

# When i had a body...

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For this part, we will take  $R = k$ , where  $k$  is a field.

**Definition 1.** Now, for a preorder  $\mathbb{P}$ , take the set  $\{C_1, \dots, C_n\}$  of connected components of the hydrography of  $V$ , and consider the set  $\mathcal{F} = \{T_1, \dots, T_n\}$ , where each  $T_i$  is a spanning tree of  $C_i$  (a *spanning tree* is a minimal tree that connects all the vertices). We call the extension of  $\mathcal{F}$  a *spanning forest* of  $\mathbb{P}$ .

Now, we start with an axiom:

**Axiom of hydrodiversity.** If  $\mathbb{T}$  is a tree (a poset in which every initial segment is totally ordered), then  $\mathcal{B} = \{[a \text{---} b] : a \leq b \text{ is an edge in } \mathbb{T}\}$  is linearly independent.

**Corollary 1.** Let  $\mathbb{P}$  be a connected partial order with  $d$  vertices. Then  $\mathcal{B}$  is a basis for  $V$ . Thus,  $\dim V = d - 1$ .

We will briefly comment this result: this proves that given connected preorders  $\mathbb{P}$  and  $\mathbb{P}'$  with  $d$  vertices, their hydrospheres are isomorphic. We may think that, for a connection  $a \sim b$  of vertices not in the preorder, we may consider  $r = \min S_a \cap S_b$  to be the maximum "ancestor" of both, and consider " $a \sim b$ " =  $[r \text{---} a] + [r \text{---} b]$ . This shows that  $V$  encodes all possible connections of vertices - when  $\mathbb{P}$  is connected, of course.

Now, for a general preorder  $\mathbb{P}$ , consider  $V_i$  to be the subspace generated by the spanning tree  $T_i$ . We pose a conjecture:

**Conjecture.** For a preorder  $\mathbb{P}$  such as above,  $V_i \cap V_j = 0$  if  $i \neq j$ .

If the above conjecture did hold, we would have  $V = \bigoplus_i V_i$ .