I believe five out of four people have trouble with fractions.
--Steven Wright

Numerators, Denominators, and Reciprocals of Fractions

A fraction indicates a portion of a whole number. There are two types of fractions:
- common fractions such as ½ or ¾ ;
- decimal fractions such as 0.5 or 0.75 (we already covered these in Chapter 1).

A fraction is an expression of division, with one number placed over another number. The bottom number, or denominator, indicates the total number of parts into which the whole is divided. The top number, or numerator, indicates how many of those parts are considered. The fraction may be read as the "numerator divided by the denominator."
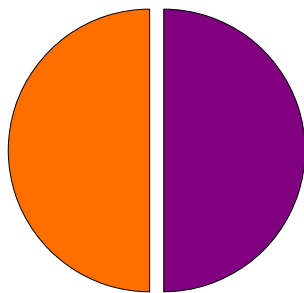
$$\frac{\textbf{Numerator}}{\textbf{Denominator}}$$

Examples

¼ = 1 part of 4 parts, or ¼ of the whole = 0.25
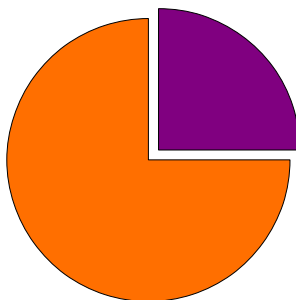
½ = 1 part of 2 parts, or ½ of the whole = 0.5

Reciprocals of fractions

To find the reciprocal of a fraction simply switch the numerator and denominator (flip it over)

- The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$

- The reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$

A whole number could be considered to have a denominator of one, so the reciprocal of a whole number would be 1 over the original whole number.

- The reciprocal of 5 is $\frac{1}{5}$

- The reciprocal of 9 is $\frac{1}{9}$

The following page is a worksheet to help reinforce this information.

Worksheet 2-1

Name:

Date:

Convert the following fractions to decimals.

1) $\frac{1}{5}$ = _____

2) $\frac{1}{2}$ = _____

3) $\frac{2}{4}$ = _____

4) $\frac{1}{10}$ = _____

5) $\frac{1}{12}$ = _____

6) $\frac{1}{100}$ = _____

7) $\frac{1}{1000}$ = _____

8) $\frac{2}{3}$ = _____

9) $\frac{3}{4}$ = _____

10) $\frac{4}{5}$ = _____

11) $\frac{3}{8}$ = _____

12) $\frac{5}{12}$ = _____

13) $\frac{7}{18}$ = _____

14) $\frac{4}{11}$ = _____

15) $\frac{2}{22}$ = _____

16) $\frac{5}{35}$ = _____

17) $\frac{9}{55}$ = _____

18) $\frac{13}{63}$ = _____

19) $\frac{15}{71}$ = _____

20) $\frac{23}{83}$ = _____

Determine the reciprocal of the following fractions.

21) $\frac{1}{5}$ = _____

22) $\frac{1}{2}$ = _____

23) $\frac{2}{4}$ = _____

24) $\frac{1}{10}$ = _____

25) $\frac{1}{12}$ = _____

26) $\frac{1}{100}$ = _____

27) $\frac{1}{1000}$ = _____

28) $\frac{2}{3}$ = _____

29) $\frac{3}{4}$ = _____

30) $\frac{4}{5}$ = _____

31) $\frac{3}{8}$ = _____

32) $\frac{5}{12}$ = _____

33) $\frac{7}{18}$ = _____

34) $\frac{4}{11}$ = _____

35) $\frac{2}{22}$ = _____

36) $\frac{5}{35}$ = _____

37) $\frac{9}{55}$ = _____

38) $\frac{13}{63}$ = _____

39) $\frac{15}{71}$ = _____

40) $\frac{23}{83}$ = _____

Reducing fractions to lowest terms

Reducing a fraction to lowest terms (also called simplifying), you need to divide the numerator and denominator by their greatest common factor.
- The greatest common factor is the largest whole number that can divide into the numerator and the denominator.
- Some fractions are already in lowest terms if there is no factor common to the numerator and the denominator.

