

Q1) The files have been submitted to connex. To run for this problem:

```
java homphily graph.csv gender.csv
```

I use two different csv files as input, graph.csv with node adjacency information. gender.csv contains information about gender. homophily.java parses both files and constructs an adjacency matrix with an extra column for gender. Homophily test is then run.

expected cross gender fraction: 0.2591

Actual Fraction of cross gender: 0.23947

Q2) Assuming changing friends to enemies can be done at no cost.

Make two groups A and B.

Fix all bad relationships (enemies) into friends in A

Fix all bad relationships (enemies) into friends in B.

Make every person from group A who is friends with group B and vice versa enemies with each other.

Q3 on next page...>

Q3)

Q3

a) Singler Problem.

If it's linear then we have two cases.

A s_1 B

$$\begin{aligned} P_k &= \Pr(\text{first step is left}) \cdot \Pr(A \text{ gets token before } B \mid \text{first step is left}) \\ &\quad + \Pr(\text{first step is right}) \cdot \Pr(A \text{ gets token first} \mid \text{first step is right}) \\ &= \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \Pr(A \text{ gets token before } B \mid \text{first step is right}) \end{aligned}$$

Now we are on this configuration:

A s_1 s_2 ... s_k B

↑
Token

s_1 now has the role of A, so probability of $s_1 = P_{k-1}$

This gives the following Recurrence.

$$P_k = \frac{1}{2} \cdot 1 + \frac{1}{2} P_{k-1} \cdot P_k$$

$$P_1 = \frac{1}{2}, \text{ then } P_k = \frac{1}{2 - P_{k-1}} \text{ for } k \geq 0$$

$$\text{Solution to Recurrence} \rightarrow P_k = \frac{k}{k+1}$$

Same applies for circle table

n 0 1 2 ... n-2 n-1

↑
token we need the probability that
n-1 gets token before n, thus $k = n-1$

∴ Speaker n gets token before n-1

$$\text{with probability } \frac{(n-1)}{(n-1)+1} = \frac{n-1}{n}$$

$$\text{we need the complement: } 1 - \frac{n-1}{n} = \frac{1}{n}$$

b) Same as calculated previously for n-1 before,
 $= \left(\frac{n-1}{n} \right)$

Q4)

a)

$$P(A|H) = \frac{P(A)P(H|A)}{P(B|A)(P(A) + P(B|A^c)P(A^c))}$$

$$P(H|A) = 3/4 \text{ and } P(A) = P(R) = 1/2, \text{ thus } P(A|H) = 3/4$$

$$P(R|H) = 1 - (3/4) = (1/4)$$

(unless you want us to assume a low signal is given. In such case it is also $(3/4)$).

b)

c) We know $\text{prob}(H/G) > 1/2$

the probability of accept if we have a high signal is higher.

The observation of this high signal thus creates the cascade of accept (since it is known that good is actually true).

$$P(A|HHH) = \frac{P(A)P(HHH|A)}{P(HHH)}$$