

ST102 Class 6 – Additional exercises

1. Show that:

$$p(x) = \begin{cases} 2x/(k(k+1)) & \text{for } x = 1, 2, \dots, k \\ 0 & \text{otherwise} \end{cases}$$

is a valid probability function for a discrete random variable X .

2. Show that:

$$f(x) = \begin{cases} 1/x^2 & \text{for } x \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

is a valid probability density function, but that X does not have a finite expected value.

3. The waiting time X (in minutes) of a customer at a taxi rank has the following probability density function:

$$f(x) = \begin{cases} 0.25e^{-0.25x} & \text{for } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

This is the exponential distribution with parameter $\lambda = 0.25$. In answering the following questions, you can use all the results for this distribution stated in the lectures, without having to derive them again.

For this distribution, calculate the probability that the waiting time is:

- (a) 5 minutes
 - (b) at most 4 minutes
 - (c) at least 8 minutes
 - (d) at most 0.5 standard deviations from its expected value.
4. Consider the dartboard shown in the picture to the right. Suppose that the diameter of the innermost circle in the middle (which has the point value of 10) is 2 units, and the width of each of the bands 1–9 is 1 unit, so that the radius of the circle up to the outer edge of the white 1-band is 10 units. Suppose that a player throwing a dart at the board has a probability of 1 of hitting this circle of radius 10, but that they are equally likely to hit any point within this circle. This means that the probability of hitting any region within the circle is proportional to the area of that region.



Let X denote the distance from the location of a randomly thrown dart to the centre of the board.

- (a) Write down the cumulative distribution function, $F(x)$, of X .
- (b) Derive the probability density function of X , and show that it satisfies the necessary conditions to be a pdf.
- (c) What is the probability that a randomly thrown dart scores 9 or 10?