The steps to simplify a fraction:
- List the whole number factors of the numerator and the denominator.
- Find the factors common to both the numerator and denominator.
- Divide the numerator and denominator by the largest common factor.

Example: $\dfrac{30}{70}$

List the factors of the numerator and the denominator:
- Numerator: 1,2,3,5,6,10,15,30
- Denominator: 1,2,5,7,10,14,35,70

The greatest common factor is 10

Divide the numerator and the denominator by 10
- Numerator: $30 \div 10 = 3$
- Denominator: $70 \div 10 = 7$

$\dfrac{3}{7}$ is the fraction when reduced to lowest terms.

Reduce the fractions in the following practice problems:

1) $\dfrac{4}{8}$

2) $\dfrac{55}{99}$

3) $\dfrac{36}{144}$

Name:

Date:

Reduce the following fractions to lowest terms.

1) $\frac{2}{4}$ = _____

2) $\frac{3}{9}$ = _____

3) $\frac{2}{16}$ = _____

4) $\frac{18}{54}$ = _____

5) $\frac{20}{240}$ = _____

6) $\frac{25}{125}$ = _____

7) $\frac{55}{75}$ = _____

8) $\frac{28}{35}$ = _____

9) $\frac{63}{90}$ = _____

10) $\frac{15}{75}$ = _____

11) $\frac{14}{36}$ = _____

12) $\frac{7}{11}$ = _____

13) $\frac{0}{18}$ = _____

14) $\frac{34}{51}$ = _____

15) $\frac{14}{63}$ = _____

16) $\frac{12}{35}$ = _____

17) $\frac{125}{500}$ = _____

18) $\frac{270}{2700}$ = _____

19) $\frac{65}{585}$ = _____

20) $\frac{82}{164}$ = _____

21) $\frac{79}{237}$ = _____

22) $\frac{17}{102}$ = _____

23) $\frac{19}{285}$ = _____

24) $\frac{18}{81}$ = _____

25) $\frac{121}{605}$ = _____

26) $\frac{63}{135}$ = _____

27) $\frac{42}{72}$ = _____

28) $\frac{33}{77}$ = _____

29) $\frac{108}{180}$ = _____

30) $\frac{110}{363}$ = _____

Adding and subtracting fractions

In order to add and subtract fractions, they must have a common denominator. Once they have a common denominator, you may add and subtract the numerators like you would in an ordinary addition or subtraction problem and then write the sum or the difference over top of the common denominator.

How to create equivalent fractions with a common denominator:
- Find a multiple for the denominator of both numbers.
- Rewrite the fractions as equivalent fractions with the common denominator.

Example: $\dfrac{4}{5} - \dfrac{1}{3}$

You can multiply the denominators to find a common denominator:
$5 \times 3 = 15$

Now create equivalent fractions:
$$\dfrac{4}{5} \times \dfrac{3}{3} = \dfrac{12}{15} \qquad \dfrac{1}{3} \times \dfrac{5}{5} = \dfrac{5}{15}$$

Now let's solve the problem with the equivalent fractions:

$$\dfrac{12}{15} - \dfrac{5}{15} = \dfrac{\mathbf{7}}{\mathbf{15}}$$

Try the following practice problems:

1) $\quad \dfrac{1}{3} - \dfrac{1}{5}$

2) $\quad \dfrac{3}{8} + \dfrac{3}{5}$

3) $\quad \dfrac{1}{19} + \dfrac{3}{38} + \dfrac{5}{76} + \dfrac{7}{152}$

1) 2/15   2) 39/40   3) 37/152

65

Name:

Date:

Solve the following problems.

