AULA 15

$\underset{\sim}{X} = \{X_1 \ldots X_n\}$, $f(\underset{\sim}{x}|\theta)$

$g(\theta)$

SIMPLES

$H_0 : g(\theta) = g_0$

$H_1 : g(\theta) \neq g_0$

$\gamma = 1 - \alpha_0$, $\alpha_0 \in (0,1)$

$\underline{w(\underset{\sim}{x})} := \{ g_0 : \delta_{g_0}$ NÃO REJEITA H_0 SE $\underset{\sim}{X} = \underset{\sim}{x} \}$

$\theta_0 \in \Omega$

$\Pr\{ g(\theta_0) \in w(\underset{\sim}{X}) \mid \theta = \theta_0 \} \geq \gamma$

$1 - \alpha_0$

PROVA (TEOREMA 9.1.1 - DE GROOT

(i) $\theta_0 \in \Omega$: $g(\theta_0) = g_0$

δ_{g_0} É UM TESTE DE TAMANHO α_0.

$$\left[\Pr(T \in R) \Longleftrightarrow \Pr(\theta \in \Omega_0) \right]$$

$\Pr(\delta_0 \text{ não rejeitar } H_0 \mid \theta = \theta_0)$

$1 - \Pr(\delta_0 \text{ rejeitar } H_0 \mid \theta = \theta_0)$

$= 1 - \alpha_0 = \gamma$

$\forall \underline{x} \quad g(\theta_0) \in W(\underline{x})$

$\Longleftrightarrow$

δ_{g_0} não rejeita H_0

$\Pr\{\delta_{g_0} \text{ não rejeitar } H_0 \mid \theta = \theta_0\}$

$= \Pr\{g(\theta_0) \in W(\underline{X}) \mid \theta = \theta_0\}$

g_0

$\geq \gamma$

Testes $\Leftrightarrow$ IC

Ex: $\quad X_1 \ldots X_n \sim N(\mu, \sigma^2)$

$\theta = (\mu, \sigma^2)$ $\quad \Omega = (-\infty, \infty) \times (0, \infty)$

$\mu \qquad \times \sigma^2$

- $g(\theta) = \mu$
- $W(\underline{x}) = (a(\underline{x}), b(\underline{x}))$

$I(\underline{X}) = (A(\underline{X}), B(\underline{X}))$

$g(\mu_0) \in W(\underline{x})$

$\Rightarrow$ Ñ REJEITO H_0

$\mu_0 \in (\underline{a(\underline{x})}, \underline{b(\underline{x})})$

$\mu \in (7.1, \ 10.2)$

$\cancel{\mu_0 = 7 \text{ mg/L}}$

$H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$

$\gamma = 1 - \alpha_0$

$0.95 \Rightarrow \alpha_0 = \frac{1 - \alpha}{2} = 0.05$

$(a, b) \leftrightarrow \gamma = 0.95$

(a', b') $\gamma' = 0.99$

$a' < a, \ b > b$

Ho é VDD

$P_r\{\, g(\theta_0) \in W(\underline{X}) \mid \theta = \theta_0 \,\} \geq \gamma$

$\Rightarrow W(\underline{X})$ é um

~~CONJUNTO DE~~

(CONFIDENCE SET) ~~CONFIANÇA~~ DE

NÍVEL γ PARA

$\theta = (\mu, \sigma^2) \qquad g(\theta)$

<u>EXEMPLO</u>:

