Modeling and Super-Twisting Control with Quaternion Feedback for a 3-DOF Inertial Stabilization Platform

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Abstract— **The majority of works in line of sight (LOS) stabilization and tracking using inertially stabilized platforms (ISP) apply simple linear controllers to achieve the required performance. Commonly, linear models are employed to describe the relationship between torque and position of the ISP joints, such as a double integrator with an inertia gain. However, these techniques do not provide ideal disturbance rejection or finite-time convergence, which are desired characteristics for these type of systems in the context of high-accuracy applications.**

In this work, we propose a novel Sliding Mode Control (SMC) strategy for both stabilization and orientation tracking for a sensor in a 3-DOF ISP installed on a moving base. Two distinct cases are considered: full state feedback and output feedback only. In the latter case, a High-Order Sliding Mode observer (HOSMO) is proposed for the estimation of the ISP joint velocities. In each case, two Super Twisting Controllers (STC) are employed in a cascade topology. The inner controller ideally rejects the *dynamic* disturbances acting on the ISP joints, uncoupling the system into an ideal double integrator. The outer controller ensures orientation tracking in quaternion space, ideally rejecting all remaining kinematic disturbances.

Numerical simulations using Matlab show the efficiency and performance of the proposed controller and observer.

I. Introduction

Line-of-sight (LOS) stabilization is a common and challenging problem in the control literature. Inertially stabilized platforms (ISP) are widely used for payload stabilization and tracking applications, when a sensor must accurately point to a target in a dynamic environment. Some examples are cameras for aerial surveying and entertainment industry [1], long-range sensors on vehicles [2], military applications [3] and thermal cameras for oil spill detection [4].

ISPs are usually gimbaled structures mounted on a mobile base (vehicle) with motors equipped with encoders/tachometers and a payload fixed on the last gimbal. Gyroscopes or inertial navigation systems (INS) are employed to close the control loop by either measuring the vehicle motion (*indirect stabilization*) or directly measuring the payload motion (*direct stabilization*) [5].

The latter is usually recommended for precision pointing applications, since the sensor location is appropriate for capturing other effects that can impact the measured angular rates, such as structure flexibility, resolvers, tachometer and/or encoder accuracy and processor sampling rate [5]. A possible drawback of this method is the larger size of the gimbals required for opposing the larger payload induced by the weight of the sensors in the inner gimbal. This drawback is usually absent in the indirect method, but since the disturbances are not measured in the LOS coordinate frame, control performance can be degraded.

It is common to find simple dynamic models describing this type of system in the literature. Usually, the joints are considered as decoupled and a double integrator with an inertia gain relates the joint torque and position. Unmodeled dynamics, such as the vehicle motion and cross-coupling effects, are treated as disturbances to be rejected.

The typical control topology is P-PI control. Usually, the inner PI velocity loop has a high bandwidth to stabilize the payload, attenuating the torque disturbances. The outer proportional orientation loop operates at a lower bandwidth and minimizes the pointing error [6], [7], [8]. However, in *high accuracy* and/or *fast dynamics* applications, unmodeled effects may add significant torque contributions, and simple linear controllers may not suffice for the required level of performance.

Some works have tackled the problem of LOS control for ISPs in a more detailed way. In [9], the effects of kinematic coupling of the base and gimbal imbalance are analyzed for a 2-DOF ISP, while [10] proposes a self-tunning PID-type fuzzy controller as an alternative to PID control used in the ISP internal stabilization loop. Recently, [11] developed an alternate adaptive control for an inertially stabilized payload with unknown inertia matrix, using the unit quaternion formalism and quaternion/angular velocity errors. In [12], unit quaternions are again used for attitude stabilization. Control is performed by means of a feedback linearization technique, taking partial advantage of the ISP Lagrangian model. In [13], it is shown that even in the presence of large parameter uncertainties, a computed-torque plus PID (CTPID) controller guarantees satisfactory performance.

Modern techniques using Sliding Mode Control (SMC) are being applied for ISP stabilization and tracking. Their attractive characteristics include: (i) exact rejection of bounded matched disturbances; (ii) finite-time convergence and (iii) ease of implementation. For example, in [14], a Non-singular Terminal Sliding Mode controller [15] is used to achieve finite-time stabilization of an ISP in the presence of bounded matched disturbances affecting its electromechanical system and output feedback only, using High-Order Sliding Mode (HOSM) observers for the estimation of the system states.

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In this work, a novel cascade control strategy based on the super-twisting algorithm (STA) is used to tackle the problem of ISP stabilization and tracking. Two cases are considered: (i) full state feedback, where the ISP joint angles and velocities are measured and (ii) output feedback, where the ISP joint velocities are estimated using an observer based on higher-order sliding mode (HOSM) theory. Stability analysis of the proposed control schemes is performed, and numerical simulations using Matlab show the efficiency and performance of the proposed control schemes.

II. ISP Modeling

In this section, a procedure for deriving the kinematic and dynamic models of an ISP installed on a moving base.

A. Notations and Conventions

This section presents the notations and conventions used in this work. Let $\mathbb{N} = \{0, 1, 2, \dots\}$ denote the set of *natural numbers* and define $\bar{\mathbb{N}} = \{\bar{0}, \bar{1}, \bar{2}, \dots\}$. Unless otherwise stated, $i, j, k \in \mathbb{N} \cup \bar{\mathbb{N}} \cup \{s, c\}$. Define the following:

