

# TRIPLE-SEPARABLE VPA (TSVP.4) OR A STONE TO BRIDGE THE GAP BETWEEN SEPARABLE COHORT MODELS AND NONSEPARABLE ONES

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## ABSTRACT

Separable cohort models (SVPA, CAGEAN, ICA. etc) are widely used in fish stock assessment since they allow to get more stable and statistically grounded solution in comparison to nonseparable models, which, as a rule, treat catch-at-age data as exact. In some cases separable cohort models are able to produce unique solution of the system of VPA equations using catch-at-age data only. This is most valuable when auxiliary information is not available.

Unfortunately the basic assumption of separable cohort models, i.e. representation of fishing mortality coefficients as a product of two factors (age-dependent selectivity factor and year (or effort)-dependent one) is sometimes too restrictive for real **stock-fishery** systems. The stability of selectivity pattern over years is often violated by variations in **fishing** regime and by natural reasons.

**One of well known “natural” reasons why selectivity pattern is not constant over years is that more abundant cohorts (generations) are of higher vulnerability for fishery than less abundant ones. Miscalculation of this factor in stock assessment, undertaken by means of separable models, may lead to biased stock size estimates.**

In the model described here, named Triple-Separable VPA (or TSVPA), an attempt is made to take into account the above mentioned factor by including the “factor of generation” in explicit form into the separability assumption. That is fishing mortality coefficient is now can be represented as a product of 3 factors: age-dependent, year-dependent and generation-dependent. Parameter estimation procedure of the TSVPA includes some principles of robust statistics and allows to estimate all the necessary parameters without auxiliary information.

**Ability** of the approach to provide more reasonable stock size estimates (in comparison to traditional “double-separable” representation) in the lack of auxiliary information is demonstrated on simulated and real data.

It seems that the approach described here may serve to bridge the gap between separable and nonseparable cohort models: triple-separabilization not only still allows not to get solution in the lack of auxiliary information (as separable models can do), but also to better reflect cohort-related effects.

## INTRODUCTION

Separability assumption is widely used in various cohort models. Separable cohort models were **first** proposed by Pope (1974), Doubleday (1976), Pope and Shepherd (1982); Fournier and Archibald (1982), and were extended by Deriso et al. (1985), Kimura (1986). Gudmundsson (1986) Patterson (1995) and others. A simple version of separable cohort model was also proposed by Kizner and Vasilyev (Kizner and Vasilyev 1993.1997: Vasilyev and Kizner, 1998; Vasilyev 1998, 1998a).

Experience of implementation of separable cohort models shows that the

separability assumption is valuable, since it helps to include errors in catch-at-age data into the analysis and to diminish the number of unknowns in the model. All of that may help in some cases to get unique solution in the deficit of auxiliary information.

Same time, the assumption of year-to-year stability of selectivity pattern, which lies behind the separable representation of fishing mortality coefficients, **often** appears to be too restrictive. For example, it does not reflect violations in separability **pattern** caused by a rather common **effect** of relatively **higher vulnerability** to **fishery** of more abundant cohorts (generations). That is why it is **often** recommended to look for separable solution only for a number of terminal years (for example, from 4 to 10 years, as it is **recommended** in ICA (Patterson, 1995)) and to look for nonseparable solution for previous years. However, a restricted **time** interval, used for estimation of selectivities  $\{s_a\}$  may decrease the stability of estimates of these parameters.

**In** the model described here the separable representation of fishing mortality coefficients is extended to include an additional generation-dependent term, which helps to take into account the above mentioned effect.

### THE MODEL

The model is similar in its main features to the previously reported one, named Instantaneous Separable VPA (or ISVPA) (**Kizner** and Vasilyev, 1997; Vasilyev, 1998). Basic assumptions of the model are the following:

- The fish stock is subject to pulse **fishing** once a **year**;
- In the intervals between these **fishing** actions, every cohort (generation) decreases exponentially due to natural mortality;
- Fishing effort, being a function of year, has **different** effects on different age groups; **this effect** is determined by age-dependent selectivity and generation-dependent factor.

Mathematically, the **above** assumptions may be expressed as:

$$C_{a,y} = \varphi_{a,y} N_{a,y} e^{-M/2}, \quad (1)$$

$$N_{a,y} = \frac{N_{a+1,y+1} e^M}{1 - \varphi_{a,y}}, \quad (2)$$

$$\varphi_{a,y} = s_a g_j f_y, \quad (3)$$

where  $N_{a,y}$  is the abundance of the  $a$ -th age group at the start of the  $y$ -th year,  $C_{a,y}$  is the catch from the  $a$ -th age group in the middle of the  $y$ -th year,  $\varphi_{a,y}$  is the fraction of the  $a$ -th age group taken as catch in the  $y$ -th year,  $f_y$  is the fishing **effort**-dependent factor in year  $y$ ,  $s_a$  is age-dependent selectivity factor, and  $g_j$  is generation-dependent factor. The selectivity vector is supposed to be normalized

$$\sum_{a=1}^m s_a = 1 \quad (4)$$

( $m$  is the number of age groups present in the catches).

