Problem Set 2

COSC 290 Spring 2018

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1 Problem 1: Binary numbers

Below is a table of all the $x_3 - x_0$ bit to base 10 number pairings.

x_3	x_2	x_1	x_0	n
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	0	8
1	0	0	1	9
1	0	1	0	10
1	0	1	1	11
1	1	0	0	12
1	1	0	1	13
1	1	1	0	14
1	1	1	1	15

1.1 DLN 3.29

x_3

1.2 DLN 3.30

$((x_3 \vee x_2) \wedge \neg(x_1 \vee x_0))$

1.3 DLN 3.31

$(x_3 \wedge x_1 \wedge x_0) \vee (\neg x_3 \wedge x_2 \wedge \neg x_1 \wedge x_0)$

1.4 DLN 3.32

$((x_2 \oplus x_1) \wedge \neg(x_3 \vee x_0)) \vee (\neg(x_2 \vee x_1) \wedge x_3 \wedge x_0)$

2 Problem 2: More binary numbers

2.1 DLN 3.33

$((x_3 \wedge y_3) \vee \neg(x_3 \vee y_3)) \wedge ((x_2 \wedge y_2) \vee \neg(x_2 \vee y_2)) \wedge ((x_1 \wedge y_1) \vee \neg(x_1 \vee y_1)) \wedge ((x_0 \wedge y_0) \vee \neg(x_0 \vee y_0))$

2.2 DLN 3.34

p checks for the equality of x and y, and q checks if y is greater than x.

$p = ((x_3 \wedge y_3) \vee \neg(x_3 \vee y_3)) \wedge ((x_2 \wedge y_2) \vee \neg(x_2 \vee y_2)) \wedge ((x_1 \wedge y_1) \vee \neg(x_1 \vee y_1)) \wedge ((x_0 \wedge y_0) \vee \neg(x_0 \vee y_0))$

$q = (y_3 \wedge \neg x_3) \vee (y_2 \wedge \neg(x_3 \vee x_2)) \vee (y_1 \wedge \neg(x_3 \vee x_2 \vee x_1)) \vee (y_0 \wedge \neg(x_3 \vee x_2 \vee x_1 \vee x_0))$

$p \vee q$

3 Problem 3: Circuits

3.1 DLN 3.84

$(q \lor r) \land \neg p$

3.2 DLN 3.85

$p \land (q \lor r)$

3.3 DLN 3.86

$(p \land q) \land \neg r$

3.4 DLN 3.87

$\neg(p \lor r)$

4 Problem 4: More circuits

4.1 DLN 3.88

There are two propositions that cannot be expressed using only two gates. These are the exclusive or and not exclusive or propositions. The truth table for each of these is below. ϕ_1 represents exclusive or, and ϕ_2 represents not exclusive or.

p	q	ϕ_1	ϕ_2
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

The shortest propositions for ϕ_1 and ϕ_2 would be:

$\phi_1 = (p \vee q) \wedge \neg(p \wedge q)$

$\phi_2 = \neg(p \vee q) \vee (p \wedge q)$