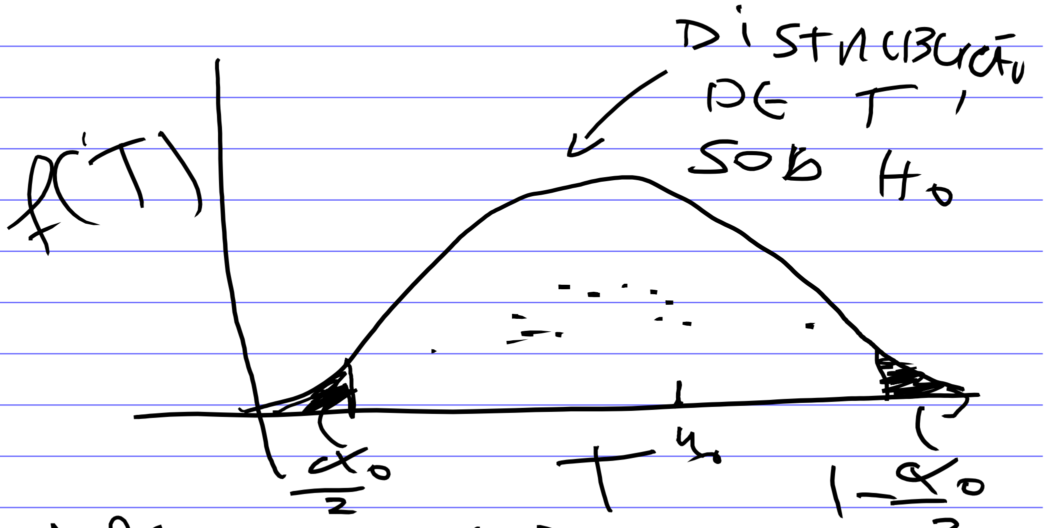
$$\delta_{\mu_0} \begin{cases} \underbrace{|\bar{X}_n - \mu_0|}_{T} \underbrace{\geq}_{\geq} \underbrace{\Phi^{-1}\left(1 - \frac{\alpha_0}{2}\right)\frac{\hat{\sigma}}{\sqrt{n}}}_{c} & \leftarrow \text{REJEITA Ho} \\ T < c \Rightarrow \text{N REJEITA Ho} \end{cases}$$

$T \geq c$

$$c = \Phi^{-1}\left(1 - \frac{\alpha_0}{2}\right)\frac{\sigma}{\sqrt{n}}$$

$\cancel{X_i} \sim N(\mu, \sigma^2)$ ← CONHECIDA

$H_0 : \mu = \mu_0$



AREA CAUDA

BICAUDAL

α_0

$$|\bar{X}_n - \mu_0| \geq c \qquad \frac{\hat{\sigma}^2}{n} \quad \sqrt{\ }$$

$$\Longleftrightarrow$$

$$\bar{X}_n \pm \underbrace{\Phi^{-1}\left(1-\frac{\alpha_0}{2}\right)\frac{\hat{\sigma}}{\sqrt{n}}}_{c}$$

$$\Longleftrightarrow$$

$$\bar{X}_n - c < \mu < \bar{X}_n + c$$

$A(X) \qquad\qquad B(X)$

$H_0: \mu = \mu_0$

$$\mu_0 \in (a(x), b(x))$$

Ñ REJEITO H_0

$\underline{\mu_0} \notin (a(\underline{x}), b(\underline{x}))$

REJEITO H_0

$\gamma \Rightarrow \alpha_0 = 1 - \gamma$

EXEMPLO

$X_i \sim N(\mu, \sigma^2)$

?

SIMPLES

$H_0: \mu = \mu_0$

COMPOSTA

$H_1: \mu \neq \mu_0$

$T = F^{-1}\left(\frac{1+\gamma}{2}; n-1\right)$

- $A(\underline{X}) = \bar{X}_n - T \frac{\hat{\sigma}'}{\sqrt{n}}$ $\quad O\left(\frac{1}{\sqrt{n}}\right)$
- $B(\underline{X}) = \bar{X}_n + T \frac{\hat{\sigma}'}{\sqrt{n}}$
- $I(X) = (A(X), B(X))$

$\hat{\sigma}' = \sqrt{\left(\frac{\sum (X_i - \bar{X}_n)^2}{n-1} \right)}$

ESTIMADOR PARA σ (DESVIO PADRÃO)

$\underset{\sim}{X} = \underline{x}$

$\mu_0 \in (a(\underline{x}), b(\underline{x}))$

$\rightarrow \overline{N}$ REJEITO

H_0

$\mu_0 \notin (a(x), b(x))$

$\rightarrow$ REJEITAR H_0

$\_\_\_\_\_\_ (\ , \_\_\_\_\_\_$

<u>OBS</u>

TESTE $\begin{cases} \text{BILATERAL} \\ \text{UNILATERAL} \end{cases}$

IC $\begin{cases} \text{BILATERAL} \\ \text{UNILATERAL} \end{cases}$

$Pr\left(\mu \in (-\infty, A(\underset{\sim}{X}))\right) = \gamma$

$(-\infty, A(\underline{x}))$ É UM

I.C. <u>EXATO</u> DE

NÍVEL γ.

