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Simulation of forest fire fronts using cellular automata

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Abstract

In this work a new model for fire front spreading based on two-dimensional cellular automata is proposed. It is a more realistic modification of the model introduced by Karafyllidis and Thanailakis (see [Karafyllidis I, Thanailakis A. A model for predicting forest fire spreading using cellular automata. Ecol Model 1997;99:87–97]), which is based on the transfer of fractional burned area. Specifically, the model proposed in this work introduces a more accurate factor of propagation from a diagonal neighbor cell and includes, in a detailed form, the rate of fire spread. Moreover, the model is useful for both homogeneous and inhomogeneous environments. Some tests have been passed in order to determine the goodness of the method.

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Keywords: Cellular automata; Fire simulation; Forest fires; Mathematical modeling

1. Introduction

Forest fires is one of the major ecological agents in forests. Each year, fire burns between six and fourteen million hectares of forest; consequently they alter the structure and composition of forests and adversely affect human health and the supply of goods and services on which communities depend.

As a consequence, fires have received increased public attention worldwide. In this way, the scientific community is responding to the challenge addresses forest fires as interesting and complex phenomena requiring a multi-disciplinary approach. Due to its destructive nature, studies involving the ignition of actual forest fires are impractical. As an alternative to studies that are field based, many scientists use computer simulation models to better understand fire behavior and fire effects. Specifically, several mathematical approaches to the study of the spreading of fire have been appeared in the literature (see, for example [1–4]).

Specifically, the efforts to model the growth of the fire front by means of mathematical models can be classified according to two approaches: Vector models and cellular automata models. Vector models assume that the fire front spreads according to a well-defined growth law, and take a standard geometrical shape. If burning conditions are uniform, a single shape can be used to determine the fire size, the perimeter over time and the area by means of the use of fractal (see [5]). More complex vector models use wave propagation techniques based on Huygen's principle (see, for example [6–8]), and also determine the temperature fields and the fire propagation simultaneously

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by performing turbulent fluid flow calculations (see [9]). On the other hand, several types of cellular automata models for fire spreading have been introduced in the literature (see, for example [10–12]).

The main goal of this work is to introduce a new model for predicting forest fire spreading. It is based on a particular type of discrete dynamical system called two-dimensional cellular automata, 2D-CA for short (see [13]). The 2D-CA are simple model of computation capable to simulate complex behavior. In this way, there are several models based on CA to model of growth processes, reaction–diffusion systems, self-reproduction, image processing, cryptography, epidemic propagation, etc.

For the previous reasons, we have choose this mathematical model which is very easy to implement in software and in hardware. Moreover, the results seem to be suitable.

Roughly speaking, 2D-CA are dynamical systems for which time and space are discrete. They consist of a collection of a finite two-dimensional array of simple objects, called cells, interacting locally with each other. Each cell can assume a state such that it changes in every time step according to a local rule whose variables are the states of some cells (its neighborhood) at previous time steps.

In this sense, a forest area is divided into a two-dimensional array of identical square areas, each one of them stands for a cell of the 2D-CA. The states considered are 0 if the cell is unburned or partially burned out, and 1 if the cell is fully burned out. Moreover, it is supposed that the state of every cell evolves tacking into account the states of its eight nearest cells and it own state at a previous time.

The model proposed in this work is a modification of the model introduced by Karafyllidis and Thanailakis in [14]. Specifically, some parameters have been modified in order to obtain a more realistic situation. In this way we have supposed that the spreading of the fire front from a diagonal neighbor cell to the main cell is circular, instead of the linear spreading proposed by Karafyllidis et al.

The rest of the paper is organized as follows: In Section 2, the basic theory of two-dimensional cellular automata is introduced. In Section 3, a review of Karafyllidis and Thanailakis model is done, and the new model is proposed. Several tests for the new model are checked and their simulations are shown in Section 4 and finally, the conclusions and further work are presented in Section 5.

