Problem Set 9999

COSC 290 Spring 2018

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1 Problem 1: G_n and F_n

There is a `\section` for each problem. The example proof is at the end. Please delete this remark and the example before you turn it in.

2 Problem 2: G_n lower bound

3 Problem 3: lower bound on height balanced binary trees

4 Problem 4: false lower bound on binary trees

5 Problem 5: bound on height

6 Example of proof by induction

This is an example proof, provided in LaTeX so that you may copy its basic formatting.

Claim: Let $P(n) := \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$. The claim is that $\forall n \in \mathrm{Z}^{\geq 1} : P(n)$.

Proof. We will prove by weak induction on n.

Base case: $n = 1$. In this case $\sum_{i=1}^{n} i = \sum_{i=1}^{1} i = 1$ and $\frac{n(n+1)}{2} = \frac{1\times(1+1)}{2} = 1$. Thus $P(1)$ is true.

Inductive case: Let $n \geq 2$. We will show $P(n-1) \implies P(n)$.

- *Given*: Assume $P(n-1)$ is true.
- *Want to show*: $P(n)$ is true.

Since $P(n-1)$ is true, we have

$$\sum_{i=1}^{n-1} i = \frac{(n-1)((n-1)+1)}{2}$$

We will use this fact to show $P(n)$:

$$\begin{aligned}
\sum_{i=1}^{n} i &= \left(\sum_{i=1}^{n-1} i\right) + n && \text{definition of summation} \\
&= \frac{(n-1)((n-1)+1)}{2} + n && \text{inductive hypothesis} \\
&= \frac{(n-1)n + 2n}{2} && \text{rearanging/simplifying terms} \\
&= \frac{n^2 - n + 2n}{2} = \frac{n^2 + n}{2} && \text{algebra} \\
&= \frac{n(n+1)}{2} && \square
\end{aligned}$$

$\square$