Generation-dependent factors are also **normalized** to give global average equal to 1:

$$\sum_{y=1}^n \sum_{a=1}^m g_{a,y} = 1/[(n-1)(m-1)] \quad (5)$$

where  $g_{a(j)l, y(j)l} = g_{a(j)l+1, y(j)l+1} = g_{a(j)l+2, y(j)l+2} = \dots = g_{a(j)k, y(j)k} = g_j$ ; (6)

$a_{(j)l}$  - index of youngest age group, and  $a_{(j)k}$  - index of oldest age group in the cohort  $j$  under consideration. (For the **terminal** year and the oldest age group values of generation-dependent factor are always hold equal to 1).

Naturally, real catch-at-age data do not strictly obey equations (1) - (3), which should be understood as a theoretical model. In order to distinguish the actual catch,  $C_{a,y}$ , from the estimated one, let us designate the latter as  $\hat{C}_{a,y} = s_a f_y g_j N_{a,y} e^{-M/2}$ .

#### Loss function.

Minimization of the median, **MD**, of squared residuals (that is, the use of the least median or LMSQ principle) instead of their sum (the classical LSQ-principle) is referred to be more **resistant** with respect to outliers, those elements of the data set which overstep considerably reasonable confidence **limits** and, hence, are suspicious of containing extremely high errors (O'Brien, 1997; Hampel et al., 1986).

According to this concept, the solution may be looked for **as** providing estimates of  $M$  and  $f_n$ , which secure minimum of the median of the distribution of the squared **logarithmic** residuals,

$$MD(SE_{a,y}) = MD\{(\ln C_{a,y} - \ln \hat{C}_{a,y}^*)^2\} \quad (7)$$

( $a = 1, \dots, m$ ;  $y = 1, \dots, n$ ). The corresponding loss function will be denoted as  $MD(M, f_n)$ .

( $n$  being the total number of years, so **that**  $f_{term} = f_n$ ). For any given pair  $(M, f_{term})$  the remaining model parameters, such as the vectors  $s_a$ ,  $f_y$ , and  $g_j$  as well as the matrices  $N_{a,y}$  and  $\hat{C}_{a,y}$ , are determined by means of a special iterative procedure derived from equations (1) - (4)

#### Iterative procedure

Within the iterative procedure the given  $M$  and  $f_n$  are not changed. The calculations start with setting the initial values of the fishing effort,  $f_y$  at  $y=1, \dots, n-1$ : selectivity,  $s_a$ ; at  $a=1, \dots, m$  (the normalizing condition (4) must be kept), and **cohort-dependent** factors  $g_j (a,y) = 1$ . Every iteration consists of the following steps.

First, the terminal vectors  $\{N_{a,n}\}$  and  $\{N_{m,y}\}$  are evaluated from the equation:

$$C_{a,y} = s_a f_y g_j N_{a,y} e^{-M/2},$$

then all other  $N_{a,y}$  are determined from (2). After that the matrix  $\|\Phi_{a,y}\|$  is evaluated from the equation

$$\Phi_{a,y} = \frac{C_{a,y}}{N_{a,y}} e^{M/2}, \quad (8)$$

and after that  $\{f_y\}$ ,  $\{s_a\}$ , and  $\{g_j\}$  are determined **as**:

$$\ln f_y = \frac{1}{m} \sum_{a=1}^m \ln \left( \frac{\Phi_{a,y}}{g_j s_a} \right), \quad (9)$$

$$\ln s_a = \frac{1}{n} \sum_{y=1}^n \ln \frac{\varphi_{a,y}}{g_j f_y}, \quad (10)$$

$$\ln s_m = \ln s_{m-1} = \frac{1}{2n} \sum_{y=1}^n \left( \ln \frac{\varphi_{m,y}}{g_j f_y} + \ln \frac{\varphi_{m-1,y}}{g_j f_y} \right), \quad (10')$$

$$\ln g_j = (\ln g_{a(j)1, y(j)1} + \ln g_{a(j)1+l, y(j)1+l} + \dots + \ln g_{a(j)k, y(j)k}) / k, \quad (11)$$

where

$$g_{a(j), y(j)} = \frac{\varphi_{a,y}}{s_a f_y}$$

When evaluating  $f_y$  from (9), selectivities and generation-dependent factors are taken from the previous iteration; when **evaluating**  $s_a$  from (10), generation-dependent factors are taken from the previous iteration. At the end of each iteration, selectivities and generation-dependent factors must be re-normalized so that to satisfy conditions (4) and (5). This iterative procedure is simply “weighed geometrical mean procedure”, as from (9)-(11) it immediately follows that  $f_y$ ,  $s_a$  and  $g_j$  equal to the **geometrical** means of  $\varphi_{a,y}$  weighed by  $g_j s_a$ ,  $g_j f_y$ , and  $s_a f_y$  respectively.

