3 \Omega_2 \tag{5.6}$$

$$\begin{bmatrix} \Omega_1 \\ T_1 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{H_3} \\ H_3 & 0 \end{bmatrix} \begin{bmatrix} \Omega_2 \\ T_2 \end{bmatrix} \tag{5.7}$$

The gyrator can then be represented by a two-port element as shown below in Figure 5.3. Recall that, for a gyrator, both ports must be branches or both ports must be links, as determined by other elements in the system.

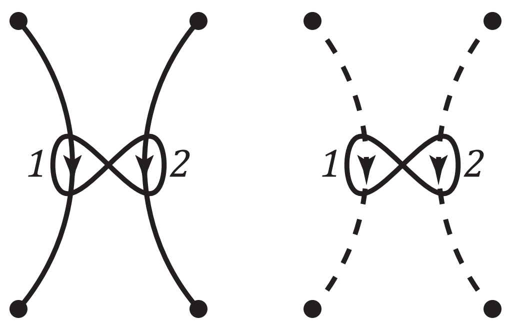

Figure 5.3: Two-port elements representing a gyroscope.

5.2 Assignment

5.2.1 Part 1

For the first part of this assignment, assume that the dampers are disconnected and the springs each have a stiffness of 500 [N/m]; that is, $b_1 = b_2 = 0$ and $k_1 = k_2 = 500$ [N/m].

1. Draw a linear graph and normal tree for the system described in the Background section; use the yaw-rate Ω_z as an across-variable source.

2. From the normal tree, develop a state space model of the system where the system outputs are θ_y, the tilt angle of the flywheel, and $\hat{\Omega}_z$, the estimated yaw-rate of the aircraft. Assume that $\hat{\Omega}_z$ is to be determined exclusively in terms of the spring deflection (and therefore it's force).

3. Determine the transfer function, $G(s) = \frac{\hat{\Omega}_z(s)}{\Omega_z(s)}$, relating the estimated yaw-rate to the actual yaw-rate. From this transfer function determine the steady-state gain of the system and the natural frequency of the system.

5.2.2 Part 2

The measurement device, as configured in the first part of the assignment, is effectively useless. It is useless because the response is undamped and will oscillate wildly when the aircraft yaws. To fix this, the spring coefficients, $k_1 = k_2$, and damping coefficients, $b_1 = b_2$, can be adjusted to tune the response characteristics.

1. Draw a new linear graph and normal tree for the modified system, now including the dampers.
2. Produce a new state-space model with the same inputs and outputs as before.
3. Find the new transfer function $G(s) = \frac{\hat{\Omega}_z(s)}{\Omega_z(s)}$ relating the estimated yaw-rate to the actual yaw-rate. From the transfer function show that the steady-state gain is the same as for the original transfer function found in Part 1.
4. Determine the values of k_1, k_2, b_1, and b_2 that result in a 2% settling time, t_s, of 0.01 [sec] and a maximum overshoot, M_P, of 3%.

   Refer to Appendix C of the lab manual for equations relating the settling time and maximum overshoot to the natural frequency and damping ratio. Then you can relate the spring and damper coefficients to the natural frequency and damping ratio.

5.3 Discussion and Deliverables

5.3.1 Discussion Questions

1. Are there any remaining assumptions other than those mentioned in this assignment?
2. Consider what may happen if the angle of deflection is large; what can be done to guarantee that the maximum deflection is less than some acceptable limit?
3. From this model would it be possible to determine the reaction torque on the clevis supporting the flywheel? If so, what would the expression be?
4. If this sensor were to be implemented in an actual aircraft what parameters would need to be measured in order to compute the estimated yaw-rate?