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COSC 480 - Learning From Data
Fall 2019
Problem Set 7

1. Smallest break point for a 3D perceptron?

   This question can be easily answered by applying the claim proven in class that the VC of a d-dimensional perceptron is equal to $d+1$. For a 3D perceptron, we have $d = 3$ and therefore $d_{vc} = 3 + 1 = 4$. We can understand the VC dimension as the highest number of points a hypothesis set is able to shatter. Therefore, if a 3D perceptron is able to shatter at most 4 points, then we must have the minimal break point at 5 points.

2. Break point for two-interval hypothesis set.

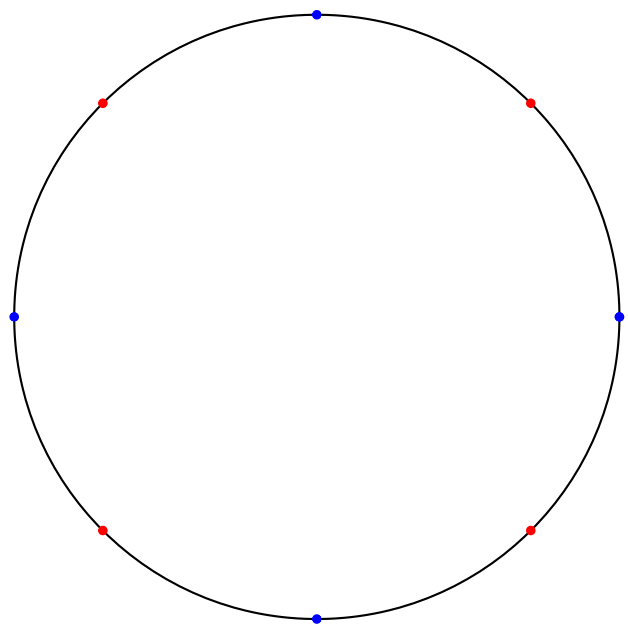
   In order to find a break point for this hypothesis set, must find a number of points where at least one dichotomy cannot be produced. Additionally, if we are seeking to show that this is the *smallest* break point, we must also show that this hypothesis set can shatter all sets of points smaller than the number in question.
   We show that the two-interval set has a break point at 5. Let $x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4, x_5 = 5$ be the 5 points to be classified as either -1 or $+1$. Given this set of points, there is no assignment of a, b, c, d such that the points x_1, x_3, x_5 can be classified as $+1$ and the remaining points x_2 and x_4. This is because the two interval set is only able to classify two consecutive sequence of points as $+1$. Since the three points can be considered as three non-consecutive intervals, this dichotomy cannot be produced and thus 5 must be a breakpoint.
   It remains to show that 5 is the smallest breakpoint. Let us consider the case of 4 points to be classified as +1 or -1. In this case, the two-interval set will have to classify either 0, 1, 2, 3, or 4 points as either +1 or -1. This is clearly possible in the cases of 0,1,2, but requires more though for 3 and 4 points. We have established that the two-interval hypothesis set can discriminate any two consutive sequences of points. In a set of 4 points, there is no way to select 3 or 4 points such that more than 2 of these are not consecutive. Therefore we have at most two consecutive sequences in a set of 4 points, and thus the two-interval hypothesis set can shatter 4 points.

3. Break point for a l-interval hypothesis set

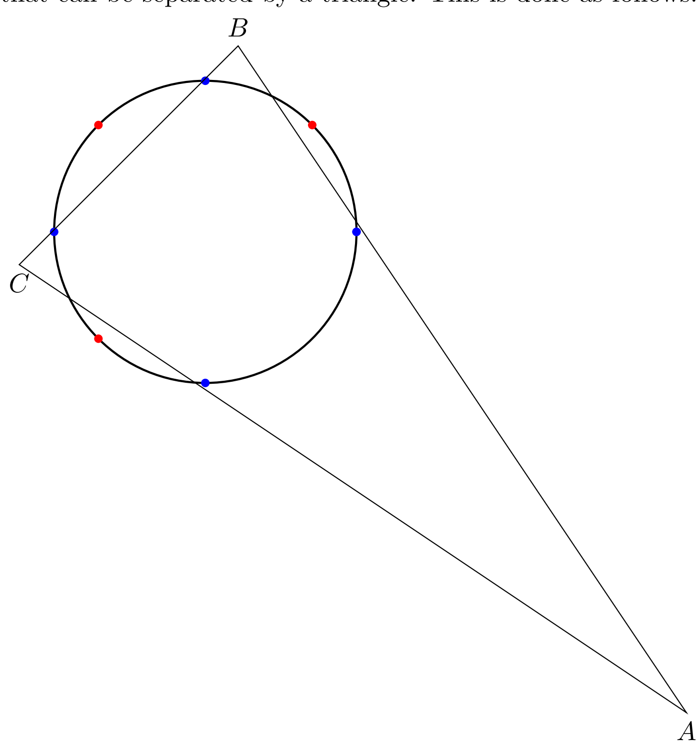
   This question requires showing a general case of the above problem. As we have shown, a l-interval hypothesis set cannot produce a dichotomy where $l+1$ nonconsecutive points are labeled as $+1$. Thus, the number of points where the l-interval hypothesis set cannot shatter will be equal to the $l+1$ points plus the number of points needed to separate these such that none are consecutive. This is clearly equal to $(l+1) + l = 2l + 1$, as each point in $l+1$ will necessarily be followed by another point not counted in $l+1$, minus the very last point which will have nothing following it. Thus the general break point for a l-interval set is $2l + 1$.

4. Break point for a triangle in 2D space

   We can visualize the points to be classified as points on a uniform circle. Let us visualize this for 8 points in the diagram as follows.



From this picture, it is clear that no triangle can create this dichotomy of points, where the blue and red colorings represent the binary classifications. Producing this dichotomy would require a 4-sided polygon. Therefore, 8 must be a break point for a triangle in 2D space. However, is it the smallest break point? In order to check for this, we show that removing a single point produces a dichotomy that can be separated by a triangle. This is done as follows:



Clearly, we are now able to create this dichotomy using only a triangle separator. Therefore all dichotomies on 7 points are possible, while this is not the case for 8 points. Thus, 8 is the break point for the 2D triangle hypothesis set.