A “Sand-Box” model for Nuclear Pairing and Alpha Correlations

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Abstract

In this research, we proposed a schematic model that describes nucleon-nucleon pairing correlations and possible quartic correlations of alpha-type in a single variational wave function. We started with investigation of the pairing correlations and developed a C++ code that solves this model and can describe the available experimental data (such as nucleon separation energies). The model was applied to some sd-shell nuclei heavier than O-16 and lighter than Ca-40. We plan to extend this model further to study the alpha and pairing condensates together: the conditions at which these condensates appear, and possibility coexist.

Atomic Nucleus and its importance

The atomic nucleus consists of protons and neutrons which are at the center of an atom. An atom is composed of a positively-charged nucleus, with a cloud of negatively-charged electrons surrounding it. Different numbers of protons in the nucleus define different atoms and chemical elements. Isotopes are due to different neutrons but the same proton.

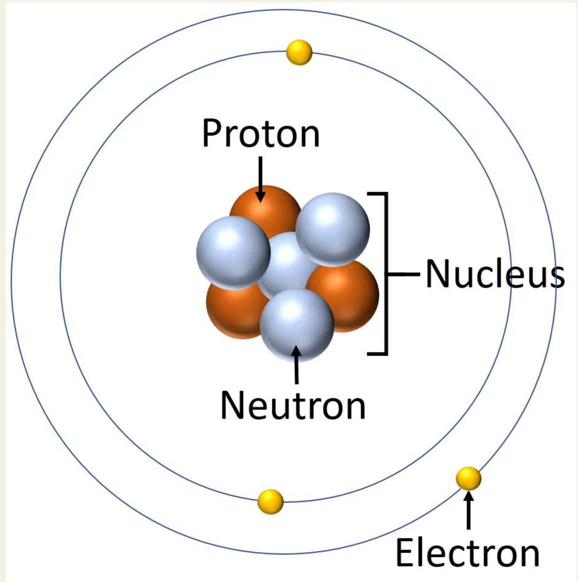

Fig 1. Structure of an atomic nucleus

Particles with opposite electric charges attract each other. Negative electrons orbit the positive nucleus. Particles with the same electric charge repel each other. This means that the positive protons in the nucleus push apart from one another. One important applicant is nuclear medical imaging and diagnostic technique. Positron emission tomography (PET) system introduces a positron-emitting radionuclide (tracer) into the body through a biologically active molecule. PET can detect gamma rays indirectly and produce three-dimension images.

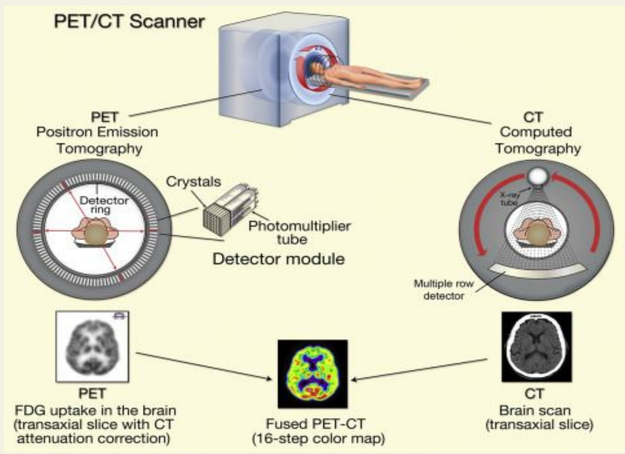

Fig 2. Schema of a PET acquisition process.

Nuclear astrophysics is an interdisciplinary part of both nuclear physics and astrophysics. Generally speaking, it aims to understand the origin of chemical elements and isotopes and the role of nuclear energy generation, in cosmic sources.

Goals

- Investigate the nuclear structure and role of the nuclear pairing forces
- Study two possible types of a Bose-Einstein condensate in nuclear systems: pairing and alpha correlations, and to investigate the conditions of the condensates appearance and their co-existence
- Assess possibilities of using the nuclear pairing correlation models as a tool for studying superfluidity in solid-state systems such as quantum dots, crystals and neutron stara

Pairing Correlations

Fig 3. The nuclear separation energies in MeV are plotted for different isotopes of Oxygen

- As observed from the graph above, the local maxima occur in cases of oxygen having even mass numbers.
- The significant drop (=8 MeV) indicates that O-16 is the most stable and bounded element amongst all elements.
- We can observe a clear zigzag pattern for the separation energies from O-17 to O-22 (see the figure above) – the systems with even number of particles (O-18, O-20, O-22) have higher separations energies compared to the odd systems (O-17, O-19, O-21). The difference in the energies is about 4 MeV..
- This pattern is due to nuclear pairing correlations: when two identical particles are close to each other on the same level, they tend to form a pair have a binding energy. To separate these particles in an even system, the pairs have to be broken first to overcome that energy. Whereas, in an odd system, at least one particle is unpaired, hence it's relatively easier to remove this particle since there is no need to break any pairs and overcome any binding energies.