1) $\dfrac{1}{3} + \dfrac{1}{3}$ = ____

2) $\dfrac{5}{8} - \dfrac{3}{8}$ = ____

3) $\dfrac{7}{8} - \dfrac{3}{8}$ = ____

4) $\dfrac{4}{25} + \dfrac{1}{5}$ = ____

5) $\dfrac{1}{8} + \dfrac{3}{16}$ = ____

6) $\dfrac{7}{8} - \dfrac{1}{4}$ = ____

7) $\dfrac{2}{7} + \dfrac{3}{7} + \dfrac{1}{7}$ = ____

8) $\dfrac{1}{8} + \dfrac{2}{8} + \dfrac{7}{8}$ = ____

9) $\dfrac{14}{38} - \dfrac{1}{19}$ = ____

10) $\dfrac{11}{12} - \dfrac{2}{3}$ = ____

11) $\dfrac{9}{24} - \dfrac{1}{6}$ = ____

12) $\dfrac{4}{5} - \dfrac{67}{100}$ = ____

13) $5 + \dfrac{7}{10} + \dfrac{3}{1000}$ = ____

14) $\dfrac{5}{8} + \dfrac{4}{27} + \dfrac{1}{48}$ = ____

15) $1 + \dfrac{7}{100} + \dfrac{3}{10}$ = ____

16) $\dfrac{1}{2} + \dfrac{1}{9} + \dfrac{1}{36}$ = ____

17) $\dfrac{1}{3} + \dfrac{1}{6} + \dfrac{1}{9}$ = ____

18) $\dfrac{11}{99} + \dfrac{4}{9} + \dfrac{11}{33}$ = ____

19) $\dfrac{1}{8} + \dfrac{1}{4} + \dfrac{1}{2} + \dfrac{3}{8}$ = ____

20) $\dfrac{12}{19} + \dfrac{14}{38} + \dfrac{1}{19} + \dfrac{1}{38}$ = ____

21) $\dfrac{1}{8} + \dfrac{2}{8} + \dfrac{7}{8}$ = ____

22) $\dfrac{10}{30} + \dfrac{13}{15} + \dfrac{1}{5} + \dfrac{17}{60}$ = ____

23) $\dfrac{1}{8} + \dfrac{2}{8} + \dfrac{7}{8}$ = ____

24) $\dfrac{1}{81} + \dfrac{2}{27} + \dfrac{1}{3}$ = ____

Multiplying fractions

- Reduce if possible
- Multiply the numerators of the fraction to get the new numerator.
- Multiply the denominators of the fractions to get the new denominator.
- Simplify the resulting fraction if possible.

Example: $\dfrac{2}{3} \times \dfrac{4}{7} = ?$

Reduce if possible

$\dfrac{2}{3} \times \dfrac{4}{7}$ no reduction is possible.

Multiply the numerators

$2 \times 4 = 8$

Multiply the denominators

$3 \times 7 = 21$

Simplify the resulting fraction if possible

$\dfrac{8}{21}$ is already in simplest terms.

$$\dfrac{2}{3} \times \dfrac{4}{7} = \mathbf{\dfrac{8}{21}}$$

Try the following practice problems:

1) $\dfrac{2}{5} \times \dfrac{1}{8} =$

2) $\dfrac{2}{3} \times \dfrac{2}{3} =$

3) $\dfrac{7}{8} \times \dfrac{9}{21} =$

1) 1/20 2) 4/9 3) 3/8

Multiplying mixed numbers

- Write mixed numbers as improper fractions.
- Reduce if possible.
- Multiply the numerators of the fraction to get the new numerator.
- Multiply the denominators of the fractions to get the new denominator.
- Simplify the resulting fraction if possible.

Example: $4\frac{1}{8} \times 1\frac{5}{11} = ?$

Write mixed numbers as improper fractions.
$$\frac{33}{8} \times \frac{16}{11}$$

Reduce if possible
$$\frac{^3\cancel{33}}{_1\cancel{8}} \times \frac{^2\cancel{16}}{_1\cancel{11}}$$

Multiply the numerators
$$3 \times 2 = 6$$

Multiply the denominators
$$1 \times 1 = 1$$

Simplify the resulting fraction if possible
$$\frac{6}{1} = 6$$

$$4\frac{1}{8} \times 1\frac{5}{11} = \frac{^3\cancel{33}}{_1\cancel{8}} \times \frac{^2\cancel{16}}{_1\cancel{11}} = \frac{6}{1} = \mathbf{6}$$

Try the following practice problems:

1) $13\frac{1}{3} \times 1\frac{4}{5} =$

2) $7\frac{1}{5} \times 3\frac{3}{4} =$

3) $20\frac{2}{3} \times 1\frac{4}{31} =$

1) 24 2) 27 3) 23 1/3

Name:

Date:

Solve the following problems.

