

Problem Set 1

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1 Problem 1

1.1

$$T = \{2, 3, 0\}$$

1.2

$$P(A) = \{\{\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

1.3

$$bitstrs = \{\langle 0, 0, 0 \rangle, \langle 0, 0, 1 \rangle, \langle 0, 1, 0 \rangle, \langle 0, 1, 1 \rangle, \langle 1, 0, 0 \rangle, \langle 1, 0, 1 \rangle, \langle 1, 1, 0 \rangle, \langle 1, 1, 1 \rangle\}$$

2 Problem 2

2.1

To map the elements of $P(A)$ using *bitstrs*, we can define a function f that treats the values 0,1 as indications of whether an element is present in the set. For example, with the sequences $\langle 0, 0, 1 \rangle$ the 0s in the first two positions indicate an absence of elements a and b in the set, while the 1 in the final position indicates the presence of c , thus resulting in the set $\{c\}$. This mapping should iterate through each bit string sequence. The table below shows each of the x, y mappings.

x	y
$\langle 0, 0, 0 \rangle$	$\{\}$
$\langle 0, 0, 1 \rangle$	$\{c\}$
$\langle 0, 1, 0 \rangle$	$\{b\}$
$\langle 0, 1, 1 \rangle$	$\{b, c\}$
$\langle 1, 0, 0 \rangle$	$\{a\}$
$\langle 1, 0, 1 \rangle$	$\{a, c\}$
$\langle 1, 1, 0 \rangle$	$\{a, b\}$
$\langle 1, 1, 1 \rangle$	$\{a, b, c\}$

2.2

To iteratively generate the power set $P(S)$ where S is a set of size n , we can use the sequences produced by $\{0, 1\}^n$. Each sequence will correspond to exactly one set. Each set is generated by iterating through the corresponding sequence, represented by $\langle x_1, x_2, \dots, x_n \rangle$. Each element x_i will either be 0 or 1. Each element of the set S is represented by s_j for the j^{th} element in S . If $i = j$, then x_i will indicate the presence of the element s_j in the set being produced, where 0 corresponds to the element *not* being present and 1 corresponds to the element existing in the set.

For example, in the set $A = 1, 2, 3$, the bit string sequence $\langle 0, 1, 0 \rangle$ will correspond to x_1 and x_3 being equal to 0, and x_2 being equal to 1. Thus, elements s_1 and s_3 will not be present, while s_2 will, resulting in the set $\{2\}$.

3 Problem 3

3.1

$$P(\emptyset) = \{\emptyset\}$$

3.2

$$P(A) = \{\{\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

$$P(A - \{a\}) = P(\{b, c\}) = \{\{\}, \{b\}, \{c\}, \{b, c\}\}$$

3.3

$$A := \{z \cup x : z \in PT\}$$

$$P(S) = PT \cup A$$

4 Problem 4

4.1 DLN 2.124

$$A, B = 2$$

$$A, C = 1$$

$$B, C = 1$$

4.2 DLN 2.125

$$A, B = \frac{2}{5}$$

$$A, C = \frac{1}{3}$$

$$B, C = \frac{1}{4}$$

4.3 DLN 2.126

This statement is not true for the cardinality measure. A simple example to disprove it would be the sets $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5, 6, 7\}$ and $C = \{4, 5, 6, 7\}$. The cardinality measure for A and B would be 3, and 0 for A and C . However, the highest cardinality measure for B would be between B and C , equal to 4. Thus, A is most similar to B , but B is most similar to C .

4.4 DLN 2.127

This statement is also not true for the Jaccard coefficient. The same set of sets described above disprove it. For A and B , the Jaccard coefficient would be $\frac{3}{7}$, and 0 for A and C . The Jaccard coefficient for B and C would be $\frac{4}{7}$. Thus, A is most similar to B , but B is most similar to C .