Chapter 6

1.

$F(z) = (c_1 + ic_2)(x + iy)(1 - x - iy)$

$= (xc_1 + i(xc_2 + yc_1) - yc_2) * (1 - x - iy)$

$= xc_1 + ixc_2 + ic_1y - yc_2 - x^2c_1 - i\,x^2c_2 - ic_1xy + xyc_2 - ixyc_1 + xyc_2 + c_1\,y^2 + iy^2c_2$

2.

Depend on Triangle inequality

$|z + w| < |z| + |w|$

If $|z| > \frac{1}{|c|} + 1$

Then $F(z) = cz(1 - z)$

Theorem. The Escape Criterion.

Suppose |z0| > max{|c|, 2}. Then |Q(z0)| →∞ as n → ∞. That is, the orbit of z0 escapes to infinity.

Proof. $|z_1| = |Q_{c1}(z_0) = |z_0^2 + c|$, so by the triangle inequality,

$$|z_1| = |z_0^2 + c| > |z_0|^2 - c$$

But since $|z_0|$ ≥ |c|, we have

$$|z_1| \geq |z_0|^2 - |z_0| = (|z_0| - 1)|z_0|.$$

Now since $|z_0|$ > 2, there must be a λ > 0 such that $|z_0|$ > 2 + λ, and so

|z0| − 1 > 1 + λ. Therefore

$$|z_1| \geq (1 + \lambda)|z_0|.$$

That means $|z_1|$ > $|z_0|$, and so Q moves points farther away. If we apply the same argument to z_1, and then to $|z_2|$, etc., we will find

$$|z_n| \geq (1 + \lambda)^n|z_0|.$$

Thus, as n → ∞, the orbit of $z_0$tends to infinity.

3.

a. c=i

b. c=2

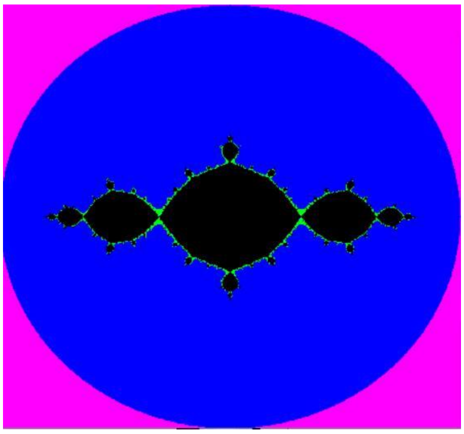
c. c=3.25

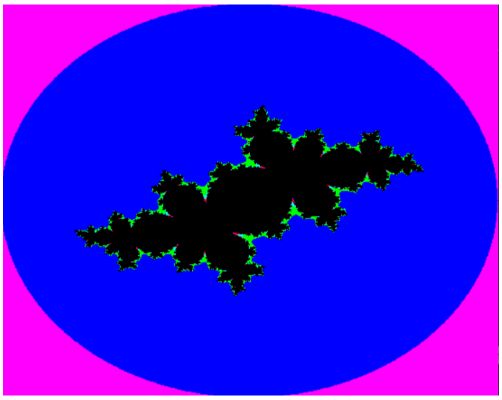
d. c=2-i

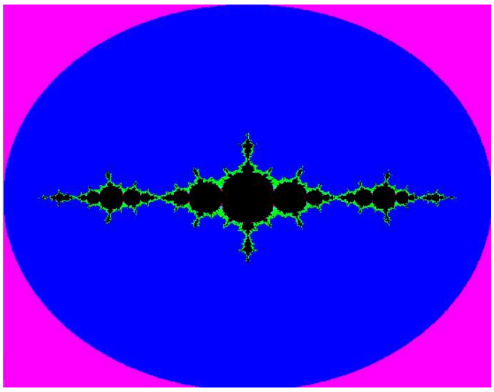
e. c=6+8i

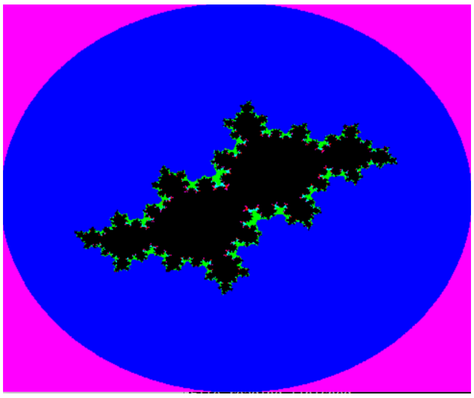
f. $\boldsymbol{c = \frac{5}{13} + \frac{12}{13}i}$

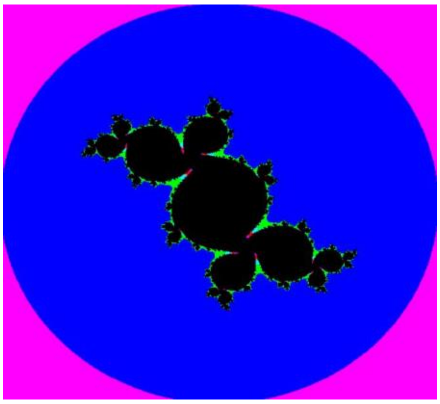
4.











5.

C = c1+ic2   z = x +iy

$z^3 = x^3 - 3xy^2 + 3ix^2y - iy^3$

$z^3 + c = x^3 - 3xy^2 + 3ix^2y - iy^3 + c1 + ic2$

Imaginary parts is $(3x^2y - y^3 + c_2)i$

6.

So this question just prove that $T_c(z) > 1$

If |z|>2 |c|<=2

Then the answer is obvious. Depend on Triangle inequality

$|z + w| < |z| + |w|$