For “short” generations (less than 3 points in the catch-at-age data matrix), as well as for the **terminal** year and oldest age group, **values** of  $g_j$  are not recalculated within iterations and remains the same (their discrepancy from initial guess (equal to 1) is due only to normalization (5)).

It is easy to show, that the iterative procedure stops when “estimated” logarithmic catches are unbiased (**residuals** have zero mean) simultaneously **within** years, age groups and generation (this will be illustrated by the results in the next section). In **order** to **understand** the statistical meaning of the convergence of the procedure, it is convenient to use the notion of estimated catch,  $\hat{C}_{a,y} = s_a f_y g_j N_a y e^{-M/2}$ , and present  $\varphi_{y,a}$  in the form:

$$\varphi_{a,y} = s_a f_y g_j \frac{C_{a,y}}{\hat{C}_{a,y}} \quad (12)$$

Let us consider the limits at  $IT \rightarrow \infty$  of all the variables participating in the model. Therefore the fractions  $\varphi_{a,y}$ , which is determined by equation (8) and figures in (9), (10) and (11), can be replaced with that given by relationship (12), where  $\hat{C}_{a,y}$  is substituted by  $\hat{C}_{a,y}^*$ , the catch estimates supplied by the iterative procedure at  $IT \rightarrow \infty$ . This substitution implies:

$$\sum_{a=1}^m [\ln C_{a,y} - \ln \hat{C}_{a,y}^*] \rightarrow 0 \quad (13)$$

and

$$\sum_{y=1}^n [\ln C_{a,y} - \ln \hat{C}_{a,y}^*] \rightarrow 0, \quad (14)$$

as well as for generations:

$$\sum_{i=1}^k [\ln C_{i,j} - \ln \hat{C}_{i,j}^*] \rightarrow 0, \quad (15)$$

the last one is valid only for generations, participating in evaluation of  $g_j$  (see above). The meaning of (13)-(15) is that the log-transformed estimates of catches are unbiased.

## RESULTS AND DISCUSSION

Advantages of “triple-separabilization” are illustrated by comparison of results of application of traditionally “**double-separable**” model ISVPA and the described above model **TSVPA** both to simulated and real data sets.

The **first** data set (referred to in Anon. (1993) as **DS6**) is the most noisy (and not separable) data among the six simulated data sets ~~provided by the ICES Workshop on~~ Methods of Fish Stock Assessments in **Reykjavik** in 1988.

The second one is data on Baltic cod in sub-divisions 25-32 (Anon. 1997). In examples of applications of ISVPA given here the estimation was based only on **catch-at-age** and **weight-at-age** in the stock (no data on effort or **cpue** was used,  $M$  was considered as unknown parameter and assumed to be independent on age).

Incorporation of generation-dependent factor **gives** obvious effect for both data sets (fig.1 and **fig.4**), but for DS6 it is more marked. Estimates of **separabilities** are almost not changed (fig.3); estimates of effort factors are somewhat **different** for ISVPA and TSVPA, but still rather similar (fig.2 and **fig.5**).

Comparison of tables 1 and 2 shows that decreased values of generation-dependent factor (in comparison to neighbour generations), as a rule, correspond to relatively more abundant generations, that is for them the given values of catches are taken by relatively lower fishing mortality than the product of effort and selectivity factors (for example, see generations being 3 years old in 1955, 1958, 1965).

Some generation are of apparent exception from this “**rule**”, but in fact the only thing we have done by “**triple-separabilization**” is that we have included a number of additional parameters **into** the model. At least, generationdependent factors can be regarded as correcting factors, diminishing the discrepancy between the separable model and nonseparable reality, and these discrepancies have a lot of other reasons besides the effect of relatively higher “availability” of more abundant generations.

The above declared unbiasedness of the TSVPA solution (“almost” zero **within-year**, -age, and -cohort averages for residuals in log-catchesj is illustrated by table 3.

Finally it is necessary to mention that the idea of “triple-separabilization” seems to be so straightforward, that it is hardly possible that it has not been used earlier. Unfortunately the author has not found such attempts in the literature.

