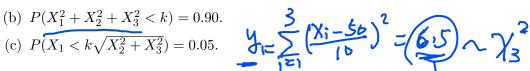
## ST102 Class 12 – Additional exercises

- 1. Suppose  $X_i \sim N(0,3)$ , for i=1,2,3,4. Assume all these random variables are independent. Derive the value of k in each of the following.
  - (a)  $P(X_1 + 3X_2 > 4) = k$ .



- 2. A random sample of size n=3 is observed such that  $x_1=65$ ,  $x_2=30$  and  $x_3=55$ . Using a chi-squared distribution, are the data consistent with being drawn from N(50, 100)?
- 3. Of the following two differences:

$$t_{0.05,n} - t_{0.10,n}$$
 and  $t_{0.10,n} - t_{0.15,n}$ 

where  $t_{\alpha,n}$  is such that  $P(T > t_{\alpha,n}) = \alpha$  for  $T \sim t_n$ , which is larger?

4. Use the fact that  $(n-1)S^2/\sigma^2 \sim \chi^2_{n-1}$  to prove that:

$$Var(S^2) = \frac{2\sigma^4}{n-1}.$$

Hint: Use the fact that the variance of a chi-squared random variable with k degrees of freedom is 2k.

5. If  $Y \sim \chi_n^2$ , it can be shown that:

$$\frac{Y-n}{\sqrt{2n}} \to N(0,1)$$

as  $n \to \infty$ . Use the asymptotic normality of  $(Y-n)/\sqrt{2n}$  to approximate the 40th percentile of a chi-squared random variable with 200 degrees of freedom.