2. Two-dimensional cellular automata

Cellular automata (CA) are discrete dynamical systems formed by a set of identical objects called cells. These cells are endowed with a state which changes at every discrete step of time according to a deterministic rule. One of the most important CA are two-dimensional finite CA. More precisely, a two-dimensional finite CA can be defined as a 4-uplet $\mathscr{A} = (C, S, V, f)$, where C is the cellular space formed by a two-dimensional array of $r \times s$ identical objects called cells: $C = \{\langle i,j\rangle, 0 \leqslant i \leqslant r-1, 0 \leqslant j \leqslant s-1\}$, such that each of them can assume a state. The state of each cell is an element of a finite or infinite *state set*, S; if S is finite and $|S| = k$ then S is taken to be $\mathbb{Z}_k = \{0, 1, \ldots, k-1\}$. The state of the cell $\langle i,j\rangle$ at time t is denoted by $a_{ij}^{(t)}$.

The set of indices of the 2D-CA is the ordered finite subset $V \subset \mathbb{Z} \times \mathbb{Z}$, $|V| = m$, such that for every cell $\langle i,j\rangle$, its neighborhood V_{ij} is the ordered set of m cells given by

$$V_{ij} = \{\langle i+\alpha_1, j+\beta_1\rangle, \ldots, \langle i+\alpha_m, j+\beta_m\rangle : (\alpha_k, \beta_k) \in V\}.$$

There are some classic types of neighborhoods, but in this work only the extended Moore neighborhood will be considered; that is, the neighborhood of every cell is given by the following set of indices:

$$V_M = \{(-1,-1),\ (-1,0),\ (-1,1),\ (0,-1),\ (0,0),\ (0,1),\\ (1,-1),\ (1,0),\ (1,1)\}.$$

Graphically the extended Moore neighborhood of a cell $\langle i,j\rangle$ can be seen as follows:

$\langle i-1,j-1\rangle$	$\langle i-1,j\rangle$	$\langle i-1,j+1\rangle$
$\langle i,j-1\rangle$	$\langle i,j\rangle$	$\langle i,j+1\rangle$
$\langle i+1,j-1\rangle$	$\langle i+1,j\rangle$	$\langle i+1,j+1\rangle$

In this case, we can distinguish two types of neighbor cells of $\langle i,j\rangle$: adjacent neighbor cells, $\{\langle i-1,j\rangle, \langle i,j+1\rangle, \langle i+1,j\rangle, \langle i,j-1\rangle\}$, which are given by:

$$V_M^{\mathrm{adj}} = \{(-1,0),\ (0,1),\ (1,0),\ (0,-1)\},$$

and diagonal neighbor cells:

$$\{\langle i-1,j+1\rangle,\ \langle i+1,j+1\rangle,\ \langle i+1,j-1\rangle,\ \langle i-1,j-1\rangle\},$$

given by the set:

$$V_M^{\mathrm{diag}} = \{(-1,1),\ (1,1),\ (1,-1),\ (-1,-1)\}.$$

The 2D-CA evolves deterministically in discrete time steps, changing the states of all cells according to a local transition function f: $S^9 \to S$. The updated state of the cell $\langle i,j\rangle$ depends on the nine variables of the local transition function, which are the previous states of the cells constituting its neighborhood, that is:

$$a_{ij}^{(t+1)} = f\left(a_{i+\alpha_1,j+\beta_1}^{(t)}, \ldots, a_{i+\alpha_9,j+\beta_9}^{(t)}\right).$$

The matrix

$$C^{(t)} = \begin{pmatrix} a_{00}^{(t)} & \cdots & a_{0,s-1}^{(t)} \\ \vdots & \ddots & \vdots \\ a_{r-1,0}^{(t)} & \cdots & a_{r-1,s-1}^{(t)} \end{pmatrix}$$

is called the configuration at time t of the 2D-CA, and $C^{(0)}$ is the initial configuration of the CA. Moreover, the sequence $\{C^{(t)}\}_{0\leqslant t\leqslant k}$ is called the evolution of order k of the 2D-CA.

As the number of cells of the 2D-CA is finite, boundary conditions must be considered in order to assure the well-defined dynamics of the CA. One can state several boundary conditions but in this work, we will consider null boundary conditions: if $(i,j) \notin \{(u,v), 0 \leqslant u \leqslant r-1, 0 \leqslant v \leqslant s-1\}$ then $a_{ij}^{(t)} = 0$.