- $\mathbf{E}_w$: world frame, arbitrarily located;
- $\mathbf{E}_{\bar{i}}$: frame fixed on body i with origin on its center of gravity (CG) ($i \in \mathbb{N}_0$);
- $\mathbf{E}_i$: fixed on body i with origin on joint i axis ($i \in \mathbb{N}_0$);
- $\mathbf{E}_c$: camera frame, fixed on the last link;
- $x_i^k, y_i^k, z_i^k \in \mathbb{R}^3$: $\mathbf{E}_i$ canonical unit vectors, written in $\mathbf{E}_k$;
- $p_{ij}^k \in \mathbb{R}^3$: position vector from the origin of frame $\mathbf{E}_i$ to the origin of $\mathbf{E}_j$, represented in $\mathbf{E}_k$;
- $v_{ij}^k \in \mathbb{R}^3$: linear velocity from $\mathbf{E}_i$ to $\mathbf{E}_j$, written in $\mathbf{E}_k$;
- $\omega_{ij}^k \in \mathbb{R}^3$: angular velocity from $\mathbf{E}_i$ to $\mathbf{E}_j$, written in $\mathbf{E}_k$;
- $h_i^k \in \mathbb{R}^3$: unit vector defining the rotation axis of joint i, represented in $\mathbf{E}_k$ ($i \in \mathbb{N}^*$);
- $m_i \in \mathbb{R}$, $I_i^i \in \mathbb{R}^{3\times3}$: mass and inertia tensor of body i represented in $\mathbf{E}_i$ ($i \in \mathbb{N}$);
- $\mathbf{S}(v) : \mathbb{R}^3 \mapsto \mathrm{so}(3)$ cross product operator;
- $\lfloor v \rceil^\alpha$: $\mathbb{R}^n \to \mathbb{R}^n$, where its elements are given by $\|v_i\|^\alpha \, sgn(v_i)$, with $v_i \in \mathbb{R}$ ($i = 1, ..., n$) being the elements of $v \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}$;
- $\mathbf{I}_n \in \mathbb{R}^{n\times n}$: identity matrix of dimension n.

Let body 0 be the moving base and bodies $1, 2, 3$ be the ISP gimbals. Also, if a superscript is omitted, the vector is written in world frame $\mathbf{E}_w$ coordinates.

B. Quaternion-Based Kinematics

Let $R \in SO(3)$ be a *rotation matrix* describing the rotation from an arbitrary frame to another. Then, R is a diffeomorphism with respect to the projective space $\mathbb{RP}^3 = \left\{ \|v\|^2 \leq \pi \mid v \in \mathbb{R}^3 \right\}$. Therefore, each point $v \in \mathbb{RP}^3$ represent a 4-parameter representation for $SO(3)$ called the *angle-axis*, where the unitary vector on the direction of v represents the rotation axis and $\|v\|$ represents the corresponding rotation angle around that axis.

Remark 1. *Note that $\mathbb{RP}^3$ covers $SO(3)$ twice, since any point on it actually represents the same rotation than the opposite point of the sphere.*

This representation can be expressed by $v = \{\theta, n\}$, where $\theta \in \mathbb{R}$ is the angle of rotation around the unit axis vector $n \in \mathbb{R}^3$, $\|n\| = 1$. Another non-minimal representation is the *unit quaternion*. The set of *quaternions* $\mathbb{H}$ is defined by:

$$\begin{aligned} &\mathbb{H} := \{\eta + i\epsilon_1 + j\epsilon_2 + k\epsilon_3 \mid \eta, \epsilon_1, \epsilon_2, \epsilon_3 \in \mathbb{R}\} \,, \\ &i^2 = j^2 = k^2 = ijk = -1 \,. \end{aligned} \tag{1}$$

A quaternion $Q \in \mathbb{H}$ can also be represented as the pair $Q := \{\eta, \epsilon\}$, where $\eta = \mathsf{Re}(Q) \in \mathbb{R}$ represents the *real* part of the quaternion and $\epsilon = \mathsf{Im}(Q) = [\ \epsilon_1 \ \ \epsilon_2 \ \ \epsilon_3\ ]^\mathsf{T} \in \mathbb{R}^3$ represents the vector part. The quaternion *conjugate* is given by $Q^* = \{\eta, -\epsilon\}$. One can also represent the quaternion in fully vector form by the notation $\bar{Q} = [\ \eta \ \ \epsilon_1 \ \ \epsilon_2 \ \ \epsilon_3\ ]^\mathsf{T} \in \mathbb{R}^4$.

Quaternions also form an algebraic *group* with respect to *multiplication*. Given two quaternions $Q_1 = \{\eta_1, \epsilon_1\}$ and $Q_2 = \{\eta_2, \epsilon_2\}$, their multiplication follow the rules established by definition (1), which results in:

$$Q_1 \cdot Q_2 = \{\eta_1\eta_2 - \epsilon_1^\mathsf{T}\epsilon_2,\, \eta_1\epsilon_2 + \eta_2\epsilon_1 + \epsilon_1 \times \epsilon_2\} \,. \tag{2}$$

Quaternion multiplication can also be performed as a linear transformation in $\mathbb{R}^4$, by:

$$\overline{Q_1 \cdot Q_2} = \mathbf{H}_+(Q_1)\, \overline{Q_2} \,, \tag{3}$$

$$= \mathbf{H}_-(Q_2)\, \overline{Q_1} \,, \tag{4}$$

where $\mathbf{H}_+$, $\mathbf{H}_-$ are *Hamilton operators* defined by:

$$\mathbf{H}_+(Q) = \left[\ \overline{Q} \quad \mathbf{h}_+(Q)\ \right], \quad \mathbf{h}_+(Q) = \begin{bmatrix} -\epsilon^\mathsf{T} \\ \eta\, \mathbf{I}_3 + \widehat{\epsilon} \end{bmatrix} \tag{5}$$

$$\mathbf{H}_-(Q) = \left[\ \overline{Q} \quad \mathbf{h}_-(Q)\ \right], \quad \mathbf{h}_-(Q) = \begin{bmatrix} -\epsilon^\mathsf{T} \\ \eta\, \mathbf{I}_3 - \widehat{\epsilon} \end{bmatrix} \tag{6}$$

The square of the quaternion *norm* is defined as the *scalar*:

$$\|Q\|^2 = Q \cdot Q^* = \{\eta^2 + \epsilon^\mathsf{T}\epsilon, 0\} \,, \tag{7}$$

and its *inverse* is the quaternion Q^{-1} such that $Q \circ Q^{-1} = \{1, 0\}$, the *unitary* quaternion.