$$\left(\bar{X}_n - T(\gamma)\frac{\hat{\sigma}'}{\sqrt{n}},\ \infty\right)$$

$$A(\underline{X})$$

$H_0: \mu \leq \mu_0$

$H_1: \mu > \mu_0$

RAZÃO DE VEROSSIMILHANÇA

- $H_0: \theta \in \Lambda_0$
- $H_1: \theta \in \Lambda_1$

ESTATÍSTICA $\Lambda_1 = \Lambda \setminus \Lambda_0$

$$\Lambda(\underline{x}) := \frac{\sup_{\theta \in \Lambda_0} f_n(\underline{x}|\theta)}{\sup_{\theta \in \Lambda_1} f_n(\underline{x}|\theta)}$$

R.V.

L.R.

$\underline{X} = \underline{x}$

Λ

δ_K: REJEITA H_0

SE

$$\Lambda(\underset{\sim}{x}) \leq K$$

SE $\Lambda(\underset{\sim}{x}) > K$

ENTÃO NÃO REJEITA

H_0

δ_K É UM TESTE DE RAZÃO DE VEROSSIMILHANÇA

δ_K TEM TAMANHO α_0

$K(\alpha_0)$

EXEMPLO

$X_i \sim \text{BERNOULLI}(p)$

$Y = \sum_i^n X_i \sim \text{BIN}(n, p)$

$Pr(Y = y) = \binom{n}{y} p^y (1-p)^{n-y}$

$f_n(\underline{x} | p) = f_n(y | p)$

HIPÓTESE

$H_0 : p = p_0$

$H_1 : p \neq p_0$

$\Lambda_0 = \{p_0\}$

$\Lambda_1 = (0,1) \setminus p_0$

$$\Lambda(\underline{x}) = \frac{\sup_{p \in \Lambda_0} f_n(\underline{x} | p)}{\sup_{p \in \Lambda_1} f_n(\underline{x} | p)}$$

$$\Lambda(\underline{x}) = \frac{\binom{n}{y} p_0^{y}(1-p_0)^{n-y}}{\sup_{p\in\Lambda_1} \binom{n}{y} p^{y}(1-p)^{n-y}}$$

$$\Lambda(\underline{x}) = \frac{\cancel{\binom{n}{y}} p_0^{y}(1-p_0)^{n-y}}{\cancel{\binom{n}{y}} \hat{p}^{y}(1-\hat{p})^{n-y}}$$

$$\hat{p} = \frac{y}{n} \quad \boxed{EMV}!$$

$$\Lambda(\underline{y}) = \left(\frac{np_0}{y}\right)^{y}\left(\frac{n(1-p_0)}{n-y}\right)^{n-y}$$

$$r(X)$$

$$\Lambda(r(\underline{X}))$$

$$Pr(Y = y \mid P = p_0)$$

$$= \binom{n}{y} p_0^{y} (1-p_0)^{n-y}$$

EXEMPLO 9.1.18 DE GROOT

RELEMBRANDO (09/22/20)

$$\Lambda(\underline{X}) = \frac{\sup_{\theta \in \Omega_0} f_n(\underline{X} \mid \theta)}{\sup_{\theta \in \Omega_1} f_n(\underline{X} \mid \theta)}$$

$\Omega_1 = \Omega \setminus \Omega_0$

RAZÃO DE VEROSSIMILHANÇAS

$H_0 : \theta \in \Omega_0$

$H_1 : \theta \in \Omega_1$

δ_c^{RV} : $\Lambda(\underset{\sim}{x}) \leq c$

$\Rightarrow$ REJEITO H_0

$\Lambda(\underset{\sim}{x}) > c \Rightarrow$ NÃO REJEITO H_0

RESULTADO ASSINTÓTICO

Λ EUCLIDEANO

$\underset{\sim}{\theta} = (\theta_1, \theta_2, \ldots \theta_k)$

$H_0: \theta_1 = \theta_0^{(1)}$

Ex: $H_0: \theta_1 = \theta_0^{(1)}$
$\theta_2 = \theta_0^{(2)}$

$\underset{\sim}{\theta} = (\mu, \sigma^2)$. $\Lambda \subseteq \mathbb{R}^2$
$(-\infty, \infty) \times (0, \infty)$

$H_0: \mu = \mu_0$.
$\sigma^2 = \sigma_0^2$.