A very important type of 2D-CA are linear 2D-CA, whose local transition function is as follows:

$$a_{ij}^{(t+1)} = \sum_{(\alpha,\beta)\in V_M} \mu_{\alpha\beta} a_{i+\alpha,j+\beta}^{(t)}, \tag{1}$$

where $\mu_{\alpha\beta} \in \mathbb{R}^+$, and $(\alpha,\beta) \in V_M$. Note that every CA endowed with a local transition function of the form given by (1), has an infinite state set: $S = [0,\infty)$. Nevertheless, if finite state sets must be considered, for example $S = \mathbb{Z}_k$, then a discretization function must be used with the local transition function as follows:

$$a_{ij}^{(t+1)} = g\left(\sum_{(\alpha,\beta)\in V_M} \mu_{\alpha\beta} a_{i+\alpha,j+\beta}^{(t)}\right),$$

with $g : [0,\infty) \to \mathbb{Z}_k$.

3. The CA-based model for fire spreading

3.1. The basic model

The basic model for fire spreading based on a two-dimensional linear cellular automata with extended Moore neighborhoods, null boundary conditions and infinite state set is described as follows.

A forest area can be interpreted as the cellular space of a 2D-CA by simple dividing it into a two-dimensional array of identical square areas of side length L. Then, each one of this areas corresponds to a cell of the CA (see Fig. 1).

The state of a cell $\langle i,j\rangle$ at a time t, is defined as follows:

$$a_{ij}^{(t)} = \frac{\text{burned out area of } \langle i,j\rangle}{\text{total area of } \langle i,j\rangle}.$$

Consequently, $0 \leqslant a_{ij}^{(t)} \leqslant 1$. If $a_{ij}^{(t)} = 0$, then the cell $\langle i,j\rangle$ is said to be unburned at time t; if $0 < a_{ij}^{(t)} < 1$, then the cell $\langle i,j\rangle$ is called partially burned out at time t; and finally, if $a_{ij}^{(t)} = 1$, the cell is said to be completely burned out at time t.

The CA used in the model will be a linear CA, that is, its dynamic basically supposes that the state of a cell $\langle i,j\rangle$ at a time $t+1$ linearly depends on the states of its neighbor cells at time t; specifically, one has:

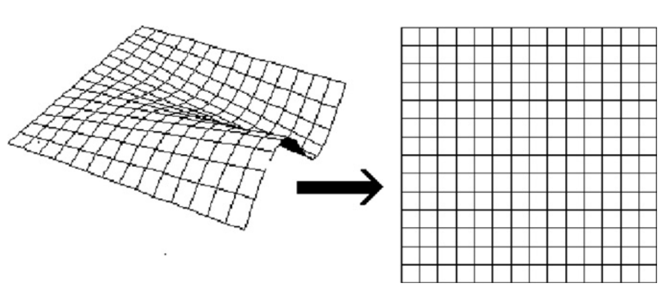

Fig. 1. The forest area as a two-dimensional array of identical cells.

$$a_{ij}^{(t+1)} = \sum_{(\alpha,\beta)\in V_M} \mu_{\alpha\beta} a_{i+\alpha,j+\beta}^{(t)}, \tag{2}$$

where $\mu_{\alpha\beta} \in \mathbb{R}^+$ are parameters involving some physical magnitudes of the cells: As each cell of the CA, $\langle i,j\rangle$, represents a small square area of the forest, then it is endowed with the three following parameters: the rate of fire spread (R_{ij}), the wind speed (W_{ij}), and the height (H_{ij}).

The rate of fire spread of $\langle i,j\rangle$, R_{ij}, determines the time needed for this cell to be completely burned out; it is assumed to be given by other model (see, for example, [15]). Note that if the cell $\langle i,j\rangle$ stands for an incombustible area, then $R_{ij} = 0$ and $a_{ij}^{(t)} = 0$ for every t.

The importance of this parameter lies in the setting-up of the size of the time step, $\tilde{t}$. Suppose that the forest is homogeneous, i.e., the value of the rate of fire spread is the same for all cells: $R_{ij} = R$, $0 \leqslant i \leqslant r-1$, $0 \leqslant j \leqslant s-1$. Then, it is easy to check that if all cells in the neighborhood of $\langle i,j\rangle$ are unburned at time t except only one adjacent neighbor cell, then the time needed for $\langle i,j\rangle$ to be completely burned out is L/R (see Fig. 2(a)).

