Problem Set 7

COSC 290 Spring 2018

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1 Problem 1: Using induction to prove algorithm correctness, DLN 5.71

Claim: Let $P(n)$ if for a sorted array $A[1...n]$ of length n, $binarySearch(A, x) \iff x \in A$. The claim is that $\forall n \in Z^{\geq 0} : P(n)$.

Proof. We will prove by strong induction on n.

Base cases: The base cases are $n = 0$ and $n = 1$. Both $P(0)$ and $P(1)$ are true, as any array of length 0 or 1 is sorted.

Inductive case: Let $n \geq 2$. We will show $P(n-1) \implies P(n)$.

- *Given*: Assume $P(n-1)$ is true.
- *Want to show*: $P(n)$ is true.

Since $P(n-1)$ is true, we have

$$\sum_{i=1}^{n-1} i = \frac{(n-1)((n-1)+1)}{2}$$

We will use this fact to show $P(n)$:

$$\begin{aligned}
\sum_{i=1}^{n} i &= \left(\sum_{i=1}^{n-1} i\right) + n && \text{definition of summation} \\
&= \frac{(n-1)((n-1)+1)}{2} + n && \text{inductive hypothesis} \\
&= \frac{(n-1)n + 2n}{2} && \text{rearanging/simplifying terms} \\
&= \frac{n^2 - n + 2n}{2} = \frac{n^2 + n}{2} && \text{algebra} \\
&= \frac{n(n+1)}{2} && \square
\end{aligned}$$

$\square$

2 Problem 2: proving a relation is a partial order

2.1 DLN 8.131

replace with your answer

2.2 DLN 8.132

replace with your answer

3 Problem 3: an equivalence relation and a partial order? DLN 8.155

replace with your answer