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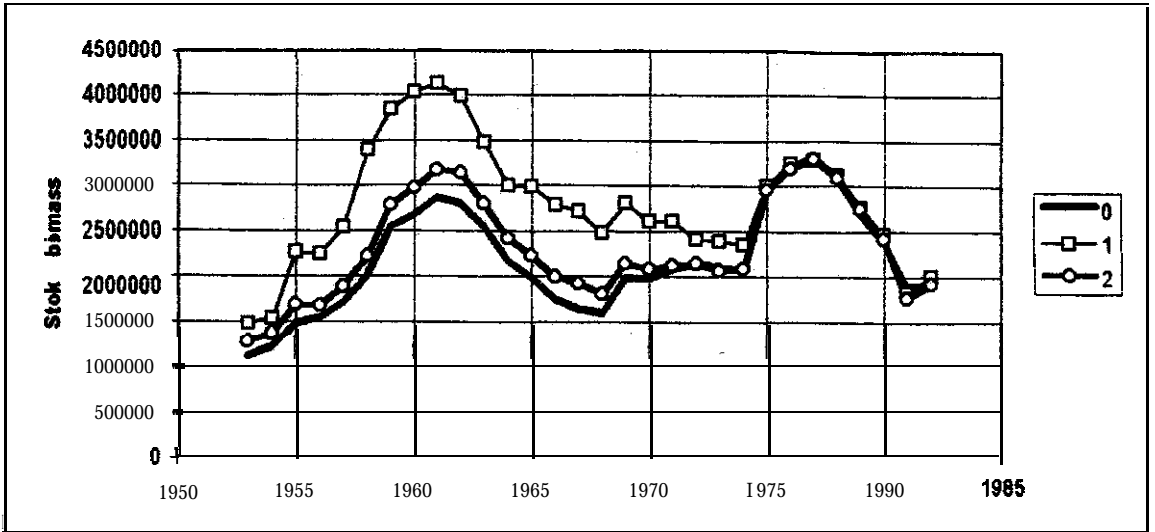


Fig. 1. Effect of introduction of generation-dependent factor into a separable cohort model (data set DS6; only catch-at-age data are used;  $\hat{M}$  is estimated.)

0 - "truth"; 1- ISVPA; 2 - TSVPA

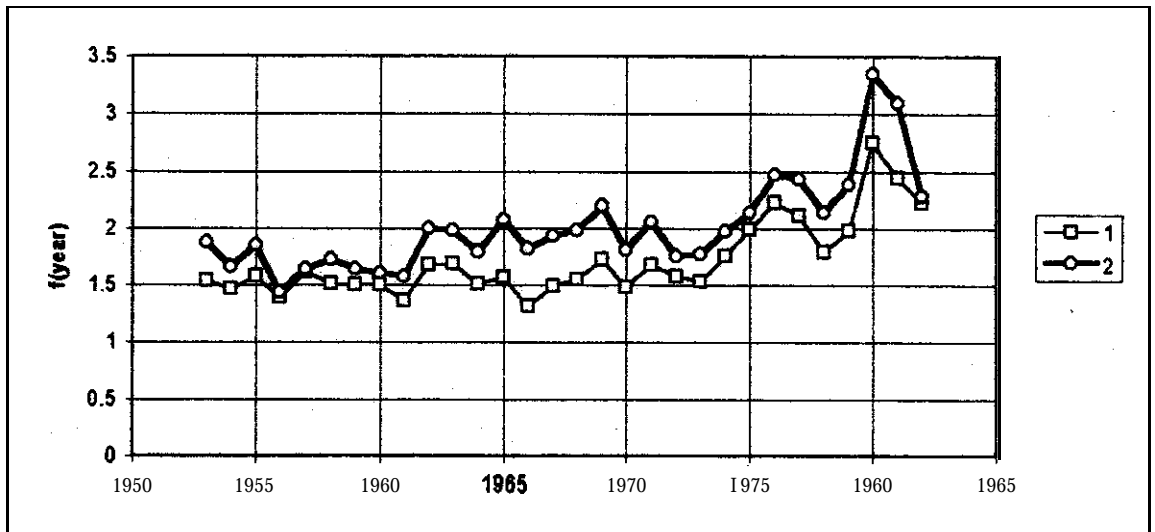


Fig.2. Estimates of effort factors  $f(y)$  for DS6: 1 - ISVPA; 2- TSVPA.

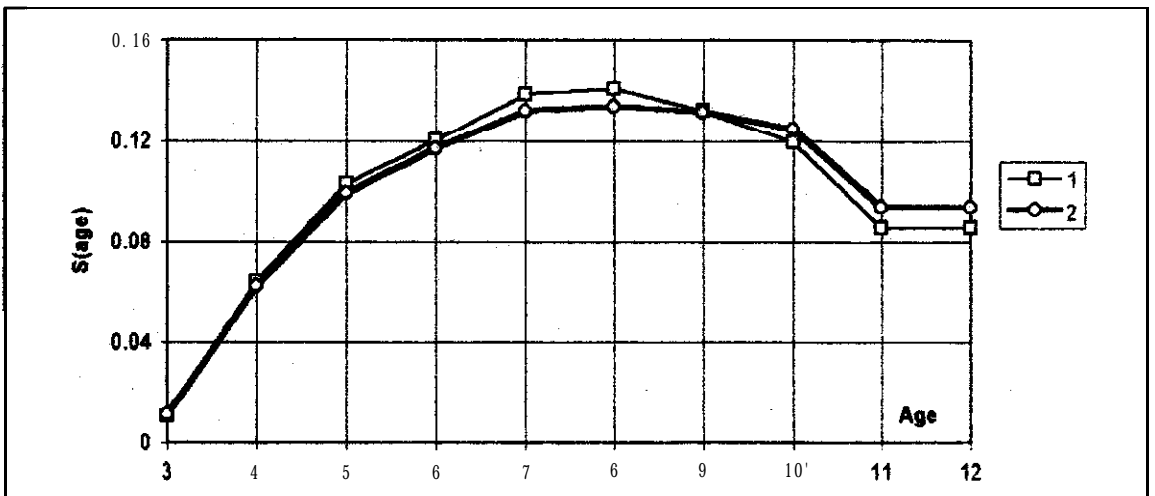


Fig.3. Estimates of selectivities for DS6: 1- ISVPA; 2- TSVPA.

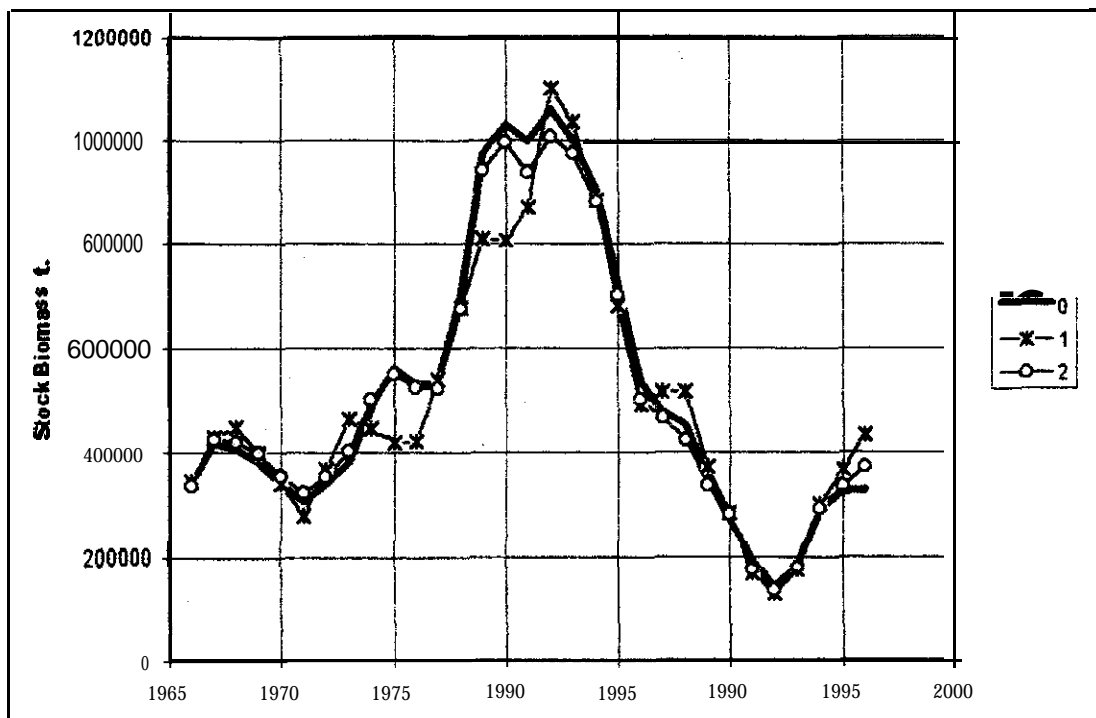


Fig. 4. Comparison of **resultst** for Baltic cod (subd.25-32) (Anon.1997)

obtained by various methods

0 - XSA (Anon.1997); 1- ISVPA; 2- TSVPA

for ISVPA and TSVPA: only catch-at-age is used;  $M$  is treated as unknown

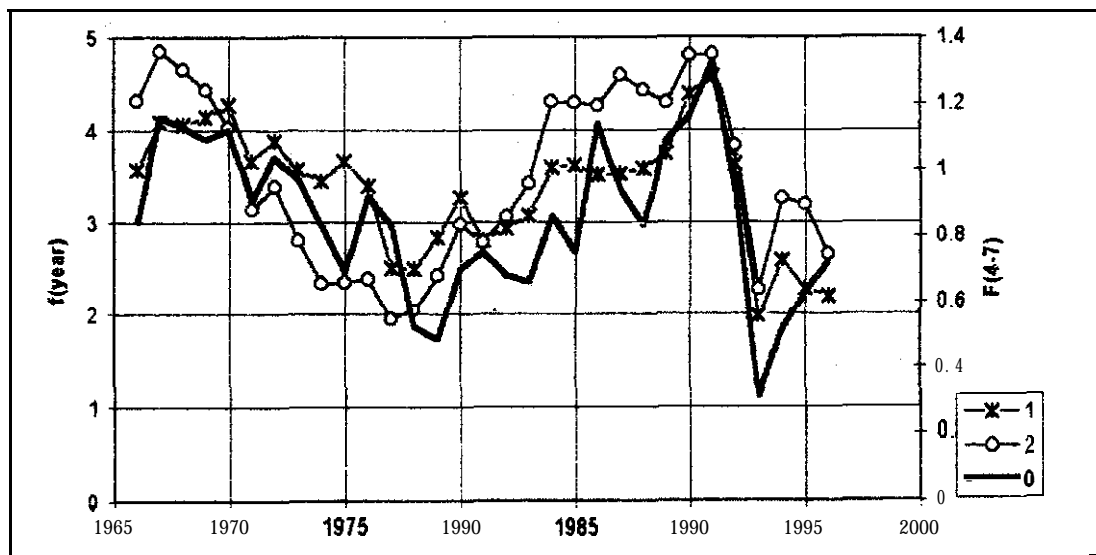


Fig. 5 Comparison of **effort factor estimates** (1- ISVPA; 2- TSVPA) with  $\{0\}$  XSA- derived  $F(4-7)$  (Anon. 1997)



YearAge	3	4	5	6	7	8	9	10	11	12
1953	248887	187344	112652	43395	29126	10459	11333	3676	1464	364
1954	127969	198929	124940	77242	28787	16412	6 2 8 5	7943	2650	1050
1955	377161	102555	144032	75543	52538	19081	9686	3957	5719	1932
1956	144685	303650	73091	93784	40723	33850	12281	5518	2453	4213
1957	260250	115883	230831	50253	62425	23315	23076	8417	3508	1725
1958	466876	208587	81830	164230	32235	38668	12421	15331	5670	2320
1959	689005	376720	149870	51513	112601	19560	23435	6494	10189	3994
1960	239751	551914	284866	99895	31251	76001	12019	14529	3584	7222
1961	255923	192450	399415	205522	64352	18262	51449	7493	9230	2236
1982	175482	206284	140801	267392	145089	40139	10685	35072	4769	6308
1983	139926	140645	150488	90486	159377	95261	22824	5643	22943	<b>3080</b>
1964	151340	111815	101878	101967	55478	90551	62482	13128	3079	15980
1965	337772	121718	80622	69543	68157	33716	53360	42122	8008	1951
1966	145835	271812	88374	51452	44023	42336	19261	29762	27801	5106
1967	144333	117171	204093	60914	32548	27921	27246	11672	17923	19869
1968	214266	115366	85392	143552	39983	19327	17305	17317	7050	11578
1969	580912	171224	81821	57368	97563	25195	11318	10708	11101	4611
1970	136362	463417	119651	51114	35943	62770	15284	6388	6513	7447
1971	140856	109149	335353	79109	32228	22970	42212	9891	3962	4486
1972	309432	112749	76860	216771	48093	18567	14059	27673	6271	2574
1973	142954	247021	82651	50975	140034	29583	11319	9082	19015	4396
1974	269335	115256	174108	56216	32273	86831	18039	6911	5929	13669
1975	1239506	215941	85491	107406	35834	18871	51686	10657	4144	4036
1976	411894	989879	154862	58601	60247	21342	10601	30013	6228	2655
1977	149941	329325	683864	97817	37360	28981	11906	5591	16757	3837
1978	136449	120087	235297	422347	58832	23011	13954	6740	3055	1 0495
1979	124739	109667	87935	158236	257811	35410	14884	7360	4099	1953
1980	247147	99793	79632	58458	99148	143091	20317	9117	3759	2635
1981	126477	196955	67242	48670	32976	50813	63493	9770	5063	1860
1982	561660	101004	139425	40692	28854	18192	27040	30373	5108	<b>3222</b>

Table 1. ISVPA-estimates of population (in numbers) for DS6

Year\Age	3	4	5	6	7	8	9	10	11	12
1953	1.12	1.59	0.87	0.87	1.25	1.06	0.58	0.70	0.70	1.00
1954	1.13	1.12	1.59	0.87	0.87	1.25	1.06	0.58	0.70	1.00
1955	0.81	1.13	1.12	1.59	0.87	0.87	1.25	1.06	0.58	1.00
1956	1.35	0.81	1.13	1.12	1.59	0.87	0.87	1.25	1.06	1.00
1957	1.14	1.35	0.81	1.13	1.12	1.59	0.87	6.87	1.25	1.00
1958	0.75	<b>1.14</b>	1.35	0.81	1.13	1.12	1.59	0.87	0.87	1.00
1959	1.17	0.75	1.14	1.35	0.81	1.13	<b>1.12</b>	1.59	0.87	<b>1.00</b>
1960	1.09	1.17	0.75	1.14	1.35	0.81	1.13	1.12	1.59	1.00
1961	0.88	1.09	1.17	0.75	<b>1.14</b>	1.35	0.81	1.13	1.12	1.00
1962	0.94	0.88	<b>1.09</b>	1.17	0.75	1.14	1.35	0.81	1.13	1.00
1963	1.07	0.94	0.88	1.09	1.17	0.75	1.14	1.35	0.81	<b>1.00</b>
1964	0.88	1.07	0.94	0.88	1.09	1.17	0.75	1.14	1.35	1.00
1965	0.74	0.88	1.07	0.94	0.88	1.09	1.17	0.75	1.14	1.00
1966	0.92	0.74	0.88	1.07	0.94	0.88	1.09	1.17	0.75	1.00
1967	1.09	0.92	0.74	0.88	1.07	0.94	0.88	1.09	1.17	1.00
1968	1.07	1.09	0.92	0.74	0.88	1.07	0.94	0.88	<b>1.09</b>	1.00
1969	1.03	1.07	1.09	0.92	0.74	0.88	1.07	0.94	0.88	1.00
1970	1.10	1.03	1.07	1.09	0.92	0.74	0.88	1.07	0.94	1.00
1971	0.96	1.10	1.03	1.07	1.09	0.92	0.74	0.88	1.07	1.00
1972	1.26	0.96	1.10	1.03	1.07	<b>1.09</b>	0.92	0.74	0.88	1.00
1973	0.77	1.26	0.98	1.10	1.03	1.07	1.09	0.92	0.74	1.00
1974	0.94	0.77	1.26	0.96	1.10	1.03	1.07	1.09	0.92	<b>1.00</b>
1975	1.02	0.94	0.77	1.26	0.96	1.10	1.03	1.07	<b>1.09</b>	1.00
1976	0.84	1.02	0.94	0.77	1.26	0.96	1.10	1.03	1.07	1.00
1977	0.80	0.84	1.02	0.94	0.77	1.26	0.96	1.10	1.03	1.00
1978	0.76	0.80	0.84	1.02	0.94	0.77	1.26	0.96	1.10	1.00
1979	0.85	0.76	0.80	0.84	1.02	0.94	0.77	1.26	0.96	1.00
1980	0.70	0.85	0.76	0.80	0.84	1.02	0.94	0.77	1.28	1.00
1981	0.70	0.70	0.85	0.76	0.80	0.84	1.02	0.94	0.77	1.00
1982	1.00	1.00	1.00	1.00	<b>1.00</b>	1.00	1.00	1.00	1.00	1.00

Table 2. Estimates of generation-dependent factors (data set DS6; TSVPA)