Moreover, if all cells in the neighborhood of $\langle i,j\rangle$ are unburned at time t except only one diagonal neighbor cell, then the time needed for $\langle i,j\rangle$ to be completely burned out is $\sqrt{2}L/R$ (see Fig. 2(b)).

In this way, the time step size is taken to be equal to $\tilde{t} = L/R$. Consequently, if all cells in the neighborhood of $\langle i,j\rangle$ are unburned at time t except only one adjacent cell, which is completely burned out, then at time $t+1$, the cell $\langle i,j\rangle$ is completely burned out: $a_{ij}^{(t+1)} = 1$. On the other hand, if the only completely burned out cell at time t is a diagonal neighbor cell of $\langle i,j\rangle$, then $a_{ij}^{(t+1)} = \lambda < 1$.

Nevertheless, almost all real forests are inhomogeneous. In this case, the size time step is taken to be the time needed for the cells with the larger rate spread to be completely burned out, i.e.,

$$\tilde{t} = \frac{L}{R}, \quad R = \max\{R_{ij}, 0 \leqslant i \leqslant r-1, 0 \leqslant j \leqslant s-1\}. \tag{3}$$

Other factor to be incorporated to the model is the wind speed and direction due to its important influence to the fire spreading. The effect of the wind on a cell $\langle i,j\rangle$ is given by the following 3×3 positive matrix, called the wind matrix of $\langle i,j\rangle$:

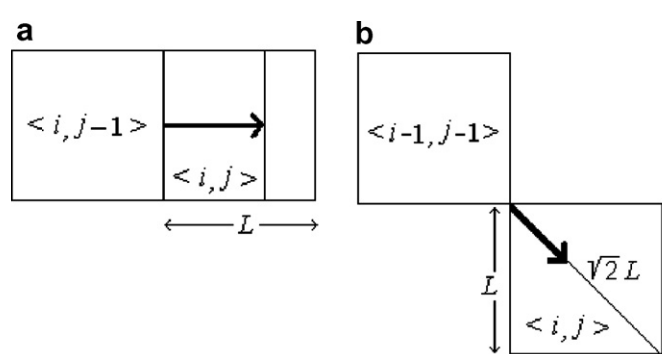

Fig. 2. Propagation from a neighbor cell to the cell $\langle i,j\rangle$.

$$W_{ij} = \begin{pmatrix} w_{i-1,j-1} & w_{i-1,j} & w_{i-1,j+1} \\ w_{i,j-1} & 1 & w_{i,j+1} \\ w_{i+1,j-1} & w_{i+1,j} & w_{i+1,j+1} \end{pmatrix},$$

in such a way that if no wind is blowing on $\langle i,j \rangle$, then $w_{i+\alpha,j+\beta} = 1$ with $(\alpha, \beta) \in V_M$; if, for example, the wind is blowing from north towards south, then the coefficients $w_{i-1,j-1}$, $w_{i-1,j}$ and $w_{i-1,j+1}$ must be larger than the rest of coefficients of W_{ij}, and so on. The values of such coefficients stand for the magnitude of the wind, and they must be determined by means of another model (see [16]).

Finally, the height differences between various points in a forest also affect to the fire spreading. As is well-known the fire front shows a higher rate of spread when they climb up an upward slope, whereas fires show a smaller rate of spread when they descend a downward slope. If H_{ij} stands for the height of the cell $\langle i,j \rangle$—H_{ij} is the height in center point of the square area which is represented by the cell, and it is supposed that this height is the same in every point of the cell—then the effect of such parameter in the fire spreading is given by the following 3×3 matrix:

$$\Phi_{ij} = \begin{pmatrix} h_{i-1,j-1} & h_{i-1,j} & h_{i-1,j+1} \\ h_{i,j-1} & 1 & h_{i,j+1} \\ h_{i+1,j-1} & h_{i+1,j} & h_{i+1,j+1} \end{pmatrix},$$

where $h_{i+\alpha,j+\beta} = \phi(H_{ij} - H_{i+\alpha,j+\beta})$, and ϕ is usually taken to be a linear function.