The Model

Based on the Hamilton Equation, we can write the energy-number function as:

$$E(N) = \varepsilon_\mu N_n - G_\mu N_p$$

Therefore,

$E(N=1) = \varepsilon$

$E(N=2) = 2\varepsilon - G$, $S(2) = |\varepsilon| + G$

$E(N=3) = 3\varepsilon - G$, $S(3) = |\varepsilon|$

$E(N=4) = 4\varepsilon - 2G$, $S(4) = |\varepsilon| + G$

…

Take oxygen atom (O) and its isotopes as an example, see Fig3 :

- When N=0, the 16-O atom stay the most stable situation, so the separation energy is the highest one. It will be hard to separate a neutron in 16-O.
- For an odd system at the same level, such as 17-O, 19-O, it has one more particle, so the total energy is ε higher than the previous isotope. In 16-O, 18-O. it is easier to remove that unpaired particle. Thus, the separation energy will drop.
- For an even system at the same level, such as 18-O, 20-O, the total energy is ε higher than the previous one; the new pair lowers the energy by pairing interaction energy, G. Thus, these isotopes become relative stable state and separation energy will be higher than the previous one.

More Serious Approach

In a more serious approach, we introduce the following Hamiltonian (the total energy)

$$\widehat{H} = \varepsilon_\mu a_\mu^{+} a_\mu - G_\mu A_\mu^{+} A_\mu$$

that will describe the nuclear pairing correlations in the system. Here εμ stands for energy of single-particle levels, μ – spans over the single-particle states, constant Gμν is the strength of the pairing interaction, the operator aμ+aμ presents the occupation number of the corresponding state nμ,

$$a_\mu^{+} a_\mu |\Psi\rangle = n_\mu |\Psi\rangle$$

In this approach we minimize the average energy of the system over the following trial wave function

$$|\Psi\rangle = \Pi_{\mu>0}\left(\mathrm{U}_\mu + \mathrm{V}_\mu A_\mu^{+}\right)|0\rangle$$

where Vμ is the amplitude of the probability of pair occupied the state μ

G/MeV	V1^2	V2^2	V3^2	ε_f	Δ
0.2	0.998807	0.963894	3.36628e-05	-3.1587	0.06
0.5	0.99426	0.976209	0.00050619	-2.8017	0.21
1.0	0.985024	0.975023	0.0149817	-1.4067	0.756

Tab 1. Collection of applied model based on G

We used C++ computer language to numerically solve the proposed model and applied it to the sd-shell nuclei (heavier than O-16 and lighter than Ca-40). The results shown above, present distributions of 8 particles over the three levels (d5/2, s1/2, and d3/2) corresponding to different strengths of the pairing interaction G, the pairing gap parameter is defined as:

$$\Delta_\mu = \Sigma_{v\neq\mu} G_{\mu v} U_v V_v$$

We can notice that the particles prefer to occupy the lowest energy levels first. In general, when the first level is occupied, the particle begin to occupy the next level, and so on. When G and Δ increase, the particles will be more likely to occupy higher levels. With greater pairing correlation strengths, it is favorable to distribute the pairs over larger space, so pairs tend to occupy higher energy levels. However, the energy of the system reduces.

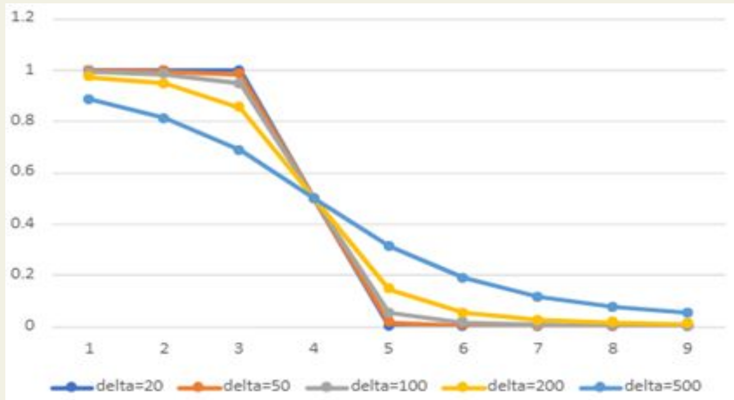

Fig 4: A graph depicting Occupation number vs. Epsilon_mu

Future Plan

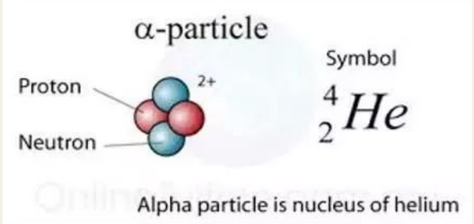

Fig 8. structure of alpha-particle

An alpha particle is a stable state of four nucleons: two protons and two neutrons (the nucleus of He-4). It is known that a particles have high binding energy (it is hard to break them) and, sometimes, when a nucleus decays, it could be more favorable to eject an alpha-particle than a proton or neutron separately (it is called alpha decay).

It is natural to study alpha-particle correlations in even-even nuclei with a similar approach that we discussed above. Similar to the pairing correlations a nuclear system can be considered as a composition of alpha particles (He-4 nuclei) that can from a condensate. For example, 0-16 can be seen as the combination of four alpha particles; Ne-20 may be seen as the combination of five alpha particles, and so on.

In the future, we plan to extend the model we used to describe the pairing correlations to alpha-particles. It can be done, for example, with the following Hamiltonian:

$$\widehat{H} = \varepsilon N - G N_p - A N_\alpha$$

where A is the strength of alpha-correlations and Na is the number of alpha-particles.

*Data from Brookhaven National Laboratory