§4.4 IPTW estimation

如何估计 marginal structural model 中的参数？

4.4.1 类比标准回归模型中的参数估计.

(1) Estimation in linear Regression Model.

$$Y = X\beta + \varepsilon$$

最小二乘： $\min_{\beta} \|Y - X\beta\|^2$

$$\Rightarrow X(Y - X^T\beta) = 0$$

$$\sum_{i=1}^{n} X_i(Y_i - X_i^T\beta) = 0$$

(2) Estimation in generalized linear Model

$$E(Y_i|X_i) = \mu_i = g^{-1}(X_i^T\beta)$$

ex: $\text{logit}(E(Y)) = X\beta$

$$\Rightarrow \sum_{i=1}^{n} \frac{\partial \mu_i^T}{\partial \beta} V_i^{-1}(Y_i - \mu_i(\beta)) = 0$$

(3) Estimation in MSMs (Linear)

$$g(E(Y^a)) = \psi_0 + \psi_1 a \Rightarrow E(Y^a) = g^{-1}(\psi_0 + \psi_1 a)$$

① 跟 generalized Linear Model 很像.

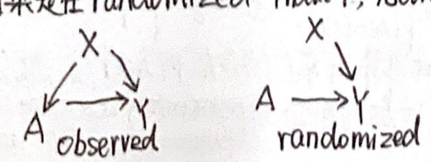
② 不等价于 regression model. 在 regression Model 中，我们以 "observed treatment A" 作为条件：

$E(Y|\underline{A}) = g^{-1}(\psi_0 + \psi_0 \underline{A})$ 这里的A是固定的，针对的是 subpopulation，而对于 MSM，a是未定的，我们可以设定它为任意值.

MSM : a-setting, 可以是任意值.

regression Model : A — conditioning

? 为什么MSM与 regression model 是不等同的？
因为有 Confounding 的存在！
如果是在 randomized trail 中，无confounding



影响，那么可以拟合 regression model，模型中的参数就表示 causal Effect.

◎ 这就给建模提供了一种思路：既然在 randomized ~~trail~~ trial 中可以用 regression model，我们可以尽量去构建 randomized trial，然后做回归.

在前几课(§4.2)中，我们知道可以使用IPTW构建 pseudo-population 来近似 randomized trial 的情形

Fig. {Estimation in MSMs}

- Pseudo-population is free from confounding (在 ignorability 和 posibivity 的假设下)
- $\sum_{i=1}^{n} \frac{\partial \mu_i^T}{\partial \beta} V_i^{-1}(Y_i - \mu_i(\beta)) = 0$ (generalized model)

$$\Downarrow$$

$$\sum_{i=1}^{n} \frac{\partial \mu_i^T}{\partial \psi} V_i^{-1} W_i (Y_i - \mu_i(\psi)) = 0 \text{ (MSMs)}$$

(μ_i是线性回归中 $E(Y|X_i)$ 的条件均值)

$$W_i = \frac{1}{A_i P(A=1|X_i) + (1-A_i) P(A=0|X_i)}.$$

$A_i=1$　示性　$A_i=0$

4.4.2 Steps in estimating parameters from MSM.

Step1: Estimating propensity score.

Step2: Create weights $\begin{cases} \frac{1}{P(A_i=1|X_i)} \leftarrow P_s \text{ for treated} \\ \frac{1}{P(A_i=0|X_i)} = \frac{1}{1-P(A_i=1|X_i)} \end{cases}$ P_s for control

Step3: Specify the MSM of interest (Linear or logit or with modification or ...)
Y contineous; Y count

Step4: Use software to fit a weighted generalized linear model.

☆ 人为地扩大了 population size，after weighting

1 treated　　9 control

⇓ weighting

● ⇓ $P_s = \frac{1}{10}$, Weight = 10

○○○○○○○○○ $P_s = \frac{1}{10}$ $\frac{9}{10}$, Weight = $\frac{10}{9}$

After weighting，两组各有10人.

Step5: Use asymptotic variance estimator (OR bootstrapping)
因为 pseudo-population > sample size

4.4 – 4.6 共3节内容

§4.5 Assessing balance

(任务：检查 covariate balance 是否实现在IPTW之后)

目标：Weighted sample ⟺ randomized trial

思路与之前 Propensity score 之后的检查方法相同，采用 standardized difference.

(Covariate balance can be checked on the weighted sample using standardized difference) 注意检验对象

! 注意是对 weighting 之后的样本做检验，而不是原样本。

(1) Two ~~ways~~ :
- Table 1 : summary statistics
- plot

① Table 1: ① mean(sd) of each covariate stratified on treated group and control group (using weights) — Weighted means

② standardized difference after weighting. (recall smd is the difference in means between groups, divided by the pooled standard deviation.)

② plot

$$smd = \left| \frac{\bar{X}_t - \bar{X}_c}{\sqrt{\frac{S_t^2 + S_c^2}{2}}} \right|$$ 绝对值

(2) Standardized difference after weighting

○ weighted means (variance) of each covariate stratified on treated group and control group.

○ Take difference in weighted means and divide by an estimate of pooled weighted standardized deviation.

! smd 越小越好， ~~>0.4~~ >0.02 imbalance | 0.2< <0.4 | <0.2

	Raw data			Weighted data		
	control	treated	smd	control	treated	smd
n						
V1						
V2						
⋮						

(3) If imbalance after weighting

① refine propensity score ~~matching~~ model

\> summary(weight)
\> tail(sort(weight))
\> head(sort(weight))

A person who was likely to be treated (given covariates) but wasn't:

Treated $\frac{1}{10}$ control $\frac{1}{10}$

○ will have a large weight

○ will have a small weight

$P(A_i=1|X_i) = \frac{9}{10}$

$P(A_i=0|X_i) = \frac{1}{10} \Rightarrow weight = \frac{1}{\frac{1}{10}} = 10$

§4.6 Distribution of weights

4.6.1 Intuition

大 weight ⟶ large standard error

• 极端的一个举例

1 person — weight 10000 ⟶ 1个人代表10000个人

outcome : 1 (10000个人的outcome都依赖于1个人，有很大的噪声，很大的误差。)

相当于10000个人的outcome都是1.

如果1个人的outcome data很显著地影响了参数估计，那么standard error就会非常大.

Prefer person to have not too large weight.

4.6.2 Bootstrapping

Estimate standard error : bootstrapping

Step1: Randomly sample from original data (有放回)

Step2: Use sample data to estimate parameters

Step3: Repeat the procedure for k times.

Step4: Take the variance of k estimators as the estimate of standard error.

问题：当一些weight非常大的sample被采样时，该样本对估计量的作用非常显著。当这些weight非常大的样本不被采样时，作用对估计量的作用非常明显，所以大weight的样本会使估计量standard error的波动很大，也会很大.

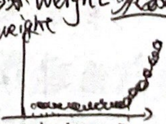
4.6.3 Relationship with positivity assumption.

$weight = \frac{1}{P(A_i=1|X_i)}$ 若weight↑，$P(A_i=1|X_i)$↓

假设weight非常非常大，那么$P(A_i=1|X_i)$就会非常趋向于0，这就可能违背positivity assumption. (near violation)

4.6.4 Checking weights

(1) Plot
- density plot (所有样本的weight 频数 频率)
- weight — index plot



(2) Summary Statistics (quantail) min/max/mean/median