SIMPLES

ASSUMINDO H_0 VERDADEIRA

$$-2\log \Lambda(X) \xrightarrow[d]{n\to\infty} \chi^2(K)$$

Ex: $\rho = \rho_0$ (H_0)

$\rho \neq \rho_0$ (H_1)

$K = 1$

$\theta = \rho$

$$-2\log \Lambda(x) \xrightarrow[d]{n\to\infty} \chi^2(1)$$

$n \gg 30$

p-VALOR

DE GROOT, Thm 9.1.4

WILKS, 1938

TESTE NÃO-VIESADO

δ É NÃO-VIESADO

SE $\theta \in \Lambda_0$ E $\theta' \in \Lambda_1$

VALE

$$\pi(\theta|\delta) \leq \pi(\theta'|\delta) \quad \forall \theta, \theta'$$

$$\pi(\theta|\delta) = Pr(\text{REJEITAR } H_0 | \theta)$$

$$\delta_c : T \geq c$$

$$\pi(\theta|\delta_c) = Pr(T \geq c | \theta)$$

TESTE t (DE STUDENT)

DE GROOT SEÇÃO 9.5

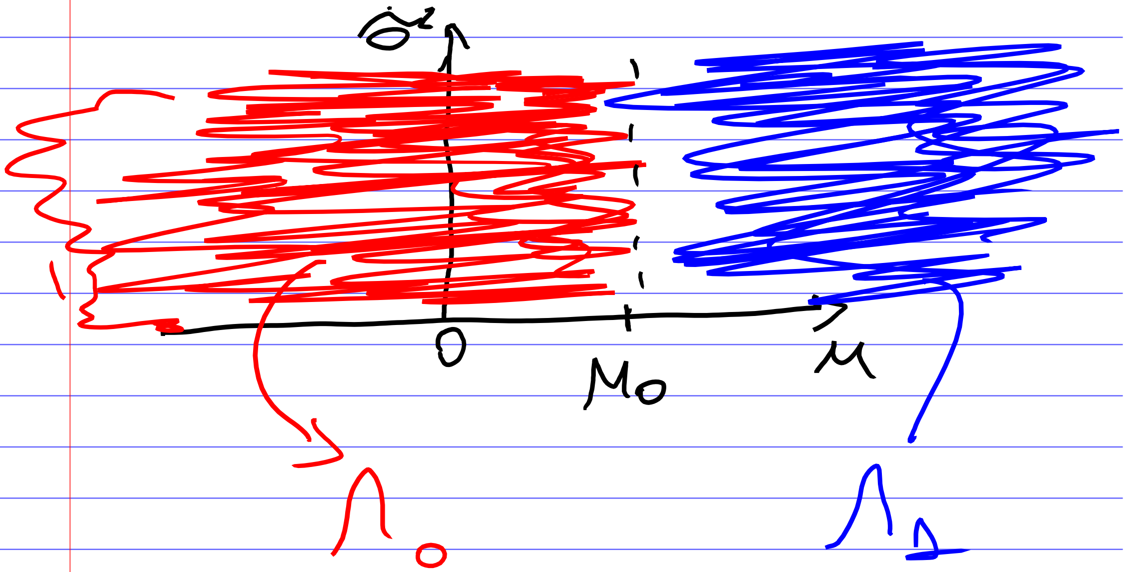
$$X_i \sim N(\mu, \sigma^2) \quad \mu = ? \quad \sigma^2 = ?$$

$$H_0 : \mu \leq \mu_0$$

$$H_1 : \mu > \mu_0$$

$$\Theta = (\mu, \sigma^2)$$

$$\Theta \in (-\infty, \infty) \times (0, \infty)$$



- $\bar{X}_n = \frac{1}{n} \sum X_i$
- $\hat{\sigma}' = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X}_n)^2}$

$$U := \frac{\sqrt{n}\,(\bar{X}_n - \mu_0)}{\hat{\sigma}'}$$

$$(A(\underline{X}),\ B(\underline{X}))$$

$\mu_0 \in$ $\quad P_r(A(X) < \underline{\mu_0} < B(X))$

δ_c : se $U \geq c$

REJEITO H_0,

se $U < c$

NÃO REJEITO

H_0

$c(\alpha_0)$

Thm (DeGroot 9.7.1)

