

When i had a body...

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For this part, we will take $R = k$, where k is a field.

Definition 1. Now, for a preorder \mathbb{P} , take the set $\{C_1, \dots, C_n\}$ of connected components of the hydrography of V , and consider the set $\mathcal{F} = \{T_1, \dots, T_n\}$, where each T_i is a spanning tree of C_i (a *spanning tree* is a minimal tree that connects all the vertices). We call the extension of \mathcal{F} a *spanning forest* of \mathbb{P} .

Now, we start with an axiom:

Axiom of hydrodiversity. If \mathbb{T} is a tree (a poset in which every initial segment is totally ordered), then $\mathcal{B} = \{[a \text{---} b] : a \leq b \text{ is an edge in } \mathbb{T}\}$ is linearly independent.

Corollary 1. Let \mathbb{P} be a connected partial order with d vertices. Then \mathcal{B} is a basis for V . Thus, $\dim V = d - 1$.

We will briefly comment this result: this proves that given connected preorders \mathbb{P} and \mathbb{P}' with d vertices, their hydrospheres are isomorphic. We may think that, for a connection $a \sim b$ of vertices not in the preorder, we may consider $r = \min S_a \cap S_b$ to be the maximum "ancestor" of both, and consider " $a \sim b$ " = $[r \text{---} a] + [r \text{---} b]$. This shows that V encodes all possible connections of vertices - when \mathbb{P} is connected, of course.

Now, for a general preorder \mathbb{P} , consider V_i to be the subspace generated by the spanning tree T_i . We pose a conjecture:

Conjecture. For a preorder \mathbb{P} such as above, $V_i \cap V_j = 0$ if $i \neq j$.

If the above conjecture did hold, we would have $V = \bigoplus_i V_i$.