

1 Sets

Sets are collections of values of one type with no internal structure. Sets have no repeated items and order does not matter.

1.1 Set Operations

- Union - $A \cup B = \{x : x \in A \text{ or } x \in B\}$
- Intersect - $A \cap B = \{x : x \in A \text{ and } x \in B\}$
- Concatenation - $AB = \{ab : a \in A \text{ and } b \in B\}$
- Cartesian Product - $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$
- Complement - $A^c = \{x : x \notin A\}$
- Powerset - $P(A) = \{A' : A' \subset A\}$

1.2 Finite and Infinite Sets

Set A is **finite** if it can be put in 1-to-1 correspondence with an initial segment of \mathbb{N} , or is the empty set \emptyset . More intuitively, a set is finite if we can list the elements of A in finite time.

Set A is **countably infinite** if it can be put in 1-to-1 correspondence with *all* of \mathbb{N} . Intuitively, a procedure can be devised to list all elements, but this procedure will never finish.

Set A is **uncountably infinite** if it is not countable.

1.3 Relations and Functions

A **function** can be defined as a binary operation between two sets that given an input in set A produces an output in set B . There are three types of functions:

1. Total function: $A \rightarrow B$ means for every input A there is exactly one output B
2. Partial function: $A \rightarrow B$ means for every input A there is at most one output B
3. Multi function: $A \multimap B$ means for every input A there are 0, 1, or many outputs in B

We also define the **identity function** such that for any type A , $f(A) = A$.

1.4 Cardinality of Sets

The cardinality of a set A , or $|A|$, is the total number of elements in A .

- Union - $|A \cup B| = |A| + |B| - |A \cap B|$
- Intersect - $|A \cap B| = |A| + |B| - |A \cup B|$
- Concatenation -

- Cartesian Product - $|A \times B| = |A| \times |B|$
- Powerset - $P(A) = 2^{|A|}$
- Total function - $|A \rightarrow B| = |B|^{|A|}$
- Partial function - $|A \rightharpoonup B| = |B + 1|^{|A|}$
- Multi function - $|A \multimap B| = |P(B)|^{|A|}$

2 Alphabets and Languages

2.1 Definitions

Alphabet A is a set of single characters. **Word** w in alphabet A is a string of characters from A . A^* denotes the set of all words in alphabet A . This operation is called *Kleene star*.

2.2 Regular Languages

3 Finite Automata

3.1 Definitions

3.2 Regular Expressions to f.a.

3.3 Simplifying

3.4 Converting ndfa to dfa

3.5 Finding Language of f.a.