As a consequence, if we incorporate all these parameters to the model defined by (2), then

$$\mu_{\alpha\beta} = w_{i+\alpha,j+\beta} h_{i+\alpha,j+\beta}, \quad \forall (\alpha, \beta) \in V_M, \tag{4}$$

and the evolution of the cell $\langle i,j \rangle$ is given by:

$$a_{ij}^{(t+1)} = a_{ij}^{(t)} + \sum_{(\alpha,\beta) \in V_M^{\mathrm{adj}}} \mu_{\alpha\beta} a_{i+\alpha,j+\beta}^{(t)} + \lambda \sum_{(\alpha,\beta) \in V_M^{\mathrm{diag}}} \mu_{\alpha\beta} a_{i+\alpha,j+\beta}^{(t)}.$$

Remark that, probably, during the evolution of the CA, some cells can assume a state greater than 1. In these cases, the states must be taken to be equal to 1.

3.2. The Karafyllidis–Thanailakis model

Karafyllidis and Thanailakis proposed in [14] that the propagation of the fire front from a diagonal neighbor cell, for example $\langle i-1, j-1 \rangle$, to the main cell, $\langle i,j \rangle$, is done in a linear front, as it is shown in Fig. 3.

Consequently, as a simple calculus shows, after a time step $\tilde{t} = L/R$, the burned out area of the cell $\langle i,j \rangle$ is given by $L^2 - [(\sqrt{2}-1)L]^2$. As a consequence, if at time t all neighbor cells of $\langle i,j \rangle$ are unburned except only one diagonal cell, for example $\langle i-1, j-1 \rangle$, then:

$$a_{ij}^{(t+1)} = \frac{L^2 - [(\sqrt{2}-1)L]^2}{L^2} = 1 - (\sqrt{2}-1)^2 \approx 0.83 = \lambda.$$

As a consequence, the evolution rule of the cell $\langle i,j \rangle$ proposed for Karafyllidis and Thanailakis is:

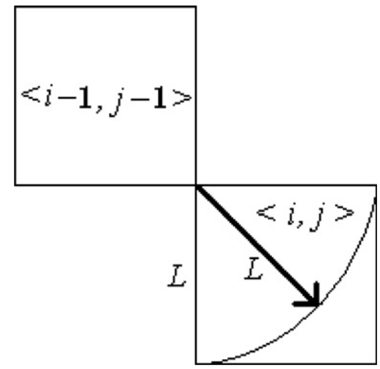

Fig. 3. Propagation from a diagonal neighbor cell to the cell $\langle i,j \rangle$ (Karafyllidis–Thanailakis model).

$$a_{ij}^{(t+1)} = a_{ij}^{(t)} + \sum_{(\alpha,\beta) \in V_M^{\mathrm{adj}}} \mu_{\alpha\beta} a_{i+\alpha,j+\beta}^{(t)} + 0.83 \sum_{(\alpha,\beta) \in V_M^{\mathrm{diag}}} \mu_{\alpha\beta} a_{i+\alpha,j+\beta}^{(t)},$$

where the parameters $\mu_{\alpha\beta}$ are defined in (4).

3.3. The new model

In this work a more realistic model is proposed. The main feature of our model is that the fire front spreading from a diagonal neighbor cell to the main cell is supposed to be circular, as it is shown in Fig. 4.

As a consequence, after a time step, the burned out area of the cell $\langle i,j \rangle$ is $\pi L^2/4$. So, if all neighbor cells of $\langle i,j \rangle$ are unburned at time t, except a diagonal neighbor, say $\langle i-1, j-1 \rangle$, which is fully burned out ($a_{i-1,j-1}^{(t)} = 1$), then

$$a_{ij}^{(t+1)} = \lambda = \frac{\pi L^2}{4L^2} = \frac{\pi}{4} \approx 0.785.$$

