ST102 Class 11 – Additional exercises

1. A perfectly-machined regular tetrahedral (pyramid-shaped) die has four faces labelled 1 to 4. It is tossed twice onto a level surface and after each toss the number on the face which is downwards is recorded. If the recorded values are x_1 and x_2 when order matters, then the observed sample mean is $\bar{x} = (x_1 + x_2)/2$. Write out the sampling distribution of the sample mean as a random quantity over repeated double tosses.

2. A random sample of size $n = 50$ is drawn from the following probability density function:

$$f(x) = \begin{cases} 3(1-x)^2 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

   Use the central limit theorem to approximate $P(0.225 \leq \bar{X} \leq 0.275)$.

3. Let $\{X_1, X_2, \ldots, X_9\}$ be a random sample from a normal distribution with $\mu_X = 2$ and $\sigma_X = 2$. Let $\{Y_1, Y_2, \ldots, Y_4\}$ be an independent random sample from a normal distribution with $\mu_Y = 1$ and $\sigma_Y = 1$. Calculate $P(\bar{X} > \bar{Y})$.