AULA 6

ESTIMADOR DE
MÁXIMA
VEROSSIMILHANÇA

$$X_1, \dots X_n \sim f(\theta)$$

$$X = x \qquad f_n(x|\theta)$$

$\theta \in \Omega$

MAXIMIZE

$$P_\theta(\underset{\sim}{X} = \underset{\sim}{x})$$

- $EM\ r$ (EXPONENTIAL)

$X_i \sim EXPO(\theta)$ IID

$f(x_i|\theta) = \theta \exp(-\theta x_i)$

A.D.P.

$$f_n(\underset{\sim}{x}|\theta) = \theta^n \exp(-S\theta)$$

$$S = \sum_{i=1}^{n} x_i \qquad \theta \in \mathbb{R}^+$$

$$\hat{\theta}_{EMV}$$

$$\max_{\theta \in \mathbb{R}^+} f_n(\underset{\sim}{x}|\theta)$$

$$l(\theta) \propto \theta^n e^{-S\theta}$$

$\log L(\theta) =: l(\theta)$

$\hat{\theta}_{\max}$ → $\log$ ↑ μονοτονία

$$l(\theta) = n \log\theta - \frac{S}{\theta}$$

$$\frac{dl}{d\theta} = \frac{n}{\theta} - S = 0$$

$$\frac{d^2 l}{d\theta^2} = -\frac{n}{\theta^2} < 0$$ ↙ l κοίλη

$$\frac{d\ell}{d\theta} = \frac{\theta}{n} - S = 0$$

$$\hat{\theta} = \frac{n}{S} = \frac{1}{\bar{x}_n}$$

$$\hat{\theta}_{MLE} = \frac{1}{\bar{x}_n}$$