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Matrícula: 21.1.4111

- Algoritmo de Kruskal:

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procedure kruskal( $G, w$ )
Input:    A connected undirected graph  $G = (V, E)$  with edge weights  $w_e$ 
Output:   A minimum spanning tree defined by the edges  $X$ 

for all  $u \in V$ :
    makeset( $u$ )

 $X = \{\}$ 
Sort the edges  $E$  by weight
for all edges  $\{u, v\} \in E$ , in increasing order of weight:
    if find( $u$ )  $\neq$  find( $v$ ):
        add edge  $\{u, v\}$  to  $X$ 
        union( $u, v$ )

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procedure union( $x, y$ )
 $r_x = \text{find}(x)$ 
 $r_y = \text{find}(y)$ 
if  $r_x = r_y$ : return
if rank( $r_x$ ) > rank( $r_y$ ):
     $\pi(r_y) = r_x$ 
else:
     $\pi(r_x) = r_y$ 
    if rank( $r_x$ ) = rank( $r_y$ ): rank( $r_y$ ) = rank( $r_y$ ) + 1

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makeset -> representação de conjuntos disjuntos: (nó, rank, pai)

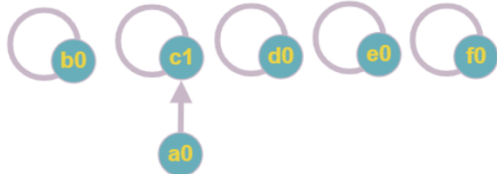
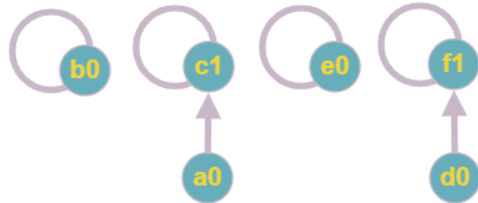
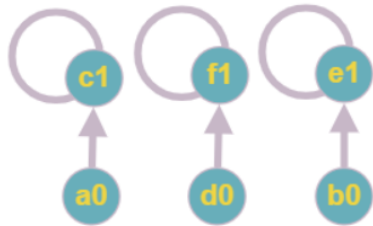
(a, 0, a), (b, 0, b), (c, 0, c), (d, 0, d), (e, 0, e), (f, 0, f)

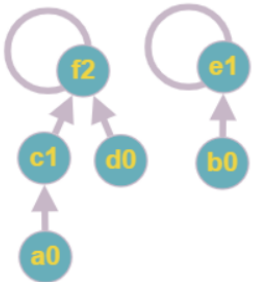
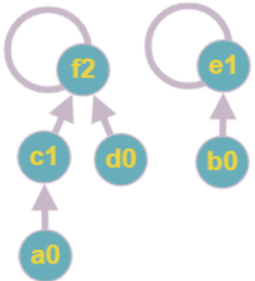
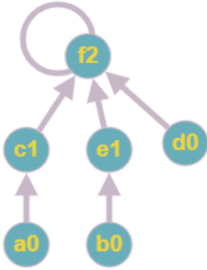


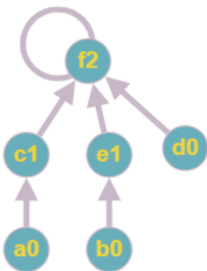
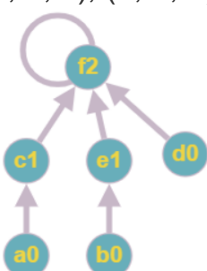
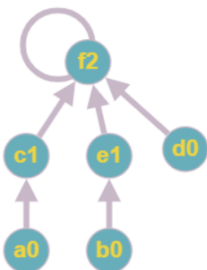
$X = \{\}$

arestas ordenadas -> $\{\{a, c\}, \{d, f\}, \{b, e\}, \{c, f\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b\}, \{c, e\}, \{e, f\}\}$

Aresta	Execução
$\{a, c\}$	representante de a = a representante de c = c representantes diferentes, faz a união: $X \leftarrow X \cup \{a, c\}$ $\text{rank}(a) = \text{rank}(c)$

	<p> $\text{pai}(a) \leftarrow c$ $\text{rank}(c) \leftarrow \text{rank}(c) + 1$ </p> <p> $(a, 0, c), (b, 0, b), (c, 1, c), (d, 0, d), (e, 0, e), (f, 0, f)$ </p>  <p> $X = \{\{a, c\}\}$ </p>
{d,f}	<p> representante de $d = d$ representante de $f = f$ representantes diferentes, faz a união: $X \leftarrow X \cup \{d, f\}$ $\text{rank}(d) = \text{rank}(f)$ $\text{pai}(d) \leftarrow f$ $\text{rank}(f) \leftarrow \text{rank}(f) + 1$ </p> <p> $(a, 0, c), (b, 0, b), (c, 1, c), (d, 0, f), (e, 0, e), (f, 1, f)$ </p>  <p> $X = \{\{a, c\}, \{d, f\}\}$ </p>
{b,e}	<p> representante de $b = b$ representante de $e = e$ representantes diferentes, faz a união: $X \leftarrow X \cup \{b, e\}$ $\text{rank}(b) = \text{rank}(e)$ $\text{pai}(b) \leftarrow e$ $\text{rank}(e) \leftarrow \text{rank}(e) + 1$ </p> <p> $(a, 0, c), (b, 0, e), (c, 1, c), (d, 0, f), (e, 1, e), (f, 1, f)$ </p>  <p> $X = \{\{a, c\}, \{d, f\}, \{b, e\}\}$ </p>
{c,f}	<p> representante de $c = c$ representante de $f = f$ representantes diferentes, faz a união: $X \leftarrow X \cup \{c, f\}$ </p>