The set of *unit quaternions* $\mathbb{H}^* = \{Q \in \mathbb{R} : \|Q\| = 1\}$ can be used as a parametrization for orientation in the following way. For an element $p = \{\theta, n\} \in \mathbb{RP}$, define:

$$Q = \left\{ cos\left(\frac{\theta}{2}\right), sin\left(\frac{\theta}{2}\right) n \right\} \in \mathbb{H}^* \,. \tag{8}$$

The inverse of an unit quaternion is given by $Q^{-1} = Q^*$, which according to (8), clearly corresponds to the opposite rotation due to negative direction of the rotation axis n.

Let $r_0, r_1, ..., r_n \in \mathbb{H}^*$ be the n absolute rotations between frames $\mathbf{E}_0, \mathbf{E}_1, ..., \mathbf{E}_n$ and the world frame $\mathbf{E}_w$, and $r_{i+1}^i \in \mathbb{H}^*$ ($i = 1, 2, ..., n-1$) represent the rotations from frame $\mathbf{E}_i$ to $\mathbf{E}_{i+1}$. Then, since the unit quaternions form a group with respect to multiplication, then:

$$r_n = r_1 \cdot r_2^1 \cdot \ldots \cdot r_n^{n-1} \in \mathbb{H}^* \,. \tag{9}$$

Now, define the set of *pure* quaternions $\mathbb{H}_p = \{v \in \mathbb{H} : \mathsf{Re}(v) = 0\}$. Note that any vector from $\mathbb{R}_3$ can be represented as the vector part of a corresponding element $v \in \mathbb{H}_p$. Besides, note that the following holds for $v, w \in \mathbb{H}_p$:

$$v \cdot w = \{-\mathsf{Im}(v)^\top \mathsf{Im}(w),\ \mathsf{Im}(v) \times \mathsf{Im}(w)\}\,. \qquad (10)$$

Let v^i and $v^j \in \mathbb{H}_p$ be representations for a vector $\vec{v}$ in frames $\mathbf{E}_i$ and $\mathbf{E}_j$, respectively, and r^i_j represents the rotation from $\mathbf{E}_i$ to $\mathbf{E}_j$, with unitary axis $n^i_j \in \mathbb{R}^3$ and rotation angle θ_{ij}. Then, the following relation is valid:

$$v^i = (r^i_j) \cdot v^j \cdot (r^i_j)^* = Ad_{r^i_j}\left[v^j\right]\,, \qquad (11)$$

where $Ad_{r^i_j}[*]$ is the adjoint *operator*. Note that, in vector algebra, $Ad_{r^i_j}$ represents the corresponding rotation matrix $R_{ij} \in SO(3)$ associated to the unit quaternion $r^i_j \in \mathbb{H}^*$. In terms of the components of r^i_j, this matrix is given by:

$$R_{ij} = n^i_j (n^i_j)^\top + s_{ij}\,\mathbf{S}(n^i_j) + c_{ij}\,(\mathbf{I}_3 - n^i_j (n^i_j)^\top)\,, \qquad (12)$$

where s_{ij} and c_{ij} are the sine and cosine functions of θ_{ij}. The rotation matrix corresponding to an absolute rotation $r_i \in \mathbb{H}^*$ is written with only one subscript, as $R_i \in SO(3)$.

Algorithm 1 (Kinematic Propagation)**.** *The algorithm is initialized with the vessel configuration $p_{00} \in \mathbb{R}^3$ and $r_0 \in \mathbb{H}^*$. Then, varying index i from 0 to $n-1$, the configuration of each frame with respect to the vessel frame $\mathbf{E}_0$ can be computed by:*

$$p_{0,i+1} = p_{0,i} + R_i\, p^i_{i,i+1}\,, \qquad (13)$$

$$\overline{r}_{i+1} = \mathbf{H}_+(r_i)\, \overline{r}^i_{i+1}\,. \qquad (14)$$

with R_i computed from r_i in (12)*. The camera pose can be computed as $p_{0c} = p_{0n} + R_c\, p^n_{nc}$, $r_c = \mathbf{H}_+(r_n)\, \overline{r}^n_c$.*

Let $\vec{v}_i$ and $\vec{\omega}_i$ be the physical linear and angular velocities of $\mathbf{E}_i$. They are represented by $v^i_i \in \mathbb{R}^3$ and $\omega^i_i \in \mathbb{R}^3$ when written in its own body frame. Let $r_i = \{\eta_i, \epsilon_i\} \in \mathbb{H}^*$ be the absolute rotation of $\mathbf{E}_i$. The time-derivative of r_i can be related to ω^i_i by:

$$\dot{\overline{r}}_i = \begin{bmatrix} \dot{\eta}_i \\ \dot{\epsilon}_i \end{bmatrix} = \frac{1}{2}\,\mathbf{h}_+(r_i)\,\omega^i_i\,, \qquad (15)$$

which is known as the *quaternion propagation* formula.

The vector $V^i_i = [\,(v^i_i)^\top\ (\omega^i_i)^\top\,]^\top \in \mathbb{R}^6$ is the *body velocity twist* associated to $\mathbf{E}_i$. Two body velocity twists associated to different frames $\mathbf{E}_i$, $\mathbf{E}_j$ located in the *same rigid-body* are related through the constant adjoint map $Ad_{g_{ij}} \in \mathbb{R}^{6\times 6}$:

$$V^i_i = Ad_{g_{ij}} V^j_j\,, \quad Ad_{g_{ij}} = \begin{bmatrix} R_{ij} & \hat{p}^i_{ij}\, R_{ij} \\ 0 & R_{ij} \end{bmatrix}. \qquad (16)$$

which has the property $Ad_{g_{ji}} = Ad^{-1}_{g_{ij}}$.

Now, given the body twists V^0_0 and $\dot{V}^0_0$ of frame $\mathbf{E}_0$ in the ship, it is possible to compute all velocities V^i_i and accelerations $\dot{V}^i_i$ associated to each link ($i = 1, 2, 3$) by means of an iterative algorithm described below. It consists in propagating the body velocity/acceleration twists of each link frame $\mathbf{E}_i$ through the system, obtaining V^i_i, $\dot{V}^i_i$, $i \in \{1, 2, 3\}$.

