

## Project Euler - Exercise 25

The Fibonacci Sequence is given by  $f_n = f_{n-1} + f_{n-2}$  where  $f_0 = f_1 = 1$ . If we define the linear transformation above

$$T = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow T \begin{bmatrix} f_{n-2} \\ f_{n-1} \end{bmatrix} = \begin{bmatrix} f_{n-1} \\ f_{n-1} + f_{n-2} \end{bmatrix} = \begin{bmatrix} f_{n-1} \\ f_n \end{bmatrix}$$

Then, if we want the  $n$ 'th term of the sequence, we just have to compute:

$$T^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} f_{n-1} \\ f_n \end{bmatrix}$$

By the Spectral Theorem, since  $T$  is a symmetric matrix, we can decompose on it's eigenvalues and eigenvectors: