2D Geometric Transformations Using Homogeneous Coordinates

Objectives

1. To understand the concept of 2D geometric transformations.
2. To study homogeneous coordinate representation.
3. To implement translation, scaling, rotation, reflection, and shearing using matrix multiplication.
4. To apply transformation matrices on 2D objects like points, lines, and polygons.
5. To understand the advantage of homogeneous coordinates in combining transformations.

Software(s) Required

Python 3, numpy, matplotlib

Theory

1. Introduction to 2D Transformations

2D geometric transformations are operations that change the position, size, or orientation of objects in a two-dimensional plane. These transformations are widely used in computer graphics, image processing, and animation.

$$Basic\ 2D\ transformations\ include:$$

- Translation
- Scaling
- Rotation
- Reflection
- Shearing

2. Homogeneous Coordinates

In normal Cartesian coordinates, a 2D point is represented as:

P(x, y)

In homogeneous coordinates, the same point is represented as:

P(x, y, 1)

This extra coordinate allows all transformations (including translation) to be represented using matrix multiplication.General form:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = T \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Where **T** is the transformation matrix (3 × 3 matrix).

Basic Transformation Matrices

1. Translation

Moves an object by tx and ty.

$$T = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$x' = x + t_x$
$y' = y + t_y$

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2. Scaling

Changes the size of an object.

$$S = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x' = S_x x$
$y' = S_y$

3. Rotation

Rotates an object by angle θ (about origin).

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x' = x\cos\theta - y\sin\theta$
$y' = x\sin\theta + y\cos\theta$

4. Reflection

- About X-axis:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- About Y-axis:

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- About Origin:

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5. Shearing

- X-direction:

$$\begin{bmatrix} 1 & sh_z & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Y-direction:

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Composite Transformation

Multiple transformations can be combined by multiplying their matrices:

Tcomposite = T1×T2×T3

This is the main advantage of homogeneous coordinates — all transformations become matrix multiplications.

Algorithm

Algorithm: 2D Transformation Using Homogeneous Coordinates

Step 1: Start
Step 2: Input the coordinates of the object (x, y).
Step 3: Convert the point into homogeneous form (x, y, 1).
Step 4: Choose the type of transformation (translation/scaling/rotation/etc.).
Step 5: Construct the corresponding 3×3 transformation matrix.
Step 6: Multiply the transformation matrix with the point matrix.
Step 7: Obtain new coordinates (x', y', 1).
Step 8: Display the transformed coordinates.
Step 9: Stop.