

## Q1)

The idea is to keep a cluster of nodes and a single path glued to the cluster in order to balance the diameter with the average distance.

The kind of cluster of nodes that would work for this is a complete graph, since the average distance is just 1 and can be more easily outscaled by the diameter.

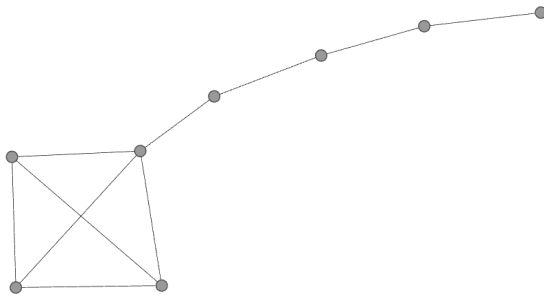
If we check the math:

$$Avg\ Distance = \frac{Sum\ of\ distances\ of\ each}{\#\ of\ pairs}$$

$$= \frac{\frac{n(n-1)}{2} + (n-1)\left(\frac{(m+1)(m+1+1)}{2} - 1\right)\left(\frac{m(m+1)}{2}\right)}{\frac{n(n-1)}{2} + mn}$$

The above equation is basically just the sum of the distances in the complete graph of size  $K_n$  and the other part is the sums of the distances to the nodes in the glued path of size  $m$ .

For example:



Is a graph with a  $K_4$  and a path of size 4 glued to it. So in that case  $n = 4$  and  $m = 4$ .

The diameter of a graph such as the one described is  $m + 1$ . This is because every node coming from the complete graph (except the one node to which the path is glued to) takes at least  $m + 1$  steps to reach the furthest node.

If we want a graph with a diameter three times the size of the average distance we would have an equation like:

$$m+1 = (3) \frac{\frac{n(n-1)}{2} + (n-1)\left(\frac{(m+1)(m+1+1)}{2} - 1\right)\left(\frac{m(m+1)}{2}\right)}{\frac{n(n-1)}{2} + mn}$$

Or

$$m+1 = (C) \frac{\frac{n(n-1)}{2} + (n-1)\left(\frac{(m+1)(m+1+1)}{2} - 1\right)\left(\frac{m(m+1)}{2}\right)}{\frac{n(n-1)}{2} + mn}$$

such that the diameter is C times as large as the average distance.

## Q2)

1. Work submitted to connex.

2. If the graph were to be undirected then there would be 21 isolated nodes. The graph is directed so I'm going to use the following definition of isolated node:

*An isolated node is a node that has no going or coming edges.*

There are 11 isolated nodes in the given directed graph.

I have included an extra method in my program for #1 that calculates the number of isolated nodes in the graph.

3. When I first implemented the diameter algorithm then the diameter turned out to be infinity. Then I compared this result with the one provided by Gephi. Gephi had 8 as the calculated diameter of the graph.

Gephi checks whether the shortest path calculated by the floyd warshall algorithm is infinity. If it is, just count it as zero. I implemented the floyd-warshall diameter algorithm this way and also got 8 as the diameter of the network.

**Q3)**

The 6 blue and red edges on the following graph represent the possible new edges according to triadic closure (the pairs of nodes to which the new edge has been added have a friend in common or two). The edges in blue have chances of the pairs becoming friends since besides having 2 friends in common, those nodes have friends that are also friends of each other.

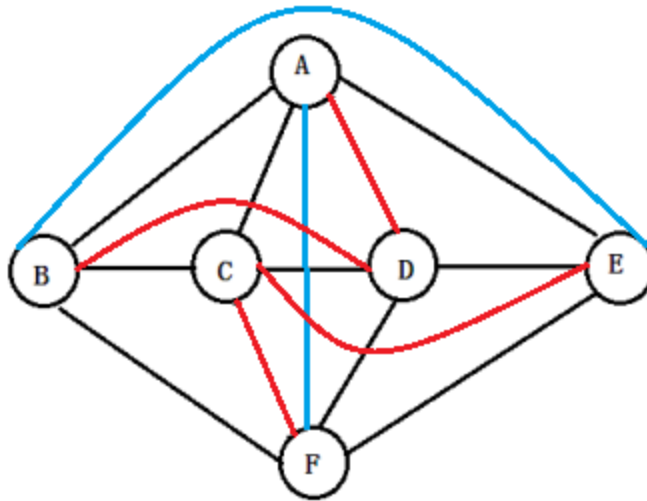


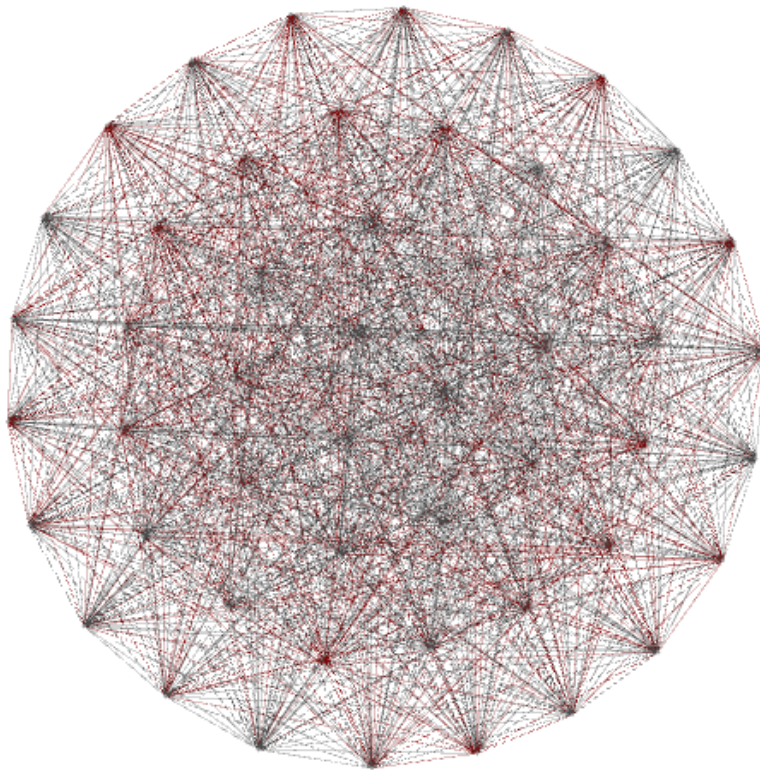
Figure 1: The network in Problem 3

**Q4)**

The generated graph is a complete graph  $K_{50}$ . This network is NOT structurally balanced. This is because you cannot split the graph into two opposing factions. The result would rather include many cases like this:

Two nodes are enemies with each other but they have a common friend, someone who is close enough to be friends of two nodes that are far apart enough to be enemies with each other.

The construction of the  $K_{50}$  Graph would look like this:



The black edges represent friendships while nodes joined by red edges are enemies. The above graph does not highlight ALL nodes that need to be red. (there were too many) but it's enough to notice the pattern.