Problem Set 2

COSC 290 Spring 2018

Caio Brighenti

February 7, 2018

1   Problem 1: Binary numbers

Below is a table of all the $x_0 - x_3$ bit to base 10 number pairings.

x_0	x_1	x_2	x_3	n
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	0	8
1	0	0	1	9
1	0	1	0	10
1	0	1	1	11
1	1	0	0	12
1	1	0	1	13
1	1	1	0	14
1	1	1	1	15

1.1   DLN 3.29

x_0

1.2   DLN 3.30

$((x_0 \lor x_1) \land \lnot(x_2 \lor x_3)) \lor (x_0 \land \lnot x_1 \land \lnot x_2 \land x_3)$

1.3   DLN 3.31

$(x_0 \land x_2 \land x_3) \lor (\lnot x_0 \land x_1 \land \lnot x_2 \land x_3)$

1.4   DLN 3.32

$((x_1 \oplus x_2) \land \lnot(x_0 \lor x_3)) \lor (\lnot(x_1 \lor x_2) \land x_0 \land x_3)$

2   Problem 2: More binary numbers

2.1   DLN 3.33

$((x_0 \wedge y_0) \vee \neg(x_0 \vee y_0)) \wedge ((x_1 \wedge y_1) \vee \neg(x_1 \vee y_1)) \wedge ((x_2 \wedge y_2) \vee \neg(x_2 \vee y_2)) \wedge ((x_3 \wedge y_3) \vee \neg(x_3 \vee y_3))$

2.2   DLN 3.34

$(y_0 \wedge \neg x_0) \vee (y_1 \wedge \neg(x_0 \vee x_1)) \vee (y_2 \wedge \neg(x_0 \vee x_1 \vee x_2)) \vee (y_3 \wedge \neg(x_0 \vee x_1 \vee x_2 \vee x_3))$

3 Problem 3: Circuits

3.1 DLN 3.84

$(q \vee r) \wedge \neg p$

3.2 DLN 3.85

$p \wedge (q \vee r)$

3.3 DLN 3.86

$(p \wedge q) \wedge \neg r$

3.4 DLN 3.87

$\neg(p \vee r)$

4   Problem 4: More circuits

4.1   DLN 3.88

There are two propositions that cannot be expressed using only two gates. These are the exclusive or and not exclusive or propositions. The truth table for each of these is below. ϕ_1 represents exclusive or, and ϕ_2 represents not exclusive or.

p	q	ϕ_1	ϕ_2
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

The shortest propositions for ϕ_1 and ϕ_2 would be:

$\phi_1 = (p \vee q) \wedge \neg(p \wedge q)$

$\phi_2 = \neg(p \vee q) \vee (p \wedge q)$