When i had a body...

Caio HLL

For this part, we will take $R = k$, where k is a field.

Definition 1. Now, for a preorder $\mathbb{P}$, take the set $\{C_1, \ldots, C_n\}$ of connected components of the hydrography of V, and consider the set $\mathcal{F} = \{T_1, \ldots, T_n\}$, where each T_i is a spanning tree of C_i (a *spanning tree* is a minimal tree that connects all the vertices). We call the extension of $\mathcal{F}$ a *spanning forest* of $\mathbb{P}$.

Now, we start with an axiom:

Axiom of hydrodiversity. If $\mathbb{T}$ is a tree (a poset in which every initial segment is totally ordered), then $\mathcal{B} = \{[a—b] : a \leq b \text{ is an edge in } \mathbb{T}\}$ is linearly independent.

Corollary 1. Let $\mathbb{P}$ be a connected partial order with d vertices. Then $\mathcal{B}$ is a basis for V. Thus, $\dim V = d - 1$.

We will briefly comment this result: this proves that given connected preorders $\mathbb{P}$ and $\mathbb{P}'$ with d vertices, their hydrospheres are isomorphic. We may think that, for a connection $a \sim b$ of vertices not in the preorder, we may consider $r = \min S_a \cap S_b$ to be the maximum "ancestor" of both, and consider "$[a \sim b]$" $= [r—a] + [r—b]$. This shows that V encodes all possible connections of vertices - when $\mathbb{P}$ is connected, of course.

Now, for a general preorder $\mathbb{P}$, consider V_i to be the subspace generated by the spanning tree T_i. We pose a conjecture:

Conjecture. For a preorder $\mathbb{P}$ such as above, $V_i \cap V_j = 0$ if $i \neq j$.