ST102 Class 14 – Additional exercises

1. The *number* of red balls and white balls in an urn is unknown, but the *proportion*, π, of red balls is either 1/3 or 1/2. A random sample of size 5, drawn with replacement, yields the sequence:

   red, white, white, red and white.

   What is the maximum likelihood estimate of π?

2. A random sample of size 8 yielded $x_1 = 1$, $x_2 = 0$, $x_3 = 1$, $x_4 = 1$, $x_5 = 0$, $x_6 = 1$, $x_7 = 1$ and $x_8 = 0$. The sample was obtained from the probability function:

$$p(x;\pi) = \begin{cases} \pi^x(1-\pi)^{1-x} & \text{for } x = 0, 1 \\ 0 & \text{otherwise.} \end{cases}$$

   Derive the maximum likelihood estimate of π.

3. A random sample $\{X_1, X_2, \ldots, X_n\}$ is drawn from the distribution with the following probability density function:

$$f(x;\theta) = \frac{\theta}{2\sqrt{x}}\,\mathrm{e}^{-\theta\sqrt{x}}$$

   for $x \geq 0$, and 0 otherwise.

   (a) Find the maximum likelihood estimator of θ.

   (b) Suppose $n = 4$ and we observe $x_1 = 4.1$, $x_2 = 7.3$, $x_3 = 6.5$ and $x_4 = 8.8$. Based on this random sample, calculate the maximum likelihood estimate of $\theta/2$.

4. A random sample $\{X_1, X_2, \ldots, X_n\}$ is drawn from the following distribution:

$$p(x;\lambda) = \frac{\lambda^{2x}\mathrm{e}^{-\lambda^2}}{x!}$$

   for $x = 0, 1, 2, \ldots$, and 0 otherwise.

   (a) Find the maximum likelihood estimator of λ.

   (b) State the maximum likelihood estimator of $\theta = \lambda^3$.