Algorithm 2 (Propagation of Velocities and Accelerations)**.** *The algorithm is initialized with given V^0_0, $\dot{V}^0_0$. Then, the velocities and accelerations are propagated upwards the kinematic chain from $i = 0$ until $i = n = 3$:*

$$V^i_i = \Omega^\top_{i-1,i}\,(\Phi_{i,i-1}\, V^{i-1}_{i-1} + H_i\, \dot{q}_i)\,, \qquad (17)$$

$$\dot{V}^i_i = \Omega^\top_{i-1,i}\,(\Phi_{i,i-1}\, \dot{V}^{i-1}_{i-1} + H_i\, \ddot{q}_i + A_i\, \dot{q}_i)\,. \qquad (18)$$

The velocity/acceleration twists of the camera are computed by $V^c_c = Ad_{g_{cn}} V^n_n$, $\dot{V}^c_c = Ad_{g_{cn}} \dot{V}^n_n$, with a constant transformation g_{cn}. The matrices in (17)*,* (18) *are given by:*

$$\Phi_{i+1,i} = \begin{bmatrix} \mathbf{I}_3 & -\mathbf{S}(p^i_{i,i+1}) \\ 0 & \mathbf{I}_3 \end{bmatrix}, \qquad \Phi^{-1}_{i+1,i} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{S}(p^i_{i,i+1}) \\ 0 & \mathbf{I}_3 \end{bmatrix},$$

$$H^\top_{i+1} = \begin{bmatrix} 0^\top\ (h^i_{i+1})^\top \end{bmatrix}, \qquad \Omega_{i,i+1} = \begin{bmatrix} R_{i,i+1} & 0 \\ 0 & R_{i,i+1} \end{bmatrix},$$

$$A_{i+1} = \begin{bmatrix} \mathbf{S}(v^i_i + \mathbf{S}(\omega^i_i)\, p^i_{i,i+1})\, h^i_{i+1} \\ \mathbf{S}(\omega^i_i)\, h^i_{i+1} \end{bmatrix}$$

where the rotation matrices $R_{i,i+1}$ are computed from $r_{i,i+1} = \left\{cos\left(\frac{1}{2}q_{i+1}\right), h^i_{i+1}\, sin\left(\frac{1}{2}q_{i+1}\right)\right\} \in \mathbb{H}^*$ using (12)*.*

Now, recall that ω^i_i can be written as the sum $\omega^i_i = \omega^i_0 + \omega^i_{0,i}$ and can be expressed in terms of $q\,, \dot{q} \in \mathbb{R}^3$ by means of the *angular body link Jacobian* $J^i_{0i}(q, \Pi_g) \in \mathbb{R}^{3\times 3}$ as $\omega^i_{0,i} = J^i_{0i}(q, \Pi_g)\, \dot{q}$:

$$\omega^i_i = J^i_{0i}(q, \Pi_g)\, \dot{q} + \omega^i_0\,, \qquad (19)$$

where Π_g is the vector of *geometric* parameters of the ISP, containing combinations of components of the axes and distance vectors of each link frame. Note that the body link Jacobian $J^i_{0i}(q, \Pi_g)$ can be computed numerically from (17), and also that the last $3-i$ columns of J^i_{0i} are null, since the velocity of link i only depends on the previous joints.

These kinematic relations can be used to describe the dependance among vehicle, ISP and camera motion by applying the group operation of $\mathbb{H}^*$, equation (19) and its time-derivative, with $\mathbf{E}_i = \mathbf{E}_c$:

$$r_c = r_0 \circ r^0_c(q, \Pi_g)\,, \qquad (20)$$

$$\omega^c_c = J^c_{0c}(q, \Pi_g)\, \dot{q} + \omega^c_0\,, \qquad (21)$$

$$\dot{\omega}^c_c = J^c_{0c}(q, \Pi_g)\, \ddot{q} + \dot{J}^c_{0c}(q, \dot{q}, \Pi_g)\, \dot{q} + \dot{\omega}^c_0\,. \qquad (22)$$

An important algebraic property is the *linearity* of (21) with respect to the *geometric* parameters:

$$\omega^c_c = W_\omega(q, \dot{q}, \omega^c_0)\, \Pi_g\,. \qquad (23)$$

where $W_\omega \in \mathbb{R}^{3\times N_g}$ is a *kinematic regressor*.

C. Dynamic Equations for Vehicle Manipulator Systems

In [16], it is shown that the equations of motion for a VMS with respect to the vehicle CG frame $\mathbf{E}_b$ can be written as:

$$M_{qq}\, \ddot{q} + C_{qq}\, \dot{q} + G_q + M_{qV}\, \dot{V}^0_0 + C^b_{qV}\, V^0_0 = \tau_q\,, \qquad (24)$$

where $\tau_q \in \mathbb{R}^n$ is the vector of generalized forces acting on the robot joints, collocated with $\dot{q}$. Matrices $M_{qq}(q, \Pi_g, \Pi_d) \in \mathbb{R}^{3\times 3}$ and $M_{qV}(q, \Pi_g, \Pi_d) \in \mathbb{R}^{3\times 6}$ are mass matrices, $C_{qq}(q, \dot{q}, V_0^0, \Pi_g, \Pi_d) \in \mathbb{R}^{3\times 3}$ and $C_{qV}(q, \dot{q}, V_0^0, \Pi_g, \Pi_d) \in \mathbb{R}^{3\times 6}$ are Coriolis matrices and $G_q(q, r_0, \Pi_g, \Pi_d) \in \mathbb{R}^3$ is the gravity vector.

It is worth mentioning that, in a similar way than in (23), (24) is also *linear* with respect to the *dynamic* parameters:

$$Y_q(q, \dot{q}, \ddot{q}, r_0, V_0^0, \dot{V}_0^0, g, \Pi_g)\, \Pi_d = \tau_q\,, \tag{25}$$

where $Y_q \in \mathbb{R}^{3\times N_d}$ is a *dynamic regressor*.

