§4.3 Marginal structual model. (MSM)

☆ a type of causal model

IPTW ⟶ simple causal effect
e.g. average causal effect.

? treatment effect modification.

· Introduction of MSM

· MSM is a model for the mean of potential outcomes. (expected value of
其中 Marginal: model the potential outcomes 不以 confounder 为条件. for whole population)
Structural: 对于 potential out comes, 而非 observed outcomes.

注意:
① 不是 conditional on X, $E(Y^0, Y^1|X)$, 而是 $E(Y^0, Y^1)$, 即 Marginal
② 不是针对 subpopulation, 而是 whole pop.

Model Setting

(1) Linear MSM:

$$E(Y^a) = \psi_0 + \psi_1 a. \quad a=0,1$$

• $E(Y^0) = \psi_0$ if $a=0$
• $E(Y^1) = \psi_0 + \psi_1$ if $a=1$

ψ_1: average causal effect $E(Y^1) - E(Y^0)$.

(2) logistic MSM for binary outcome.

$$\text{logit}[E(Y^a)] = \psi_0 + \psi_1 a, \quad a=0,1.$$

$\Rightarrow E(Y^a) = e^{\psi_0 + \psi_1 a} \quad a=0,1$

$\Rightarrow \exp(\psi_1)$ is the causal odds ratio. 新定义

$E(Y^0) = e^{\psi_0}$
$E(Y^1) = e^{\psi_0+\psi_1} \Rightarrow e^{\psi_1} = \frac{E(Y^1)}{E(Y^0)}$

$$\frac{\dfrac{P(Y^1=1)}{1-P(Y^1=1)}}{\dfrac{P(Y^0=1)}{1-P(Y^0=1)}}$$

⟵ odds that $Y^1=1$ (whole population 中所有人都接受处理的 odd: $Y=1$)

⟵ odds that $Y^0=1$ (whole population 中所有人都接受控制 control 的 odd: $Y=0$)

○ 把 $E(Y^a)$ 当作一种概率. 故 $E(Y^a) = P(Y^a=1)$

(3) MSM with effect modification. (include effect modifiers)

V is a variable that modifies the effect of A.

■ Linear MSM with effect modification.

$$E(Y^a|V) = \psi_0 + \underbrace{\psi_1 a}_{\text{A主效应}} + \underbrace{\psi_3 V}_{\text{V效应}} + \underbrace{\psi_4 aV}_{\text{A和V交互效应}}. \quad a=0,1.$$

注意: ①只以V为条件, 而非所有 confounders.

只是希望 control 的

影响TE的可能会影响TE的V

△ TE vary across subpopulation.

V: diabetes, sex, race, ... 之类的变量, 可能改变 treatment 的 effect. 在V的不同取值之间, 有不同的处理效应 (TE).

○ $E(Y^1|V) - E(Y^0|V) = \psi_0 + \psi_1 + \psi_3 V + \psi_4 V - (\psi_0 + \psi_3 V) = \psi_1 + \psi_4 V.$

如果已经知道了模型, 给定V的值, 就可以计算出 causal effect.

$$CE = \hat{\psi}_1 + \hat{\psi}_4 \underline{V} \text{ (plug-in)}$$

(4) General MSM

$$g\{E(Y^a|V)\} = h(a, V; \psi)$$

· g( ) is a link function
· h( ): a function specify parametric form of a, V. (additive, linear), 可以有二次项.

Key issue:

? Potential outcome ≠ Observed outcome.
如何在只给定 observed data 时 estimate model parameters?