Simulation Design for a Partially Linear Model

Let

$$
\begin{aligned}
y_i &= d_i + x_i'(c_y\theta_0) + u_i, \\
d_i &= \frac{\exp\{x_i'(c_d\theta_0)\}}{1+\exp\{x_i'(c_d\theta_0)\}} + v_i,
\end{aligned}
$$

where $v_i \sim N(0,1)$, $u_i \sim N(0,1)$, u_i and v_i are independent, $p = dim(x_i) = 250$, the covariates $x_i \sim N(0,\Sigma)$ with $\Sigma_{kj} = (0.5)^{|j-k|}$, and the sample size $n = 200$. θ_0 is a $p \times 1$ vector with elements set as $\theta_{0,j} = (1/j)^2$ for $j = 1,\ldots,p$. c_d and c_y are scalars that control the strength of the relationship between the controls, the outcome, and the treatment variable d_i. We can try several different combinations of c_d and c_y, setting

$$
c_d = \sqrt{\frac{(\pi^2/3)R_d^2}{(1-R_d^2)\theta_0'\Sigma\theta_0}},\ c_y = \sqrt{\frac{R_y^2}{(1-R_y^2)\theta_0'\Sigma\theta_0}},
$$

for different combinations of $R_d^2 \in \{0, 0.1, 0.5, 0.9\}$ and $R_y^2 \in \{0, 0.1, 0.5, 0.9\}$.