§ 5.3 Compliance classes

- Principal stratification according to potential treatment. (Complier, always taker, never taker, defier)
- Population causal effects and local causal effects

§ 5.3.1 Potential value of treatment

A^0	A^1	label (Subpopulation)
0	0	never takers
0	1	Compliers
1	0	Defiers
1	1	always takers

(1) Never-takers: Encouragement does not work.

✗ We can not learn any thing about the effect of treatment in this subpopulation, because there is no variation in treatment received (A). We would never observe an outcome under treatment for anyone in this subpopulation.

(2) Compliers: treatment received is equal to treatment assignment. based on treatment assignment ⟹ is randomized

在Compliers这个群体中可以实现treatment received的randomization

(3) Defiers: Do ~~dep~~ opposite of what they are encouraged to do.

在Defiers这个群体中，A (treatment received) 仍然是randomized，只不过跟assignment刚好相反.

- 通常假设这个群体在whole population中不存在，我们不希望有这样的人做违反assignment的treatment.
- 如果这种群体存在的话，通常也会假设这部分人很少很少.

(4) Always-takers: Always take treatment.

No variation in treatment received, no information about causal effect.

§ 5.3.2 Causal effects

理想: causal effect ⟵ whole population

! 当存在unmeasured confounding的时候，很难获得whole population的causal effect.

☆ IV methods的motivation: unmeasured confounding

如果存在unmeasured confounding, cannot marginalize over all confounders via matching, iptw, etc.

☆ IV methods do not focus on the average causal effect for the whole population.

IV methods的目的是在存在unmeasured confound的情况下估计出一个有效的causal effect (要看针对的是哪个群体，但肯定不是whole population了)

☆ IV methods focus on a local average treatment effect.

§ 5.3.3 Local average treatment effect

The target of inference is

$$E(Y^{Z=1} \mid A^0=0, A^1=1) - E(Y^{Z=0} \mid A^0=0, A^1=1)$$

Tip: Mean difference of potential outcome in the same subpopulation of people, is a valid causal effect.

注意到上述的 "the same subpopulation" 就是Compliers，于是有 $E(Y^{Z=1} \mid A^0=0, A^1=1) - E(Y^{Z=0} \mid A^0=0, A^1=1)$

$$= E(Y^{Z=1} - Y^{Z=0} \mid \text{Compliers})$$

- 如何理解local? $= E(Y^{A=1} - Y^{A=0} \mid \text{Compliers})$

local表示an inference about a subpopulation, and the subpopulation happens to be compliers.

Local average treatment effect (LATE)
||
complier average causal effect (CACE)

§ 5.3.4 Observational data.

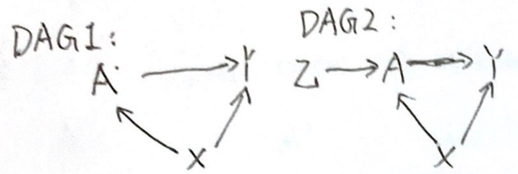
实际上我们只能观察到Z和A，而不是A^0, A^1.

Z	A	A^0	A^1	Class
0	0	0	?	Complier or Never taker
0	1	1	?	Always taker or defier
1	0	?	0	Never taker or defier
1	1	?	1	Complier or always taker.

- 给定observational data，我们只能把each subject限定在两种可能的subpopulation中.
- 除非给出additional assumption，需要假设 否则不能确切知道each subject属于哪类

§ 5. Instrumental variables methods

§ 5.1 Introduction



DAG1: $E(Y^{A=1}) - E(Y^{A=0})$ ⇓ A的causal Effect

?那么使用工具变量如何求得处理A的causal Effect.

DAG2: $E(Y^{Z=1}) - E(Y^{Z=0})$ ⇓ Z的Causal Effect

① Z直接影响A(处理)，并通过A间接影响Y.

② Z与X无关.

Z通常是一种encouragement.
比如鼓励对受试者吸烟，喝酒.

③ Z是randomized trial.
- IV is randomly assigned.
- IV is believed to be randomized in the nature.

§ 5.2 Randomized trials with noncompliance

Difference
- assigned treatment (as an instrument)
- treatment received (observed treatment in a randomized trial.)

§ 5.2.1

Z — treatment assigned
Z: ask sb. to take the treatment? (是否接受鼓励去take the treatment?)
randomization to treatment
就是把Z看成是treatment的分配.

A: treatment received (实际上是否接受treatment)

Y: outcome.

(不完全服从)
Non-compliance: Not everyone assigned treatment ($Z=1$) will acctually receive the treatment (A不一定为1) (happens in randomized trial)

Z: treatment assignment
A: actual treatment

☆ Hypothesis:
Z treatment assignment does not directly affect Y.

§ 5.2.2 Potential treatment

Observed data: (Z, A, Y) ?Z怎么观察到.

Z可能与A不同，意味着对于某些subject $Z = 1-A$, $A = 1-Z$.