1) $\frac{1}{3}$ of $\frac{1}{3}$ = _____

2) $\frac{1}{8}$ of $\frac{3}{8}$ = _____

3) $\frac{2}{3}$ of $\frac{1}{3}$ = _____

4) $\frac{5}{8}$ of $\frac{3}{8}$ = _____

5) $\frac{1}{5}$ of $\frac{3}{5}$ = _____

6) $\frac{7}{8}$ of $\frac{3}{8}$ = _____

7) $\frac{0}{4}$ of $\frac{5}{6}$ = _____

8) $\frac{1}{2}$ of $\frac{3}{5}$ = _____

9) $\frac{1}{9}$ of $\frac{1}{2}$ = _____

10) $\frac{2}{7}$ of $\frac{1}{5}$ = _____

11) $\frac{9}{10}$ of $\frac{3}{4}$ = _____

12) $\frac{3}{11}$ of $\frac{3}{4}$ = _____

Multiply the following fractions.

13) $\frac{3}{10} \times \frac{1}{4}$ = _____

14) $\frac{13}{15} \times \frac{7}{8}$ = _____

15) $\frac{21}{25} \times \frac{2}{5}$ = _____

16) $\frac{49}{50} \times \frac{1}{4}$ = _____

17) $\frac{6}{7} \times 21$ = _____

18) $25 \times \frac{3}{10}$ = _____

19) $\frac{8}{15} \times \frac{5}{6}$ = _____

20) $\frac{11}{14} \times \frac{2}{33}$ = _____

Multiply the following mixed numbers.

21) $4\frac{4}{5} \times 1\frac{1}{6}$ =

22) $7\frac{1}{2} \times 2\frac{2}{3}$ =

23) $1\frac{17}{18} \times 1\frac{1}{5}$ =

24) $1\frac{5}{7} \times 10\frac{1}{2}$ =

Dividing Fractions

Dividing by fractions is just like multiplying fractions with one exception, you need to "flip" (find the reciprocal of) the fraction you are dividing by.

The following are the steps to dividing fractions:

- Find the reciprocal of the fraction you are dividing by.
- Reduce if possible.
- Multiply the numerators of the fraction to get the new numerator.
- Multiply the denominators of the fractions to get the new denominator.
- Simplify the resulting fraction if possible.

A simpler way of remembering this is to simply think "Flip and Multiply"

Example: $\dfrac{2}{3} \div \dfrac{4}{7}$

"Flip and Multiply"
$$\dfrac{\overset{1}{2}}{3} \times \dfrac{7}{\underset{2}{4}} = \dfrac{7}{6} = \mathbf{1\dfrac{1}{6}}$$

Try the following practice problems:

1) $\dfrac{2}{5} \div \dfrac{1}{8} \ =$

2) $\dfrac{2}{3} \div \dfrac{2}{3} \ =$

3) $\dfrac{7}{8} \div \dfrac{9}{21} \ =$

Name:

Date:

Divide the following fractions:

1) $\dfrac{1}{3} \div \dfrac{1}{3} =$ _____

2) $\dfrac{1}{8} \div \dfrac{3}{8} =$ _____

3) $\dfrac{1}{8} \div \dfrac{3}{16} =$ _____

4) $\dfrac{4}{25} \div \dfrac{1}{5} =$ _____

5) $\dfrac{2}{3} \div \dfrac{1}{2} =$ _____

6) $\dfrac{16}{27} \div \dfrac{8}{9} =$ _____

7) $\dfrac{5}{15} \div 5 =$ _____

8) $\dfrac{9}{10} \div \dfrac{9}{10} =$ _____

9) $\dfrac{7}{5} \div \dfrac{5}{7} =$ _____

10) $12 \div \dfrac{3}{8} =$ _____

11) $3\dfrac{2}{3} \div 1\dfrac{1}{4} =$ _____

12) $1\dfrac{5}{6} \div 7\dfrac{1}{3} =$ _____

13) $15\dfrac{3}{4} \div 5\dfrac{1}{7} =$ _____

14) $15 \div 1\dfrac{1}{5} =$ _____

Solve the following word problems:

15) How many 6 ¼ milligram capsules of metoprolol tartrate can be made from 500 mg of metoprolol tartrate?

