GramSchmidt Procedure

Sonny Monti

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Assume we have linearly independent vectors a,b,c,..., we want to construct orthogonal vectors A,B,C... such that they span the same subspace as a,b,c.
Next we want to have orthonormal basis of this subspace q1,q2,q3...

Procedure

1. Take $\underline{A} = \underline{a}$

2. it is likely that vector b is not orthogonal to A. we want to subtract vector b its projection doing the line defined by A. $\underline{B} = \underline{b} = \frac{A^T \cdot b}{A^T A}\underline{A}$
   Quick Check: $< \underline{A}, \underline{B} >=< \underline{b}, \underline{a} > - \frac{<A,b>}{\cancel{<A,A>}} \cdot \cancel{< \underline{A}, \underline{A} >}$

3. we take c and we subtract its projection along the lineas defined by A,B
   C$= \underline{C} - \frac{A^T C}{A^T A} \cdot \underline{A} - \frac{B^T C}{B^T B}\underline{B}$ and so on

4. finally we define $\underline{q}_1 = \frac{A}{||\underline{A}||}, \underline{q}_2 = \frac{B}{||\underline{B}||}$
   Example:
   $\underline{a} = \begin{vmatrix} 1 \\ -1 \\ 0 \end{vmatrix}, \underline{b} = \begin{vmatrix} 2 \\ 0 \\ -2 \end{vmatrix}, \underline{c} = \begin{vmatrix} 3 \\ -3 \\ 3 \end{vmatrix}$
   find A,B,C,q1,q2,q3
   $\underline{A} = \underline{a} = \begin{vmatrix} 1 \\ -1 \\ 0 \end{vmatrix}$
   $\underline{B} = \underline{b} - \frac{A^T b}{A^T A}\underline{A} = \begin{vmatrix} 2 \\ 0 \\ -2 \end{vmatrix} - \frac{2}{2} \begin{vmatrix} 1 \\ -1 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \\ -2 \end{vmatrix}$

$$\underline{C} = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}$$

$$\underline{q1} = \frac{1}{\sqrt{2}} \begin{vmatrix} 1 \\ -1 \\ 0 \end{vmatrix}, \underline{q2} = \frac{-1}{\sqrt{6}} \begin{vmatrix} 1 \\ 1 \\ -2 \end{vmatrix}, \underline{q3} = \frac{1}{\sqrt{3}} \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}$$