• 类比potential outcome，可以定义potential treatment:

(i) $A^{Z=1} = A^1$，如果randomized to $Z=1$ (处理的分配为1)，真实的treatment为A^1 (其值可能为0，也可能为1)

(ii) $A^{Z=0} = A^0$，如果randomized to $Z=0$ (处理的分配为0)，真实的treatment为A^0.

一旦assign完成，就只能观察到A'或者A^0. 其实它们都是潜在的.

§ 5.2.3 Causal effect of Z on A

Think of average causal effect of treatment assignment on actual treatment: $E(A^1 - A^0)$ [proportation treated]

如果whole population都assigned to treatment ($Z=1$)，它们的received treatment的减去如果whole population都assigned to control ($Z=0$)的received treatment, (A^0) 再取期望.

In perfect compliance (绝对服从)，每个人都按照assignment去做.
$A^1=1$, $A^0=0$ (for everyone)，那么 $E(A^1-A^0)=1$.

☆ Estimable from observed data (需要两个条件 Randomization, consistency)

Consistency: $A^1 = A$ if $Z=1 \Rightarrow A^1 = A|Z=1$
A的值
(类比 $Y^a = Y|Z=1$)

Randomization: Z是随机分配的.

$E(A^1) = E(A|Z=1)$
$E(A^0) = E(A|Z=0)$

在assign to treatment的这部分subpopulation中，A的值就等于whole population都assign to treatment中A的均值. 因为Z是随机的，所以部分与总体的某些特征是一致的.

Take the subpopulation $Z=1$ and take their sample mean of A.

§ 5.2.4 Causal effect of Z on outcome

Think of average causal effect of treatment assignment on outcome: $E(Y^{Z=1} - Y^{Z=0})$ [average value of Y]

If everyone assigned to received treatment, their average outcome minus the average outcome if no one had been assigned to receive the treatment.

In perfect compliance, average causal effect of treatment assignment on outcome will be equal to the causal effect of treatment on outcome.

☆ Estimable from observed data (同样两个条件 Randomization, consistency)

$E(Y^{Z=1}) = E(Y|Z=1)$, $E(Y^{Z=0}) = E(Y|Z=0)$

§ 5.2.5 Causal effect of A on outcome

☆ 重点关注的Causal effect

把Z看作一种strong encouragement that most people will do what they have been told to receive the treatment. (Z is an IV)

5.1, 5.2

§5.4 Assumptions

Important assumptions that are necessary for identifying causal effect.

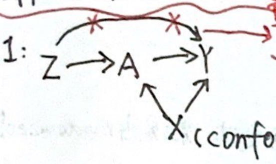
§5.4.1 Assumptions about IVs.

(1) It associated with treatment

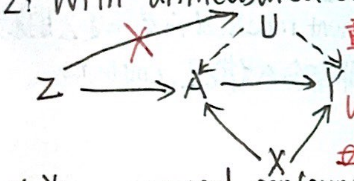
(2) It affects the outcome only through its effect on treatment. Exclusion restriction.

重点1: Must not directly affect Y

DAG1:



DAG2: With unmeasured confounding U



重点2: Z must not affect unmeasured confounding, ~~directly or indirectly~~

- U作为unmeasured confounding, affects Y.
- 如果 $Z \to U$ 成立，那么Z就可以通过U affects Y.

$Z \to U \to Y$

也就是说Z可以不通过 treatment A来影响Y，违反了Exclusion restriction: Z can not affect Y through its impact on some unmeasured confounders. 如果Z对Y有影响，也只能通过A.

(假设的实际性 realistic) If Z is randomized treatment assignment, are IV assumptions met?

1st. Z affects A $\Rightarrow$ check this through data.

2nd. Z: coin flip $\Rightarrow$ not affect the outcome or unmeasured confounders.

☆ subject: knowledge of treatment assignment.

§5.4.2

Recall the identification of causal effect is we don't know who the compliers are.! 必须作额外的假设.

(Monotonicity assumption) There are no defiers.

? Q: 为什么称为 Monotonicity (单调性)?

因为随着 encouragement 程度的增大，take treatment 的程度也增大. The assumption is that probability of treatment should increase with more encouragement.

With monotonicity assumption:

Z	A	A^0	A^1	class
0	0	0	?	Complier or Never
0	1	1	1	Always taker
1	0	0	0	Never taker
1	1	?	1	Complier or Always

对于某部分subjects来说，我们可以确定他们属于哪一个subpopulation，于是问题就简化了很多. we can actually identify the causal effect among compliers. (monotonicity assumption的作用)

§5.5 Causal Effect Identification and Estimation

- How to estimate complier average causal effect use observational data?
- How the CACE relates to intention-to-treat-effect

Recall: Our goal is to estimate

$$CACE = E(Y^{A=1} - Y^{A=0} \mid complier)$$

$\Rightarrow$ Begin with sth. that we can identify: ITT

$$E(Y^{Z=1} - Y^{Z=0}) = E(Y|Z=1) - E(Y|Z=0) \quad (5\text{-}1)$$

Intention-to-treat-effect

Q: 为什么 $E(Y^0) = E(Y|Z=0)$

因为Z是randomization 所以在Y的条件分布和potential Y的分布相同一致.