As a consequence, the transition function of the 2D-AC is given by:

$$a_{ij}^{(t+1)} = a_{ij}^{(t)} + \sum_{(\alpha,\beta) \in V_M^{\mathrm{adj}}} \mu_{\alpha\beta} a_{i+\alpha,j+\beta}^{(t)} + 0.785 \sum_{(\alpha,\beta) \in V_M^{\mathrm{diag}}} \mu_{\alpha\beta} a_{i+\alpha,j+\beta}^{(t)},$$

where $0 \leqslant i \leqslant r-1, 0 \leqslant j \leqslant s-1$. Note that this transition function is only valid for homogeneous forests. In the case of inhomogeneous forests, the size of the time step is given

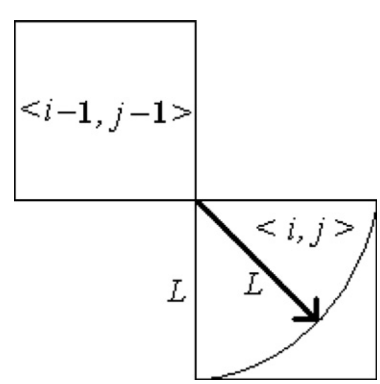

Fig. 4. Propagation from a diagonal neighbor cell to the cell $\langle i,j \rangle$ (new proposed model).

by the expression (3) and consequently we have to incorporate in the transition function a new factor: the rate of fire spread R_{ij}. It is calculated as follows: Let $\langle i,j\rangle$ be a cell and suppose $R_{ij} \neq R$; if all neighbor cells are unburned at time t except only one adjacent cell, then after a time step the burned out area of $\langle i,j\rangle$ is $R_{ij}L^2/R$; as a consequence $a_{ij}^{(t+1)} = R_{ij}/R < 1$. If the burned out neighbor cell is a diagonal cell, then the burned out area of $\langle i,j\rangle$ after a time step is $\pi R_{ij}^2 L^2/(4R^2)$, and $a_{ij}^{(t+1)} = \pi R_{ij}^2/(4R^2)$. Consequently, this new fact is incorporated in the model as follows:

$$a_{ij}^{(t+1)} = \frac{R_{ij}}{R} a_{ij}^{(t)} + \sum_{(\alpha,\beta)\in V_M^{\mathrm{adj}}} \mu_{\alpha\beta} \frac{R_{i+\alpha,j+\beta}}{R} a_{i+\alpha,j+\beta}^{(t)} + \sum_{(\alpha,\beta)\in V_M^{\mathrm{diag}}} \mu_{\alpha\beta} \frac{\pi R_{i+\alpha,j+\beta}^2}{4R^2} a_{i+\alpha,j+\beta}^{(t)},$$

with $0 \leqslant i \leqslant r-1, 0 \leqslant j \leqslant s-1$.

Furthermore, it is also possible to incorporate, in a very simple manner, changes in both wind speed and direction. It can be modeled by simple varying the wind matrix in time:

$$W_{ij}^{(t)} = \begin{pmatrix} w_{i-1,j-1}^{(t)} & w_{i-1,j}^{(t)} & w_{i-1,j+1}^{(t)} \\ w_{i,j-1}^{(t)} & 1 & w_{i,j+1}^{(t)} \\ w_{i+1,j-1}^{(t)} & w_{i+1,j}^{(t)} & w_{i+1,j+1}^{(t)} \end{pmatrix},$$

as a consequence, the evolution of the cell $\langle i,j\rangle$ with non-constants wind conditions is given by:

$$a_{ij}^{(t+1)} = \frac{R_{ij}}{R} a_{ij}^{(t)} + \sum_{(\alpha,\beta)\in V_M^{\mathrm{adj}}} w_{i+\alpha,j+\beta}^{(t)} h_{i+\alpha,j+\beta} \frac{R_{i+\alpha,j+\beta}}{R} a_{i+\alpha,j+\beta}^{(t)} + \sum_{(\alpha,\beta)\in V_M^{\mathrm{diag}}} w_{i+\alpha,j+\beta}^{(t)} h_{i+\alpha,j+\beta} \frac{\pi R_{i+\alpha,j+\beta}^2}{4R^2} a_{i+\alpha,j+\beta}^{(t)}.$$

