

# **Problem Set 2**

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# 1 Problem 1: Binary numbers

Below is a table of all the  $x_0 - x_3$  bit to base 10 number pairings.

$x_0$	$x_1$	$x_2$	$x_3$	$n$
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	1	8
1	0	1	0	9
1	0	1	1	10
1	1	0	0	12
1	1	0	1	13
1	1	1	0	14
1	1	1	1	15

## 1.1 DLN 3.29

$x_0$

## 1.2 DLN 3.30

$$((x_0 \vee x_1) \wedge \neg(x_2 \vee x_3)) \vee (x_0 \wedge \neg x_1 \wedge \neg x_2 \wedge x_3)$$

## 1.3 DLN 3.31

$$(x_0 \wedge x_2 \wedge x_3) \vee (\neg x_0 \wedge x_1 \wedge \neg x_2 \wedge x_3)$$

## 1.4 DLN 3.32

$$((x_1 \oplus x_2) \wedge \neg(x_0 \vee x_3)) \vee (\neg(x_1 \vee x_2) \wedge x_0 \wedge x_3)$$

## 2 Problem 2: More binary numbers

### 2.1 DLN 3.33

$$((x_0 \wedge y_0) \vee \neg(x_0 \vee y_0)) \wedge ((x_1 \wedge y_1) \vee \neg(x_1 \vee y_1)) \wedge ((x_2 \wedge y_2) \vee \neg(x_2 \vee y_2)) \wedge ((x_3 \wedge y_3) \vee \neg(x_3 \vee y_3))$$

### 2.2 DLN 3.34

$$(y_0 \wedge \neg x_0) \vee (y_1 \wedge \neg(x_0 \vee x_1)) \vee (y_2 \wedge \neg(x_0 \vee x_1 \vee x_2)) \vee (y_3 \wedge \neg(x_0 \vee x_1 \vee x_2 \vee x_3))$$

### 3 Problem 3: Circuits

#### 3.1 DLN 3.84

$$(q \vee r) \wedge \neg p$$

#### 3.2 DLN 3.85

$$p \wedge (q \vee r)$$

#### 3.3 DLN 3.86

$$(p \wedge q) \wedge \neg r$$

#### 3.4 DLN 3.87

$$\neg(p \vee r)$$

## 4 Problem 4: More circuits

### 4.1 DLN 3.88

There are two propositions that cannot be expressed using only two gates. These are the exclusive or and not exclusive or propositions. The truth table for each of these is below.  $\phi_1$  represents exclusive or, and  $\phi_2$  represents not exclusive or.

$p$	$q$	$\phi_1$	$\phi_2$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

The shortest propositions for  $\phi_1$  and  $\phi_2$  would be:

$$\phi_1 = (p \vee q) \wedge \neg(p \wedge q)$$

$$\phi_2 = \neg(p \vee q) \vee (p \wedge q)$$