It is well known that the *Newton Euler* method is a computationally efficient algorithm that can be used to numerically solve the *inverse dynamics* problem for (24). Given q, $\dot{q}$, $\ddot{q}$, r_0, V_0^0, $\dot{V}_0^0$ and $g \in \mathbb{R}$, the *Newton-Euler* algorithm for the inverse dynamics is expressed by:

$$\tau_q = NE(q, \dot{q}, \ddot{q}, r_0, V_0^0, \dot{V}_0^0, g, \Pi)\,, \tag{26}$$

where $\Pi^{\mathsf{T}} = \begin{bmatrix} \Pi_g^{\mathsf{T}} & \Pi_d^{\mathsf{T}} \end{bmatrix}$ contains combinations of the geometric and dynamic parameters (masses and inertias).

The Newton-Euler algorithm is composed of two steps. The first one is the *propagation of velocities and accelerations* upwards the kinematic chain, summarized in Algorithm 2. The second one consists in solving the dynamic equations of motion for each rigid body in the system, starting from the n-th link and ending up on $\mathbf{E}_b$.

Algorithm 3 (Backward Propagation of Wrenches)**.** *Solving the Newton-Euler equations for the contact body wrenches $F_i^i \in \mathbb{R}^6$ between the VMS bodies (links and vehicle), yields:*

$$F_i^i = \Phi_{i+1,i}^{\mathsf{T}}\, \Omega_{i,i+1}\, F_{i+1}^{i+1} + M_i\, \dot{V}_i^i + B_i\,, \tag{27}$$

$$M_i = \begin{bmatrix} m_i\, \mathbf{I}_3 & -m_i\, \mathbf{S}(p_{i\bar{i}}^i) \\ m_i\, \mathbf{S}(p_{i\bar{i}}^i) & I_i^i \end{bmatrix},$$

$$B_i = \begin{bmatrix} m_i\, \mathbf{S}(\omega_i^i)\, (\mathbf{S}(\omega_i^i)\, p_{i\bar{i}}^i + v_i^i) \\ m_i\, \mathbf{S}(p_{i\bar{i}}^i)\, \mathbf{S}(\omega_i^i)\, v_i^i + \mathbf{S}(\omega_i^i)\, I_i^i\, \omega_i^i \end{bmatrix},$$

where the parameters $p_{i\bar{i}}^i$, m_i and I_i^i compose Π_d in (26)*.*

These equations must be solved from $i = n = 3$ to $i = 1$, using the velocity and acceleration twists V_i^i and $\dot{V}_i^i$ previously computed in Algorithm 2. Also, we set here $n+1 = c$ and we do not consider external wrenches acting on the camera frame $\mathbf{E}_c$, so that $F_{n+1}^{n+1} = 0$.

Finally, the joint torques can be computed projecting the wrenches acting on frames $\mathbf{E}_i$ into their rotation axis by:

$$\tau_{q_i} = H_i^{\mathsf{T}}\, \Omega_{i-1,i}\, F_i^i\,. \qquad i = 1, ..., n\,. \tag{28}$$

Note that (27) does not take into account the gravity forces acting on the links. The effect of gravity (in $-z_0$ direction) is introduced by modifying $\dot{V}_i^i$ in (27) for each i-th link with:

$$\dot{V}_i^i \leftarrow \dot{V}_i^i - g \begin{bmatrix} R_i^{\mathsf{T}}\, z_0 \\ 0 \end{bmatrix}. \tag{29}$$

This algorithm can be used to compute the terms and some matrices of (24) separately: the mass matrices M_{qq}, M_{qV}, the gravity vector G_q and the Coriolis term $C_{qq}\,\dot{q} + C_{qV}\, V_0^0$.

III. Super-Twisting Control

In this section, a novel second-order sliding mode (SOSM) controller based on super-twisting algorithm (STC) will be developed for the stabilization and tracking of the LOS. Two cases are considered: STC with full state feedback and STC with output feedback.

Consider (24), rewritten here as:

$$M_{qq}\, \ddot{q} + \tau_d = \tau_q\,, \tag{30}$$

where $\tau_q \in \mathbb{R}^3$ is the vector of generalized forces acting on the robot joints, collocated with $\dot{q}$ and $\tau_d = C_{qq}\,\dot{q} + G_q + M_{qV}\,\dot{V}_{0b}^b + C_{qV}\, V_{0b}^b \in \mathbb{R}^3$ is a disturbance vector. The dynamic model (30) can be rewritten as:

$$\begin{aligned} \dot{x}_1 &= x_2\,, \\ \dot{x}_2 &= M_{qq}^{-1}(x_1, \Pi)\, \tau_q + x_3(x_1, x_2, \Pi, t)\,, \end{aligned} \tag{31}$$

where the states $x_1 = q$, $x_2 = \dot{q}$ are the ISP joint angles and velocities and $x_3 = -M_{qq}^{-1}(x_1)\, \tau_d$ is a state-dependent disturbance.

Remark 2. *Note that, under assumption of torque control $u(t) = \tau_q$, state-space model* (31) *is a double-integrator with a nonlinear high-frequency gain and a matched disturbance $x_3 \in \mathbb{R}^3$.*

Now, in a similar way than in (31), (15) and (22) can be rewritten as:

$$\begin{aligned} \dot{y}_1 &= \frac{1}{2}\, \mathbf{h}_+(y_1)\, y_2\,, \\ \dot{y}_2 &= J_{0c}^c(x_1, \Pi_g)\, \dot{x}_2 + y_3(x_1, x_2, \Pi_g, t)\,. \end{aligned} \tag{32}$$

where the state $y_1^{\mathsf{T}} = \bar{r}_c^{\mathsf{T}} = \begin{bmatrix} y_{11} & y_{12}^{\mathsf{T}} \end{bmatrix}$ is the vector representation of the camera orientation $r_c \in \mathbb{H}^*$, with $y_{11} = \eta_c$ and $y_{12} = \epsilon_c$ being the scalar and vector components. State $y_2 = \omega_c^c$ is the camera angular velocity, while $y_3 = \dot{J}_{0c}^c\, \dot{q} + \dot{\omega}_0^c$ is another state-dependent disturbance.