Finally, we can discretize the states of every cell of the 2D-CA in order to obtain a new 2D-CA with discrete state set. As our goal is to study the spread of the fire front, we will consider a 2D-CA whose state set is $S = \mathbb{Z}_2$, by setting:

$$s_{ij}^{(t)} = \begin{cases} 0, & \text{if } 0 \leqslant a_{ij}^{(t)} < 1 \\ 1, & \text{if } a_{ij}^{(t)} \geqslant 1 \end{cases}$$

for $0 \leqslant i \leqslant r-1$, $0 \leqslant j \leqslant s-1$. That is, the local transition function is

$$a_{ij}^{(t+1)} = g\left(\frac{R_{ij}}{R} a_{ij}^{(t)} + \sum_{(\alpha,\beta)\in V_M^{\mathrm{adj}}} w_{i+\alpha,j+\beta}^{(t)} h_{i+\alpha,j+\beta} \frac{R_{i+\alpha,j+\beta}}{R} a_{i+\alpha,j+\beta}^{(t)} + \sum_{(\alpha,\beta)\in V_M^{\mathrm{diag}}} w_{i+\alpha,j+\beta}^{(t)} h_{i+\alpha,j+\beta} \frac{\pi R_{i+\alpha,j+\beta}^2}{4R^2} a_{i+\alpha,j+\beta}^{(t)} \right),$$

where

$$g : [0,\infty) \to \mathbb{Z}_2,$$
$$t \mapsto g(t) = \begin{cases} 0, & \text{if } 0 \leqslant t < 1, \\ 1, & \text{if } t \geqslant 1. \end{cases}$$

4. Testing the proposed model

A good model for predicting forest fire spreading must passes some tests. In this work, we will consider four basic tests, which are divided into two classes: homogeneous forest tests and inhomogeneous forest tests. In both classes of tests, we must consider flat and non-flat forests and weather conditions (wind speed and direction).

If an homogeneous flat forest without wind conditions is considered, the model must yield a circular fire front. If there are some weather conditions, the wind speed and direction must affect the forest fire front. Furthermore, if the homogeneous forest is non-flat, the topography conditions must be reflected in the dynamics of fire front since, as it is mentioned above, fires show a higher rate of spread when they climb up an upward slope, whereas fires show a smaller rate of spread when they descend a downward slope.

On the other hand, if the forest is inhomogeneous, the fire front must be of circular shape. It advances with the same speed in all directions, in the areas whose rate of fire spread is equal to R (see expression (3)); and this speed must decreases in the areas with another rate of fire spread.

An algorithm using the C++ language has been implemented for the computational and graphical representations of the fire fronts. The hypothetical forests used are modeled by means of a bidimensional array of 1024×1024 cells. In the initial configuration, there is a circular burned area of radius 10 whereas the rest cells are unburned, and 500 evolutions of the cellular automata are calculated. In the following figures, only the fire fronts at times $t = 10k$, with $k \in \mathbb{Z}, 0 \leqslant k \leqslant 50$ are shown.

First of all, suppose that the fire is spreading in a hypothetical homogeneous forest, then $R_{ij} = R \in \mathbb{R}$ for every i,j.

If the forest is flat and no wind is blowing, then one can suppose that

$$\Phi_{ij} = \begin{pmatrix} h & h & h \\ h & 1 & h \\ h & h & h \end{pmatrix}, \quad W_{ij} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

where $h \in \mathbb{R}$, and $0 \leqslant i \leqslant 1023$, $0 \leqslant j \leqslant 1023$, and for the sake of simplicity, we can also consider $h = 1$. In this case the fire front is circular as it is shown in Fig. 5(a).

Now, suppose that the forest is non-flat and there is wind blowing according to the following matrices:

$$\Phi_{ij} = \begin{pmatrix} 1.5 & 1 & 0.5 \\ 1.5 & 1 & 0.5 \\ 1.5 & 1 & 0.5 \end{pmatrix}, \quad W_{ij} = \begin{pmatrix} 0.5 & 0.5 & 0.5 \\ 1 & 1 & 1 \\ 1.5 & 1.5 & 1.5 \end{pmatrix}, \tag{5}$$

with $0 \leqslant i \leqslant 1023$, $0 \leqslant j \leqslant 1023$. Then, the evolution of the fire front is shown in Fig. 5(b).

