# ST102 Class 7 – Additional exercises

1. Suppose $X$ has a discrete uniform distribution, such that:

$$p(x) = \begin{cases} 1/k & \text{for } x = 1, 2, \ldots, k \\ 0 & \text{otherwise.} \end{cases}$$

Show that its moment generating function is:

$$M_X(t) = \frac{\mathrm{e}^t(1 - \mathrm{e}^{kt})}{k(1 - \mathrm{e}^t)}.$$

(Do not attempt to find the mean and variance.)

2.* Suppose that $X \sim \mathrm{Bin}(n, \pi)$ and $Y \sim \mathrm{Poisson}(\lambda)$. Show that if $n \to \infty$ and $\pi \to 0$ in such a way that $n\pi = \lambda$ remains constant, then, for any $x$, we have:

$$P(X = x) \to P(Y = x) \quad \text{as } n \to \infty.$$

Hint 1: Since $n\pi = \lambda$ remains constant, substitute $\lambda/n$ for $\pi$ from the beginning.

Hint 2: One step of the proof uses the limit definition of the exponential function which states that, for any number $y$, then:

$$\lim_{n\to\infty} \left(1 + \frac{y}{n}\right)^n = \mathrm{e}^y.$$

3. Cars independently pass a point on a busy road at an average rate of 150 per hour.

   (a) Assuming a Poisson distribution, find the probability that no cars pass in a given *two-minute* period.

   (b) What is the expected number of cars passing in *one minute*?

   (c) Find the probability that more than the expected number of cars actually pass in a given one-minute period.

4. Common probability distributions are built into R. Here, we consider the binomial distribution – specifically, let $X \sim \mathrm{Bin}(20, 0.4)$.

   (a) Plot the probability function of $X$, using:

```
> x <- 0:20
> plot(x,dbinom(x,size=20,prob=0.4),type="h")
```

   (b) Calculate $P(X \leq 5)$, using:

```
> pbinom(5,size=20,prob=0.4)
```

   and hence also calculate $P(X = 5)$.

(c) Obtain a random sample of 10 observations from $X$, using:

```
> rbinom(10,size=20,prob=0.4)
```

5. Here, we consider the Poisson distribution – specifically, let $X \sim \text{Pois}(7)$.

   (a) Plot the probability function of $X$, using:

   ```
   > x <- 0:20
   > plot(x,dpois(x,7),type="h")
   ```

   (b) Calculate $P(X \leq 5)$, using:

   ```
   > ppois(5,7)
   ```

   and hence also calculate $P(X = 5)$.

   (c) Obtain a random sample of 10 observations from $X$, using:

   ```
   > rpois(10,7)
   ```