AULA 4

PRIORIS CONJUGADAS

$X_1, \ldots, X_n$

$X_i \sim B(p)$

$$f(\underline{x}|\underline{\theta}) = \begin{cases} \theta, & x_i = 1 \\ 1-\theta, & x_i = 0 \end{cases}$$

$$\prod_{i}^{n} f(x_i|\theta) = \underline{\theta^y (1-\theta)^{n-y}}$$

$1 < 1$

$$f(\theta) \propto \theta^{\alpha-1}(1-\theta)^{\beta-1}$$

$$f(\theta|x) \propto \theta^{y+\alpha-1}(1-\theta)^{(n-y)+\beta-1}$$

COMPLETANDO O QUADRADO

VAMOS TRANSFORMAR

ax^2+bx+c

EM

$$a(x-h)^2+k$$

$$a(x^2-2hx-h^2)+k$$

$$ax^2-2hax-h^2+k$$

$$ax^2+bx+c$$

- $h = -\frac{b}{2a}$

- $k = c + h^2 = c + \frac{b^2}{4a}$

APLICAÇÃO:
DISTRIBUIÇÃO
NORMAL

$$\sum_{i=1}^{n} (x_i - \theta)^2 =$$

$$\sum_{i=1}^{n}(x_i^2 - 2\theta x_i + \theta^2)$$

$$= \sum_{i=1}^{n} x_i^2 - 2\theta\sum_{i=1}^{n} x_i + n\theta^2$$

POLINÔMIO

EM θ

$$n\theta^2 - 2S\theta + S_2$$

$$(\theta - \quad)^2$$

$$n(\theta - h) + K$$

CTE

C.R.