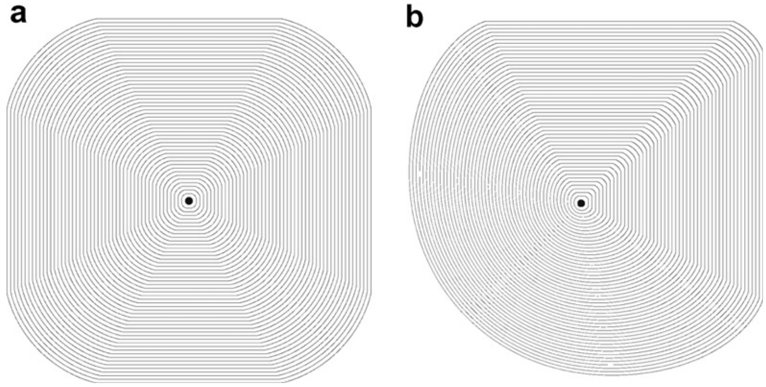

Fig. 5. Fire front spreading in an homogeneous forest.

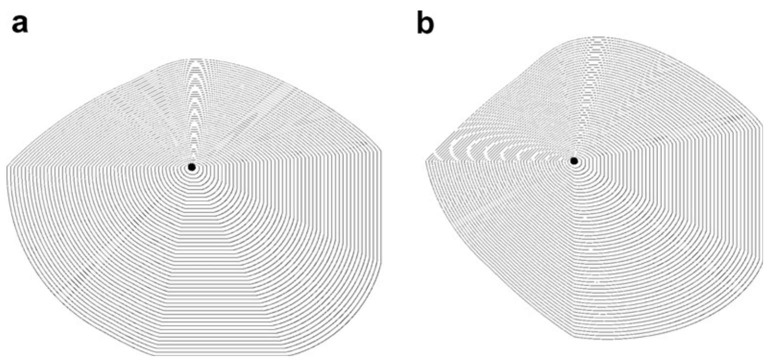

Fig. 6. Fire front spreading in an inhomogeneous forest.

Finally, suppose that the forest fire is spreading in an inhomogeneous environment, where the rates of spreading are as follows:

$$R_{ij} = \begin{cases} 1, & \text{if } 0 \leqslant i \leqslant 511, \quad 0 \leqslant j \leqslant 1023, \\ 3, & \text{if } 512 \leqslant i \leqslant 1023, \quad 0 \leqslant j \leqslant 511, \\ 4.5, & \text{if } 512 \leqslant i \leqslant 1023, \quad 512 \leqslant j \leqslant 1023. \end{cases}$$

Then, in Fig. 6(a) the evolution of a forest fire front without weather and topography conditions is shown. If the wind is blowing and the forest is non-flat according to (5), the evolution of the fire front is shown in Fig. 6(b).

5. Conclusions and further work

In this work, a CA-based model for the study of the dynamic of a forest fire front has been presented. It is a modification of the Karafyllidis and Thanailakis model in order to obtain a more realistic modelization. Basically, we have proposed a circular spreading of the fire front, when it come from a diagonal cell, instead of the linear front presented by Karafyllidis et al. The model determines the dynamic of the fire front in both homogeneous and inhomogeneous forests with weather and land topography conditions. Moreover, several tests have been checked in order to determine the goodness of the proposed method.

Further work aimed at designing a CA-based model to simulate forest fire front spreading using hexagonal cellular automata. Moreover, some changes in the notion of the state of a cell can be studied. In this sense, a similar cellular automata model will be designed in which the states of the cells will be defined by means of the transfer of heat and energy, instead of the transfer of fractional burned area.

Furthermore, due to the fact that the behavior of some discrete models has discrepancies with the corresponding continuous model, it will be of interest to study how to decrease these discrepancies. Then, further works to consider are:

(1) Consider new temporal coordinates and not only spatial coordinates. Then, new definitions of the discrete step of time are necessary.
(2) Study the influence of new variables, like humidity or air temperature in fire propagation.
(3) Finally, it can be interesting to explore another cases like spontaneous combustion, using probabilistic updating rules.

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