Problem Set 7

COSC 290 Spring 2018

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1 Problem 1: Using induction to prove algorithm correctness, DLN 5.71

Claim: Let $P(n)$ if for a sorted array $A[1...n]$ of length n, $binarySearch(A, x) \iff x \in A$. The claim is that $\forall n \in \mathrm{Z}^{\geq 0} : P(n)$.

Proof. We will prove by strong induction on n.

Base cases: The base cases are $n = 0$ and $n = 1$. Both $P(0)$ and $P(1)$ are true, as any array of length 0 or 1 is sorted.

Inductive case: Let $n \geq 2$. We will show $P(0) \wedge ... \wedge P(n-1) \iff P(n)$.

- *Given*: Assume $P(0) \wedge ... \wedge P(n-1)$ is true.
- *Want to show*: $P(n)$ is true.

We proceed by cases. There are three ways in which x can exist with relation to $A[1...n]$ and $middle$. These cases are as such:

$$A[1...x = middle...n]$$
$$A[1...x...middle...n]$$
$$A[1...middle...x...n]$$

In Case 1, we have that $x = middle$. In Case 2, we have that $1 \leq x < middle \leq n$. In Case 3, we have that $1 \leq middle < x \leq n$.

Case 1: In the first case, the algorithm successfully found the item x it was searching for. Thus, the function returns true.
Case 2: In the second case, the function will be called recursively on $A[1...middle - 1]$. From there
Case 3: In the third case, the function will be called recursively on $A[middle + 1...n]$. From there

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2 Problem 2: proving a relation is a partial order

2.1 DLN 8.131

replace with your answer

2.2 DLN 8.132

replace with your answer

3 Problem 3: an equivalence relation and a partial order? DLN 8.155

replace with your answer