Remark 3. *Note that the state-space model* (32) *is a double integrator with a nonlinear high-frequency gain and a matched disturbance y_3 with respect to a control input $\dot{x}_2$.*

This structure strongly suggests the use of a *cascade controller* for both stabilization and tracking. An inner controller acts on $u(t)$ in (31) to control $\dot{x}_2$, providing dynamic stabilization for the system, while an outer tracking controller acts on $\dot{x}_2$ in (32), controlling the camera orientation y_1.

Given an orientation reference $r_{c_d}(t) \in \mathbb{H}^*$ for the camera, it can be represented in vector form by $\bar{r}_{c_d}^{\mathsf{T}}(t) = y_{1_d}^{\mathsf{T}}(t) = \begin{bmatrix} y_{11_d}(t) & y_{12_d}^{\mathsf{T}}(t) \end{bmatrix}$. The angular velocity of the camera is also given as $\omega_{c_d}^c(t) = y_{2_d}(t)$. The quaternion and angular velocity errors can be defined as:

$$e_c = r_{c_d}(t) \cdot r_c^*\,, \tag{33}$$

$$e_\omega = y_{2_d}(t) - y_2\,. \tag{34}$$

Note that when $r_c = r_{c_d}(t)$, the orientation error (33) is zero.

A. Super-Twisting Control with Full State Feedback

Suppose that both ISP states x_1 and x_2 are available. The following theorem provides an stability analysis for the proposed sliding mode cascade controller.

Theorem 1 (*Cascade STC with Full State Feedback*)**.** *Let* (31) *and* (32) *be the system dynamic and kinematic models. Assume the following:*

(i) the ISP joint velocities and accelerations are uniformly norm-bounded;

(ii) the zero, first and second order time-derivatives of the vehicle velocity twists are uniformly norm-bounded;

Defining the super-twisting control expression:

$$S_t(s, A, B) = A\lfloor s\rceil^{1/2} + B\int_0^t sgn(s)d\tau\,,$$

with matrices $A, B > 0$, the super-twisting-based controllers can be defined as follows. The outer sliding surface is:

$$s_y = e_\omega + K_c\,\mathsf{Im}(e_c)\,,\quad K_c > 0\,, \tag{35}$$

where $K_c > 0$. The corresponding outer control law is:

$$w(t) = \widehat{J}^c_{0c}(x_1)^{-1}\left[\dot{y}_{2_d}(t) + K_c\,\psi + S_t(s_y, \Lambda_3, \Lambda_4)\right]\,. \tag{36}$$

where $\widehat{J}^c_{0c}(x_1) = J^c_{0c}(x_1, \widehat{\Pi}_g)$ and ψ is a function of y_1, y_2 and r_{c_d}. The inner sliding surface is defined as:

$$s_x = x_2 - \int_0^t w(\tau)\,d\tau\,, \tag{37}$$

and the corresponding inner control law is:

$$u(t) = \widehat{M}_{qq}(x_1)\,\left[w(t) - S_t(s_x, \Lambda_1, \Lambda_2)\right]\,, \tag{38}$$

where $\widehat{M}_{qq}(x_1) = M_{qq}(x_1, \widehat{\Pi}_g, \widehat{\Pi}_d)$. Then, control laws (38) *and* (36) *ensure finite-time exact convergence of the sliding variables s_x and s_y as defined in* (37) *and* (35)*. Furthermore, the quaternion and angular velocity errors e_c, e_ω are asymptotically stable under the dynamics of $s_y = 0$:*

$$e_\omega + K_c\,\mathsf{Im}(e_c) = 0\,. \tag{39}$$

Proof. Using (31) and Assumption 3, the dynamics of the sliding variable s_x is given by:

$$\dot{s}_x = \dot{x}_2 - w(t) = M_{qq}^{-1}\,u(t) + x_3 - w(t)\,. \tag{40}$$

Substituting (38) into (40), it becomes:

$$\dot{s}_x = -(\mathbf{I}_3 - M_{qq}^{-1}\,\Delta M_{qq})\,S_t(s_x, \Lambda_1, \Lambda_2) + x_3\,, \tag{41}$$

where $\Delta M_{qq} = M_{qq} - \widehat{M}_{qq}$. Using (25), $\Delta M_{qq}\,S_t = Y_q^*\,\widetilde{\Pi}_d + \Delta Y_q^*\,\widehat{\Pi}_d$, with $\Delta Y_q^* = Y_q^* - \widehat{Y}_q^*$, where $Y_q^* = Y_q(x_1, 0, S_t(s_x, \Lambda_1, \Lambda_2), 0, 0, 0, 0, \Pi_g)$ and $\widehat{Y}_q^* = Y_q(x_1, 0, S_t(s_x, \Lambda_1, \Lambda_2), 0, 0, 0, 0, \widehat{\Pi}_g)$. Then, it is possible to rewrite (41) as:

$$\begin{aligned}\dot{s}_x &= -\Lambda_1\,\lfloor s_x\rceil^{1/2} + w_x\,,\\ \dot{w}_x &= -\Lambda_2\,\lfloor s_x\rceil^{0} + d_x\,,\end{aligned} \tag{42}$$

where $d_x = \nabla(M_{qq}^{-1}\,Y_q^*)\,\widetilde{\Pi}_d + \nabla(M_{qq}^{-1}\,\Delta Y_q^*)\,\widehat{\Pi}_d + \dot{x}_3$ is clearly dependent on the base motion and on the errors on the geometric and dynamic parameters. Here, the operator ∇ denotes time differentiation.