YearAge	3	4	5	6	7	8	9	10	11	12	AgeMean	GenMean
1953	0.431959	-0.67295	-1.60E-02	-5.93E-02	-0.45034	0.190077	5.76E-02	0.616867	-9.04E-02	-3.12E-17	7.42E-04	
1954	6.87E-02	0.190408	-0.28496	-3.13E-02	0.51099	-0.29124	-0.01499	0.267226	-0.40688	5.77E-17	7.91E-04	
1955	0.472121	-0.18158	-0.17982	-0.41518	0.295815	0.190878	-2.36E-02	0.17478	-0.32515	2.95E-17	8.27E-04	
1956	-0.51757	9.38E-02	0.153219	0.160929	-0.12498	0.178519	0.208394	0.207468	-0.35119	7.68E-17	8.62E-04	-8.94E-05
1957	0.457431	-0.25285	0.30789	-0.23821	-6.49502	-0.30391	-0.19891	-0.2536	0.555572	-3.12E-17	8.50E-04	-2.64E-04
1958	0.420914	-9.71E-02	-7.80E-02	-2.33E-02	-0.20729	9.12E-02	0.654566	-0.15354	-0.59909	-3.90E-18	8.42E-04	-3.58E-04
1959	-0.28181	0.114301	-0.10019	0.137953	5.66E-02	-5.61E-03	-0.26756	q.431484	-7.67E-02	-7.81E-18	8.48E-04	-2.48E-04
1960	-0.1179	-0.28619	0.136412	5.97E-02	-6.07E-02	-0.12455	1.01E-02	-0.31962	0.711242	4.29E-17	8.48E-04	-2.65E-04
1961	0.219864	-6.56E-02	-0.20148	0.267036	-3.16E-02	0.333549	-0.45405	-1.30E-02	-4.65E-02	2.65E-17	8.14E-04	-5.22E-04
1962	0.643234	-0.15367	-0.30537	-8.65E-02	1.60E-02	-0.25616	-4.43E-02	-0.21486	0.409422	-1.34E-17	7.76E-04	-3.99E-04
1963	3.65E-02	-0.16815	4.02E-02	8.07E-03	7.87E-02	4.20E-02	5.02E-02	0.109915	-0.11686	4.68E-17	7.55E-04	-4.25E-04
1964	-0.30052	7.36E-02	-0.16188	0.206932	0.22066	-5.04E-03	0.118614	-0.36261	0.368821	-3.04E-17	7.38E-04	-3.15E-04
1965	-8.52E-02	-8.95E-02	-0.17454	0.156446	-7.73E-02	0.064002	0.199728	-0.2825	0.275687	1.04E-17	6.82E-04	-5.17E-04
1966	-8.14E-02	7.04E-02	0.238994	0.16642	0.120228	-6.07E-02	9.16E-03	0.397222	-0.85406	-6.81E-17	6.32E-04	-4.66E-04
1967	0.136153	2.98E-02	1.67E-02	-0.14637	4.00E-02	-9.47E-02	-0.21711	6.05E-02	0.180719	-8.41E-17	5.73E-04	-3.29E-04
1968	0.149469	-0.12217	0.169444	0.14433	4.89E-02	9.39E-02	3.63E-03	-0.41684	0.122117	-1.73E-18	4.96E-04	-4.65E-04
1969	2.49E-02	-0.32454	-0.16283	4.36E-02	0.134921	-5.18E-02	0.195484	-0.31032	0.454738	9.54E-18	4.07E-04	-4.41E-04
1970	-4.18E-02	-2.60E-02	0.113122	-0.1799	2.30E-02	-6.97E-02	0.232179	0.144615	-0.1924	2.95E-17	3.16E-04	-3.85E-04
1971	-4.34E-02	-0.18438	0.199935	8.00E-02	-4.51E-02	-0.07441	-9.42E-02	0.335601	-0.17195	5.77E-17	2.03E-04	-3.88E-04
1972	-0.43531	0.237092	4.30E-03	-0.16027	4.20E-02	0.344321	0.275688	-3.60E-02	-0.27096	8.15E-17	8.41E-05	-3.96E-04
1973	-0.52783	7.47E-02	0.116258	-4.76E-02	0.170106	0.296822	-0.27767	0.278976	-8.44E-02	-6.03E-17	-6.39E-05	-3.51E-04
1974	0.158172	0.391291	-0.31176	0.161488	-6.13E-02	0.153162	-4.20E-02	0.216564	-0.66772	-6.46E-17	-2.16E-04	-3.18E-04
1975	0.304237	0.252822	0.673204	-7.34E-02	-0.20231	-0.11247	-0.34837	-0.58521	8.81E-02	5.38E-17	-3.67E-04	-3.03E-04
1976	3.54E-02	-0.08928	5.19E-03	-4.29E-02	-0.41417	-0.1691	0.253679	0.157953	0.26843	8.54E-17	-5.14E-04	2.57E-04
1977	0.132074	0.118378	-6.16E-02	-0.04089	0.238064	-2.88E-02	1.61E-02	-0.2072	-0.17282	3.47E-18	-6.57E-04	-1.96E-04
1978	-0.64529	3.47E-03	-3.65E-02	7.43E-02	-0.11705	0.389966	-0.30748	0.23418	0.396563	3.47E-17	-7.81E-04	-1.21E-04
1979	-9.77E-02	0.478534	0.16095	-2.17E-02	-6.63E-02	-0.23283	-0.11723	0.237113	-0.34955	-3.56E-17	-8.74E-04	2.49E-05
1980	2.66E-02	0.282681	-6.83E-02	-0.33662	-0.11198	-0.31283	-1.99E-02	-0.72933	1.2612	-3.21E-17	-8.52E-04	7.52E-05
1981	-0.47193	0.448206	-0.18283	0.238004	4.38E-02	2.07E-02	0.156088	1.03E-02	-0.27022	5.25E-17	-7.91E-04	2.13E-04
1982	-3.47E-18	-5.20E-17	8.41E-17	-5.55E-17	-5.72E-17	1.73E-17	3.47E-17	1.65E-17	2.17E-17	-1.65E-17		5.05E-04
YearMean:	-9.9E-05	-6.1E-05	-2.1E-05	5.64E-05	1.45E-04	2.53E-04	3.86E-04	5.35E-04	1.46E-03			5.62E-04
GenMean:					5.28E-04	5.85E-04	6.11E-04	6.14E-04	5.79E-04	5.62E-04		

Table 3 TSVPA residuals in lnC (dataset - DS6)