16) The OR likes to use phenylephrine syringes, each containing 4/5 of a milligram. How many phenylephrine syringes can I make if I have a 20 mg vial on hand?

Name:

Date:

_____

Convert these fractions into equivalent decimals (you only need to solve them out to the thousandths position).

1) $\dfrac{7}{15}$ = _____

4) $\dfrac{13}{15}$ = _____

7) $\dfrac{9}{27}$ = _____

2) $\dfrac{13}{40}$ = _____

5) $\dfrac{17}{30}$ = _____

8) $\dfrac{17}{25}$ = _____

3) $\dfrac{5}{8}$ = _____

6) $\dfrac{7}{8}$ = _____

9) $\dfrac{13}{37}$ = _____

_____

Determine the reciprocals of the following fractions.

10) $\dfrac{3}{4}$ = _____

12) $\dfrac{15}{24}$ = _____

14) $\dfrac{65}{585}$ = _____

11) $\dfrac{1}{5}$ = _____

13) $\dfrac{2}{8}$ = _____

15) 1 = _____

_____

Reduce the following fractions.

16) $\dfrac{6}{8}$ = _____

19) $\dfrac{2}{8}$ = _____

22) $\dfrac{27}{45}$ = _____

17) $\dfrac{12}{48}$ = _____

20) $\dfrac{12}{36}$ = _____

23) $\dfrac{17}{31}$ = _____

18) $\dfrac{15}{24}$ = _____

21) $\dfrac{18}{24}$ = _____

24) $\dfrac{52}{148}$ = _____

_____

Perform the following additions and subtractions.

25) $\dfrac{1}{6} + \dfrac{3}{6}$ = _____

28) $\dfrac{5}{6} - \dfrac{2}{3}$ = _____

26) $\dfrac{7}{6} + \dfrac{5}{24}$ = _____

29) $\dfrac{1}{8} + \dfrac{2}{3}$ = _____

27) $\dfrac{7}{8} - \dfrac{7}{12}$ = _____

30) $\dfrac{11}{16} - \dfrac{1}{2}$ = _____

_____

31) $\dfrac{1}{6} + \dfrac{4}{7}$ = _____   32) $\dfrac{9}{11} - \dfrac{2}{5}$ = _____

Perform the following multiplications and divisions.

33) $\dfrac{1}{6} \times \dfrac{4}{7}$ = _____   37) $10 \div \dfrac{1}{2}$ = _____

34) $4 \times \dfrac{7}{8}$ = _____   38) $\dfrac{7}{16} \times 7$ = _____

35) $\dfrac{3}{4} \div \dfrac{1}{6}$ = _____   39) $\dfrac{3}{4} \times \dfrac{1}{3}$ = _____

36) $\dfrac{7}{16} \div 7$ = _____   40) $\dfrac{3}{4} \div \dfrac{1}{3}$ = _____

Solve the following word problems.

41) What is the total volume of a mixture of 1/8 milliliter of solution X mixed with 2 and 1/4 milliliters of solution Y?

42) A patient is to receive 3 liters of an intravenous solution. If 2 1/8 liters have already been administered, how much solution remains?

43) How many milligrams of drug are needed to make 20 tablets of 1/4 milligrams each?

44) A bottle of Children's Tylenol contains 20 teaspoons of liquid. If each dose for a 2 year-old child is 1/2 teaspoon, how many doses are available in this bottle?

45) A patient is on strict recording of fluid intake and output, including measurements of liquid medications. A nursing student gave the patient 1/4 ounce of medication at 8 AM and 1/3 ounce of medication at 12 noon. What is the total amount of medication to be recorded on the Intake and Output sheet?