Note that (42) is STA, and therefore is finite-time stable for bounded disturbances. It is evident that, if the nominal parameters are well known, system (41) is only perturbed by $d_x \approx \dot{x}_3$. Due to Assumptions (i) and (ii), the following inequalities hold:

$$\left\|\nabla(M_{qq}^{-1}\,Y_q^*)\,\widetilde{\Pi}_d\right\| < L_{x_1}\,, \tag{43}$$

$$\left\|\nabla(M_{qq}^{-1}\,\Delta Y_q^*)\,\widehat{\Pi}_d\right\| < L_{x_2}\,, \tag{44}$$

$$\|\dot{x}_3\| < L_{x_3}\,. \tag{45}$$

Then, $\|d_x\| < L_{x_1} + L_{x_2} + L_{x_3}$, and according to [17], it is possible to chose Λ_1 and Λ_2 so that (42) achieves SOSM in finite-time. It means that after a time $T_1 > 0$, $s_x = \dot{s}_x = 0$ and due to (40), $\dot{x}_2 = w(t)\ \forall t > T_1$, even in the presence of the bounded disturbance d_x.

Next, using (32), (33) and (34), the dynamics of the outer sliding variable (35) is given by

$$\dot{s}_y = \dot{y}_{2_d} - J^c_{0c}(x_1)\,\dot{x}_2 - y_3 + K_c\,\psi\,, \tag{46}$$

where $\psi(y_1, y_2, r_{c_d}) = y_{11}\,\dot{y}_{12_d} - 0.5\,y_{12}^\mathsf{T}\,y_2\,y_{12_d} - \dot{y}_{11_d}\,y_{12} - \dot{\widehat{y}}_{12_d}\,y_{12} - 0.5\,y_{11_d}\,(y_{11}\,\mathbf{I}_3 - \widehat{y}_{12})\,y_2 - 0.5\,\widehat{y}_{12_d}\,(y_{11}\,\mathbf{I}_3 - \widehat{y}_{12})\,y_2$, with $\dot{y}_{1_d} = \mathbf{h}_-(y_1)\,y_{2_d}$.

Since $\dot{x}_2 = \dot{s}_x + w(t)$, substituting (36) into (46) yields:

$$\begin{aligned}\dot{s}_y &= -\Lambda_3\,\lfloor s_y\rceil^{1/2} + w_y\,,\\ \dot{w}_y &= -\Lambda_4\,\lfloor s_y\rceil^{0} + d_y\,,\end{aligned} \tag{47}$$

where $d_y = -\dot{y}_3 - \nabla(J^c_{0c}\,\dot{s}_x) - \nabla(W_\omega^*)\widetilde{\Pi}_g$, with $W_\omega^* = W_\omega(x_1, w(t), 0)$, according to (23). Again, due to Assumptions (i) and (ii):

$$\|\nabla(J^c_{0c}\,\dot{s}_x)\| < L_{y_1}\,, \tag{48}$$

$$\left\|\nabla(W_\omega^*)\widetilde{\Pi}_g\right\| < L_{y_2}\,, \tag{49}$$

$$\|\dot{y}_3\| < L_{y_3}\,. \tag{50}$$

Note that (48) is reasonable, since $\ddot{s}_x$ is bounded, but constant L_{y_1} clearly depends on the initial conditions of (31). Also, in (49), $\nabla(W_\omega^*)$ depends on x_1, x_2, $w(t)$ and $\dot{w}(t)$, which are also bounded. Then, $\|d_y\| < L_{y_1} + L_{y_2} + L_{y_3}$, again guaranteeing finite-time stabilization of (47) after a time $T_2 > 0$. It means that for all $t \geq T_2$, the system is sliding and therefore, it follows the nonlinear dynamics of the sliding variable (35), which is asymptotically stable [18]. Therefore, the quaternion errors (33) and (34) tend to zero asymptotically after a time $max(T_1, T_2)$.

□

B. HOSMO Observer for STC with Output Feedback

If state $x_2 \in \mathbb{R}^3$ is not available, an observer could be used to estimate the joint velocity state $x_2(t)$ using the measurements of $x_1(t)$. Because of its desired characteristics such as finite-time exact convergence, sliding mode observers could be used for this purpose, such as the *super-twisting* observer (STO) [17]. However, according to [19], it is not possible to achieve SOSM using *continuous* control when STC is implemented based on STO. A solution is to use STC with HOSM-based observers to achieve continuous control.

Remark 4. *Two HOSMOs could be designed: one for the joint velocities $x_2(t)$, and other for the camera angular velocity $y_2(t)$. However, usually the camera orientation $y_1(t)$ is obtained from an Inertial Measurement Unit (IMU), a device that combines measurements of gyroscopes (which measure angular velocity) and magnetometers (which measure magnetic fields), providing an accurate estimate for $y_1(t)$. Therefore, trustworthy direct measurements of $y_2(t)$ are usually already available.*

Theorem 2 (*Cascade STC with Output State Feedback*)**.** *Let* (31) *and* (32) *be the system dynamic and kinematic models. Assume the following:*

(i) the ISP joint velocities and accelerations are uniformly norm-bounded;

(ii) the zero, first and second order time-derivatives of the vehicle velocity twists are uniformly norm-bounded;

Defining the estimation error $e_{x_1} = x_1 - \hat{x}_1$, the HOSM observer for x_2 is the third-order system:

$$\begin{aligned}\dot{\widehat{x}}_1 &= K_1 \lfloor e_{x_1} \rceil^{2/3} + \widehat{x}_2 \,, \\ \dot{\widehat{x}}_2 &= K_2 \lfloor e_{x_1} \rceil^{1/3} + \widehat{x}_3 + \widehat{M}_{qq}^{-1}(x_1)\, u \,, \\ \dot{\widehat{x}}_3 &= K_3 \lfloor e_{x_1} \rceil^{0} \,. \end{aligned} \tag{51}$$

where K_1, K_2 and K_3 are positive-definite matrices. The outer sliding variable and control law are defined in the same way as (35) *and* (36)*. The modified inner sliding variable is:*

$$\widehat{s}_x = \widehat{x}_2 - \int_0^t w(\tau)\, d\tau \,, \tag{52}$$

and the corresponding inner control law is:

$$u(t) = \widehat{M}_{qq}(x_1) \left[w(t) - K_2 \lfloor e_{x_1} \rceil^{1/3} - S_t(\widehat{s}_x, \Lambda_1, \Lambda_2) \right] . \tag{53}$$