$\pi(\theta|\delta) = Pr(\boxed{U \geq c} | \theta)$

REJEITAR H_0

$\theta = (\mu, \sigma^2)$

(i) $\pi(\mu, \sigma^2 | \delta_c) = \underline{\alpha_0}$ and $\mu = \mu_0$

(ii) $\pi(\mu, \sigma^2 | \delta_c) < \alpha_0$ and $\underline{\mu < \mu_0}$

(iii) $\pi(\mu, \sigma^2 | \delta_c) > \alpha_0$ and $\mu > \mu_0$

(iv) $\lim_{\mu \to -\infty} \pi(\mu, \sigma^2 | \delta_c) = 0$

(v) $\lim_{\mu \to \infty} \pi(\mu, \sigma^2 | \delta_c) = 1$

$$U = \frac{\sqrt{n}\,(\bar{X}_n - \mu_0)}{\hat{\sigma}'}$$

$\bar{X}_n \to -\infty \quad (n >> n_0)$

$U \to -\infty$

$\Pr(U \geq c \mid \theta) = 0$

$\mu \to -\infty$

$\Pr(U \geq c \mid \theta) = 1$, $\theta = (\mu, \hat{\sigma}^2)$

$\mu \to \infty$

PROVA (i) PIVOTAL

$$U := \frac{\sqrt{n}\,(\bar{X}_n - \mu)}{\hat{\sigma}'}$$

$$W := \frac{\sqrt{n}\,(\mu_0 - \mu)}{\hat{\sigma}'}$$

$\mu = \mu_0 \quad W \in 0$ Q.C.

$$U = U^{\circ} - W$$

$$\pi(\theta \mid \delta_c) = Pr(U \geq c \mid \theta)$$

$$= Pr(U^{\circ} - W \geq c \mid \theta)$$

$$= Pr(U^{\circ} \geq W + c \mid \theta)$$

$$U^{\circ} \sim T(n-1)$$

$$\to Pr(U^{\circ} \geq W + c \mid \theta)$$

$$< Pr(U^{\circ} \geq c \mid \theta) = \alpha_0$$

$$\mu = \mu_0$$

$$\to Pr(U^{\circ} \geq W + c \mid \theta) < \alpha_0$$

$$Pr(U \geq c \mid \theta) < \boxed{\alpha_0}$$

$$\mu < \mu_0$$

$$\mu > \mu_0 \Rightarrow Pr(U \geq c \mid \theta)$$

$$> \alpha_0$$

$$\to \pi(\mu, \sigma^2 \mid \delta_c) > \alpha_0$$

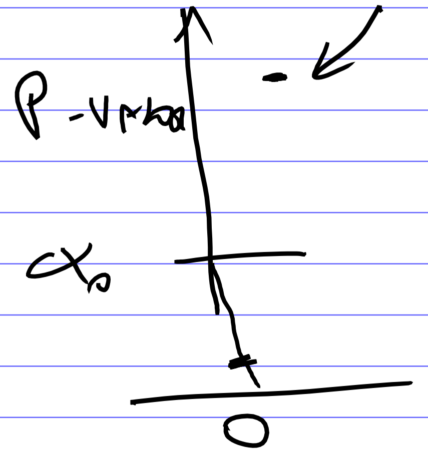
P-VALOR

$$Pr(T \geq t ; H_0) = p$$

$\underset{\sim}{X} = \underline{x}$ $\qquad \boxed{t = r(\underline{x})}$

$T = r(\underset{\sim}{X})$

$p > \alpha_0$



MAIS COMPATÍVEIS SÃO OS DADOS COM H_0

$Pr($ OBSERVAR t TÃO EXTREMA $) \ll \alpha_0$

$\Rightarrow$ DADOS SÃO POUCO COMPATÍVEIS C/ H_0

$\alpha_0 = 0.05$

$\frac{1}{20}$

- $p = 0.048$
- $p < 0.0000$

$\alpha_0 = 0.001$