(1) $E(Y|Z=1) = E(Y|Z=1, \text{always takers}) P(at)$
$+ E(Y|Z=1, \text{never takers}) P(nt)$
$+ E(Y|Z=1, \text{compliers}) P(\text{compliers})$

a weighted average of the expected value of Y given Z=1 in 3 subpopulations

$= E(Y|\text{always takers}) P(\text{always takers})$
$+ E(Y|\text{never takers}) P(\text{never takers})$
$+ E(Y|Z=1, \text{compliers}) P(\text{compliers}) \quad (5\text{-}2)$

相等

(2) $E(Y|Z=0) = E(Y|\text{always takers}) P(\text{always takers})$
$+ E(Y|\text{never takers}) P(\text{never takers})$
$+ E(Y|Z=0, \text{compliers}) P(\text{compliers}) \quad (5\text{-}3)$

(3) $E(Y^{Z=1} - Y^{Z=0}) = E(Y|Z=1) - E(Y|Z=0)$
$= E(Y|Z=1, \text{compliers}) P(\text{compliers})$
$- E(Y|Z=0, \text{compliers}) P(\text{compliers}) \quad (5\text{-}4)$

$\Longrightarrow$ 由式 (5-4) 可知，

$$\boxed{\frac{E(Y|Z=1)-E(Y|Z=0)}{P(\text{compliers})}}$$

$= E(Y|Z=1, \text{compliers}) - E(Y|Z=0, \text{compliers})$

$= E(Y^{z=1} - Y^{z=0} \mid \text{compliers})$

$= E(Y^{a=1} - Y^{a=0} \mid \text{compliers})$

$\boxed{= \text{CACE (complier average causal effect)}}$ (5-5)

式(5-6)的另一种理解：
由Z的变化带来的A的期望变动，就是由randomized Z影响的那部分A，而对于always-takers和never-takers来说，他们的treatment received A是不受Z的影响的，因此受Z影响的那部分A只能是compliers.

Note: $P(\text{compliers}) = E(A|Z=1) - E(A|Z=0)$ (5-6)

$E(A|Z=1)$: proportion of people who are always takers or compliers.

$E(A|Z=0)$: a probability of taking treatment if you were assigned $z=0$. = the proportion of people who are always takers.

A is binary, expected value of binary variables is just the probability of it. $A_i = 1$ or $A_i = 0$, $E(A|Z=1) = (A_1 + A_2 + \cdots + A_n | Z=1) \times \frac{1}{n} = P(A=1|Z=1)$

$E(A|Z=1) = P(A=1|Z=1)$ — always takers / compliers

$E(A|Z=0) = P(A=1|Z=0)$ — always takers

$\Longrightarrow P(\text{compliers}) = E(A|Z=1) - E(A|Z=0)$

$\boxed{P(\text{compliers}) + P(\text{always takers})} - P(\text{always takers})$

由 (5-5) 和 (5-6) 可得

$$\text{CACE} = \frac{E(Y|Z=1) - E(Y|Z=0)}{E(A|Z=1) - E(A|Z=0)} \quad (5\text{-}7)$$

$\to$ itt: causal effect of treatment assignment Z on the outcome

$\to$ causal effect of treatment assignment Z on the treatment received A

- 在 Perfect compliance 的情形下，no noncompliers，$P(\text{compliers}) = 1$

  $\text{CACE} = E(Y|Z=1) - E(Y|Z=0) = \text{ITT}$

- $\text{CACE} \geq \text{ITT}$，因为 $P(\text{compliers}) \in (0, 1)$

  从另一角度解释，因为有部分人assigned to treatment，但是最终没有receive treatment，这在一定程度上减弱了真实的causal effect of treatment

- 为什么complier average causal effect 大于 intention-to-treat-effect？

  $E(Y^{a=1} - Y^{a=0} \mid \text{compliers})$ $\qquad$ $E(Y^{z=1} - Y^{z=0})$

  比如，100个病人，2种药，比较疗效，Y为治愈人数。

  有50个人在分配到Drug A $^{z=1}$，50个人分配到Drug B $(z=0)$

  在分配到Drug A的人群中，有20人没有实际服用Drug A，这20个人 $A=0$

  在分配到Drug B的人中有25个人实际服用了Drug A

  ① 假设 Drug A 比 Drug B 好.

  $Y^{z=1}\downarrow$ $\quad Y^{z=0}\uparrow$ $\quad E(Y^{z=1} - Y^{z=0})\downarrow$ $\qquad$ X

  ② 若 Drug A < Drug B. $\quad Y^{z=1}\uparrow$ $\quad Y^{z=0}\downarrow$ $\quad E(Y^{z=1} - Y^{z=0})\uparrow$