	<p> $\text{rank}(c) = \text{rank}(f)$ $\text{pai}(c) \leftarrow f$ $\text{rank}(f) \leftarrow \text{rank}(f) + 1$ </p> <p> $(a, 0, c), (b, 0, e), (c, 1, f), (d, 0, f), (e, 1, e), (f, 2, f)$ </p>  <p> $X = \{\{a, c\}, \{d, f\}, \{b, e\}, \{c, f\}\}$ </p>
{a,d}	<p> representante de $a = f$ representante de $d = f$ representantes iguais, não faz a união </p> <p> $(a, 0, c), (b, 0, e), (c, 1, f), (d, 0, f), (e, 1, e), (f, 2, f)$ </p>  <p> $X = \{\{a, c\}, \{d, f\}, \{b, e\}, \{c, f\}\}$ </p>
{b,c}	<p> representante de $b = e$ representante de $c = f$ representantes diferentes, faz a união: $X \leftarrow X \cup \{b, c\}$ $\text{rank}(e) < \text{rank}(f)$ $\text{pai}(e) \leftarrow f$ </p> <p> $(a, 0, c), (b, 0, e), (c, 1, f), (d, 0, f), (e, 1, f), (f, 2, f)$ </p>  <p> $X = \{\{a, c\}, \{d, f\}, \{b, e\}, \{c, f\}, \{b, c\}\}$ </p>

<p>{c,d}</p>	<p>representante de c = f representante de d = f representantes iguais, não faz a união</p> <p>(a, 0, c), (b, 0, e), (c, 1, f), (d, 0, f), (e, 1, f), (f, 2, f)</p>  <p>$X = \{\{a,c\}, \{d,f\}, \{b,e\}, \{c,f\}, \{b,c\}\}$</p>
<p>{a,b}</p>	<p>representante de a = f representante de b = f representantes iguais, não faz a união</p> <p>(a, 0, c), (b, 0, e), (c, 1, f), (d, 0, f), (e, 1, f), (f, 2, f)</p>  <p>$X = \{\{a,c\}, \{d,f\}, \{b,e\}, \{c,f\}, \{b,c\}\}$</p>
<p>{c,e}</p>	<p>representante de c = f representante de e = f representantes iguais, não faz a união</p> <p>(a, 0, c), (b, 0, e), (c, 1, f), (d, 0, f), (e, 1, f), (f, 2, f)</p>  <p>$X = \{\{a,c\}, \{d,f\}, \{b,e\}, \{c,f\}, \{b,c\}\}$</p>
<p>{e,f}</p>	<p>representante de e = f representante de f = f</p>



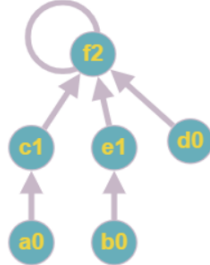
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BCC241 - Projeto e Análise de Algoritmos
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representantes iguais, não faz a união

(a, 0, c), (b, 0, e), (c, 1, f), (d, 0, f), (e, 1, f), (f, 2, f)



$X = \{\{a, c\}, \{d, f\}, \{b, e\}, \{c, f\}, \{b, c\}\}$

- Algoritmo de Prim

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procedure prim( $G, w$ )
Input:    A connected undirected graph  $G = (V, E)$  with edge weights  $w_e$ 
Output:   A minimum spanning tree defined by the array prev

for all  $u \in V$ :
    cost( $u$ ) =  $\infty$ 
    prev( $u$ ) = nil
Pick any initial node  $u_0$ 
cost( $u_0$ ) = 0

 $H = \text{makequeue}(V)$     (priority queue, using cost-values as keys)
while  $H$  is not empty:
     $v = \text{deletemin}(H)$ 
    for each  $\{v, z\} \in E$ :
        if cost( $z$ ) >  $w(v, z)$ :
            cost( $z$ ) =  $w(v, z)$ 
            prev( $z$ ) =  $v$ 
            decreasekey( $H, z$ )

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Set S	a	b	c	d	e	f
$\{\}$	0/nil	∞ /nil	∞ /nil	∞ /nil	∞ /nil	∞ /nil
$\{a\}$		6/a	1/a	5/a	∞ /nil	∞ /nil
$\{a, c\}$		5/c		5/a	6/c	4/c
$\{a, c, f\}$		5/c		2/f	6/c	
$\{a, c, f, d\}$		5/c			6/c	
$\{a, c, f, d, b\}$					3/b	
$\{a, c, f, d, b, e\}$						