θ

$$h = -\frac{b}{2a}$$

$$\cancel{-\,-}\ {}^{+}\ \frac{\cancel{2}S}{\cancel{2}n} = h$$

$$n\left(\theta - \boxed{\frac{S}{n}}\right)^2 + K$$

$$\bar{x}_n \equiv x_n$$

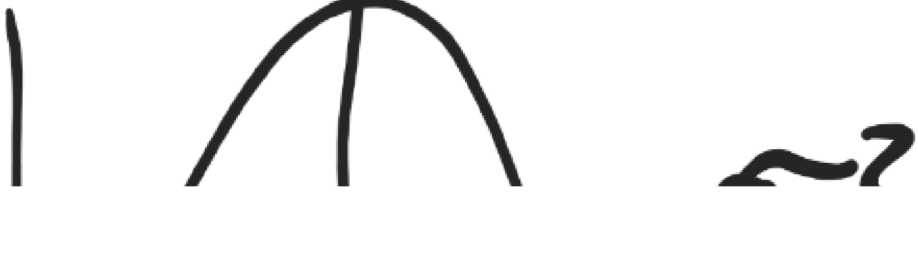
$$n(\theta - \bar{x}_n)^2 \boxed{+ K}$$

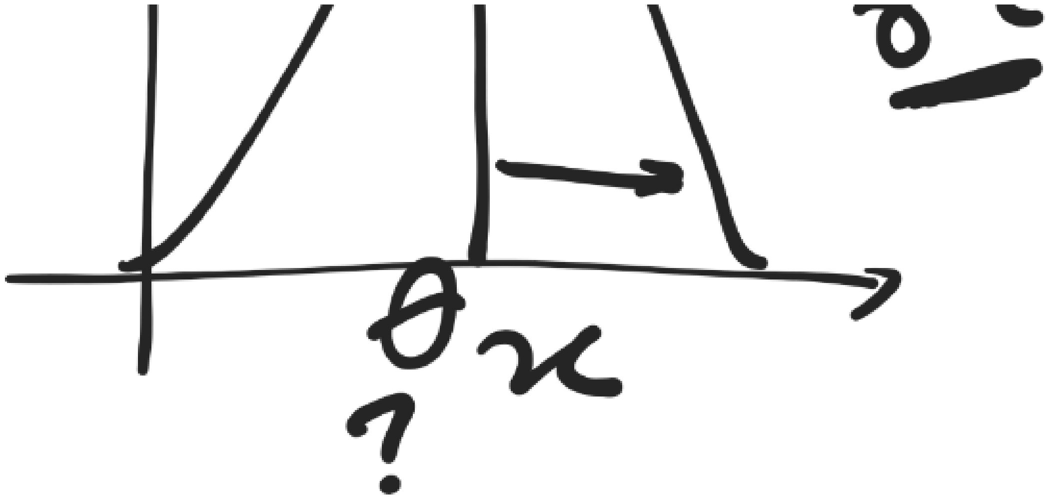
NÃO DEPENDE DE θ

$$L(\theta) \propto \exp\left(-n(\theta - \bar{x}_n)^2\right)$$

$$\xi(\theta) \propto \exp\left(-\frac{(\theta - \mu_0)^2}{2V_0^2}\right)$$

$$-\left(n(\theta - \bar{x}_n)^2 + (\theta - \mu_0)^2\right)$$





$$\theta \sim N(\mu_0, V_0^2)$$

VARIANCIA DE θ

$$E[L(\theta, a) \mid \underset{\sim}{x}] =$$

$$\int_{\Lambda} L(\theta, a)\, \xi(\theta \mid \underline{x})\, d\theta$$

$$= \int_{\Lambda} L(\theta, a)\, f(\underline{x} \mid \theta)\, \xi(\theta)\, d\theta$$

$$\delta(\underline{x}) = E[\theta \mid \underline{x})$$

$$\int_{\Lambda} \theta\, \xi(\theta \mid \underline{x})\, d\theta$$

/ l

$$E_{\theta|x}\left[L(\theta,\delta^*)\right] =$$

$$E_{\theta|x}\left[(\theta-\delta^*)^2\right] =$$

$$E_{\theta|x}\left[\theta^2 - 2\theta\delta^* - (\delta^*)^2\right] =$$

$$\underline{E[\theta^2] - 2\delta^* E[\theta] + (\delta^*)^2}$$

POLINÔMIO EM δ^*

$$d = 2\delta^* - 2E[\theta] = 0$$

$\partial\delta$

$$\delta^* = E[\theta] \quad //$$

PERDA ABSOLUTA

$$\underset{\theta|\underline{x}}{E}\left[\,|\theta - \delta^*|\,\right] =$$

$$\int_{\Omega} |\theta - \delta|\, f(\theta|\underline{x})\, d\theta$$

$$= \int^{\delta} (\delta - \theta)\, f(\theta|\underline{x})\, d\theta \; +$$

$-\infty$ $\qquad\qquad I_1$

$$\int_{\delta}^{\infty} (\theta-\delta) f(\theta|x)\, d\theta$$

I_2

LEIBNITZ

$$\phi(a) = \int_{m}^{M} f(x,a)\, dx$$

$$\frac{d\phi(a)}{da} = \int_{m}^{M} \frac{d f(a,x)}{da}\, dx$$

$$+ f(M,a)\boxed{\frac{dM}{da}}$$

$$- f(m,a)\boxed{\frac{dm}{da}}$$

$$E[L(\theta,\delta)|\underline{x}] = I_1 + I_2$$

$$\frac{d\, E[L(\theta,\delta)|\underline{x}]}{d\,\delta}$$

$$= \frac{d I_1}{d\delta} + \frac{d I_2}{d\delta}$$

$$+ \int^{\delta} (\ldots)$$

$$\mathcal{L}_1 = \int_{-\infty} (\delta - \theta)\, \underbrace{f(\theta \mid x)\, d\theta}_{dF}$$

$$\frac{d\mathcal{L}_1}{d\delta} = \int_{-\infty}^{\delta} 1 \; \overset{\delta-\theta}{dF} + \underbrace{(\delta - \delta)\cdot 1}_{0}$$

$$= \int_{-\infty}^{\delta} 1 \, dF$$

$$\frac{d\mathcal{L}_2}{d\delta} = -\int_{\delta}^{\infty} 1 \, dF$$

$$\frac{d\,E[L(\theta,\delta^{\cdot})\,|\,\underline{x}]}{d\,\delta^{\cdot}} =$$

$$\int_{-\infty}^{\delta^{\cdot}} 1\,dF - \int_{\delta^{\cdot}}^{\infty} 1\,dF$$

$$= 0$$

$$\int_{-\infty}^{\delta^{\cdot}} 1\,dF = \int_{\delta^{\cdot}}^{\infty} 1\,dF$$

$$\int^{\delta^{\cdot}} f(\theta|x)\,d\theta \quad \int^{\infty} f(\theta|x \ldots$$

$$\int_{-\infty} E(\theta \mid x)\, d\theta = \int_{\delta} \ldots$$

$\Rightarrow$ δ É A MEDIANA