⇒ 由式(5-4)可知,

$$\frac{E(Y|Z=1) - E(Y|Z=0)}{P(\text{compliers})}$$

$= E(Y|Z=1, \text{compliers}) - E(Y|Z=0, \text{compliers})$

$= E(Y^{Z=1} - Y^{Z=0} | \text{compliers})$

$= E(Y^{a=1} - Y^{a=0} | \text{compliers})$

$= CACE$ (complier average causal effect) (5-5)

Note: $P(\text{compliers}) = E(A|Z=1) - E(A|Z=0)$ (5-6)

$E(A|Z=1)$: proportion of people who are always takers or compliers.

$E(A|Z=0)$: a probability of taking treatment if you were assigned $z=0$. = the proportion of people who are always takers.

A is binary, expected value of binary variables is just the probability of it.

$A_i = 1$ or $A_i = 0$, $E(A|Z=1) = (A_1 + A_2 + \cdots + A_n|Z=1) \times \frac{1}{n} = P(A=1|Z=1)$

$E(A|Z=1) = P(A=1|Z=1)$ — always takers, compliers

$E(A|Z=0) = P(A=1|Z=0)$ — always takers

$\Rightarrow P(\text{compliers}) = E(A|Z=1) - E(A|Z=0)$

$P(\text{compliers}) + P(\text{always takers}) - P(\text{always takers})$

式(5-6)的另一种理解:
由Z的变化带来的A的期望变动. 就是由randomized Z影响的那部分A, 而对于always-takers和never-takers来说, 他们的treatment received A是不受Z的影响的, 因此受Z影响的那部分A只能是compliers.

由(5-5)和(5-6)可得

$$CACE = \frac{E(Y|Z=1) - E(Y|Z=0)}{E(A|Z=1) - E(A|Z=0)} \quad (5-7)$$

→ itt: causal effect of treatment assignment Z on the outcome

→ causal effect of treatment assignment Z on the treatment received A

- 在Perfect compliance的情形下, no noncompliers, $P(\text{compliers}) = 1$

  $CACE = E(Y|Z=1) - E(Y|Z=0) = ITT$

- $CACE \geq ITT$, 因为 $P(\text{compliers}) \in (0, 1)$

  从另一角度解释, 因为有部分人assigned to treatment, 但是最终没有receive treatment, 这在一定程度上减弱了真实的causal effect of treatment

- 为什么complier average causal effect 大于 intention-to-treat-effect?

  $E(Y^{a=1} - Y^{a=0} | \text{compliers})$ $\quad E(Y^{Z=1} - Y^{Z=0})$

比如, 100个病人, 2种药, 比较疗效. Y为治愈人数.
有50个人在分配到Drug A (Z=1) 另50个人分配到Drug B (Z=0)
在分配到Drug A的人群中, 有20人没有实际服用Drug A, 这20个人 A=0
① 假设 Drug A 比 Drug B 好. 在分配到Drug B的人中有这个人实际服用了Drug A
$Y^{Z=1}$↓ $Y^{Z=0}$↑ $E(Y^{Z=1} - Y^{Z=0})$↓ X
② 若 Drug A < Drug B. $Y^{Z=1}$↑ $Y^{Z=0}$↓ $E(Y^{Z=1} - Y^{Z=0})$↑