Then, control laws (53) *and* (36) *with observer* (51) *ensure finite-time exact convergence of the sliding variables s_x and s_y as defined in* (52) *and* (35)*, and of the estimation errors e_{x_1}, $e_{x_2} = x_2 - \widehat{x}_2$ and $e_{x_3} = x_3 - \widehat{x}_3$. Furthermore, the quaternion and angular velocity errors e_c, e_ω are asymptotically stable under the dynamics of* (39)*.*

Proof. Using (31) and (51), the dynamics of the estimation errors is:

$$\begin{aligned}\dot{e}_{x_1} &= -K_1 \lfloor e_{x_1} \rceil^{2/3} + e_{x_2} \,, \\ \dot{e}_{x_2} &= -K_2 \lfloor e_{x_1} \rceil^{1/3} + e_{x_3} + (M_{qq}^{-1} - \widehat{M}_{qq}^{-1})\, u \,, \\ \dot{e}_{x_3} &= -K_3 \lfloor e_{x_1} \rceil^{0} + \dot{x}_3 \,. \end{aligned} \tag{54}$$

By using transformation $e_{x_4} = e_{x_3} + (M_{qq}^{-1} - \widehat{M}_{qq}^{-1})\, u$, it is possible to rewrite (54) as:

$$\begin{aligned}\dot{e}_{x_1} &= -K_1 \lfloor e_{x_1} \rceil^{2/3} + e_{x_2} \,, \\ \dot{e}_{x_2} &= -K_2 \lfloor e_{x_1} \rceil^{1/3} + e_{x_4} \,, \\ \dot{e}_{x_4} &= -K_3 \lfloor e_{x_1} \rceil^{0} + d_e \,. \end{aligned} \tag{55}$$

where $d_e = \dot{x}_3 + (M_{qq}^{-1} - \widehat{M}_{qq}^{-1})\, \dot{u} + \nabla(M_{qq}^{-1} - \widehat{M}_{qq}^{-1})\, u$. Due to Assumption (i) and (53), two constants $L_{e_1}, L_{e_2} > 0$ exist, such that:

$$\left\| (M_{qq}^{-1} - \widehat{M}_{qq}^{-1})\, \dot{u} \right\| < L_{e_1} \,, \tag{56}$$

$$\left\| \nabla(M_{qq}^{-1} - \widehat{M}_{qq}^{-1})\, u \right\| < L_{e_2} \,. \tag{57}$$

Also, by Assumption (ii), $\|\dot{x}_3\| < L_{x_3}$ also holds. Then, $\|d_e\| < L_{e_1} + L_{e_2} + L_{x_3}$, and therefore the disturbance d_e is norm-bounded. According to (), it is possible to chose K_1, K_2 and K_3 so that the states on (55) are finite-time stable.

Remark 5. *Since $M_{qq}^{-1} - \widehat{M}_{qq}^{-1} \neq 0$ due to parametric uncertainty, the estimation error e_{x_3} is expected to be norm-bounded only. Therefore, $x_3 = \widehat{x}_3 + \beta(\widetilde{\Pi})$, where $\beta(\widetilde{\Pi})$ is a small residue dependent on the parametric uncertainty.*

The dynamics of the modified sliding variable is given by:

$$\dot{\widehat{s}}_x = K_2 \lfloor e_{x_1} \rceil^{1/3} + \widehat{x}_3 + \widehat{M}_{qq}^{-1}(x_1)\, u(t) - w(t) \,. \tag{58}$$

Using the *continuous* control law (53), yields:

$$\begin{aligned}\dot{\widehat{s}}_x &= -\Lambda_1 \lfloor \widehat{s}_x \rceil^{1/2} + \widehat{w}_x \,, \\ \dot{\widehat{w}}_x &= -\Lambda_2 \lfloor \widehat{s}_x \rceil^{0} + K_3 \lfloor e_{x_1} \rceil^{0} \,. \end{aligned} \tag{59}$$

Since the disturbance $K_3 \lfloor e_{x_1} \rceil^0$ is obviously norm-bounded, the STA (59) is finite-time stable. Therefore, after a finite time $\bar{T}_1 > 0$, $\dot{x}_2 = w(t)$.

To prove the stability of the outer controller, a similar procedure is performed. Since $\dot{x}_2 = \dot{\widehat{s}}_x + \dot{e}_{x_2} + w(t)$, substituting (36) into (46) yields:

$$\begin{aligned}\dot{s}_y &= -\Lambda_3 \lfloor s_y \rceil^{1/2} + w_y \,, \\ \dot{w}_y &= -\Lambda_4 \lfloor s_y \rceil^{0} + \bar{d}_y \,, \end{aligned} \tag{60}$$

where $\bar{d}_y = -\dot{y}_3 - \nabla(J_{0c}^c\, \dot{\widehat{s}}_x) + \nabla(J_{0c}^c \dot{e}_{x_2}) - \nabla(W_\omega^*)\widetilde{\Pi}_g$. Again, due to Assumptions (i) and (ii), (55) and (59), two positive constants $\bar{L}_{y_1}$, $\bar{L}_{y_2}$ exist, such that:

$$\left\| \nabla(J_{0c}^c\, \dot{\widehat{s}}_x) \right\| < \bar{L}_{y_1} \,, \tag{61}$$

$$\|\nabla(J_{0c}^c \dot{e}_{x_2})\| < \bar{L}_{y_2} \,, \tag{62}$$

Then, $\|\bar{d}_y\| < \bar{L}_{y_1} + \bar{L}_{y_2} + L_{y_2} + L_{y_3}$, again guaranteeing finite-time stabilization of (60) after a time $\bar{T}_2 > 0$. Therefore, the quaternion errors (33) and (34) tend to zero asymptotically after a time $max(\bar{T}_1, \bar{T}_2)$.

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IV. SIMULATION RESULTS

V. CONCLUSION AND FUTURE WORKS

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