probability Theory

- Set theory
  - sets
  - operators $\cap$, $\cup$, ^{c}, Venn graph
  - relation
  - interpetation
- probability
  - 3 Axioms
    - $P(A) \geq 0$, $A \subseteq S$ $S=\{1,2,3,4,5,6\}$
    - $P(S)=1$
    - $P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$ $A_i \cap A_j = \phi$, $i \neq j$
  - properties — $P(A \cup B)$, $P(A^c)$
  - counting def $P(A) = \frac{\#\{A\}}{\#\{S\}}$
    - ways : listing
      : Combinatorial method
  - Conditional probability
    - independent $P(A\cap B) = P(A)P(B)$
    - def : $P(A|B) = \frac{P(A\cap B)}{P(B)}$ ... ①
    - total Probability $P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$
    - Bayes $P(B_j|A) = \frac{P(A\cap B_j)}{P(A)} = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{n} P(A|B_i)P(B_i)}$ ... ②
- random variable
  - Discrete $p(x) = P(X=x)$, $\begin{cases} p(x) \geq 0 \\ \sum_i p(x_i) = 1 \end{cases}$
    - cdf: $P(X \leq x) = F(x) = \sum_{x_i \leq x} p(x_i)$
    - Discrete Uniform $p(x) = \frac{1}{n}$,
    - Bernoulli $Bin(1,\pi) \sim Y_i$, $p(x) = \pi^x (1-\pi)^{1-x}$
    - Binomial $Bin(n,\pi) = \sum_{i=1}^{n} Y_i$, $p(x) = C_n^x \pi^x (1-\pi)^{n-x}$
    - Poisson $Pois(\lambda)$; $p(x) = e^{-\lambda} \frac{\lambda^x}{x!}$,
  - waiting time
  - Continuous $f(x)$; $P(X=x)=0$. $\begin{cases} f(x) \geq 0 \\ \int_{-\infty}^{\infty} f(x)\,dx = 1 \end{cases}$
    - $P(X \leq x) = F(x) = \int_{-\infty}^{x} f(t)\,dt$
    - uniform $U(a,b)$ $f(x) = \frac{1}{b-a}$, $x \in [a,b]$
    - Exponential $Exp(\lambda)$ $f(x) = \lambda e^{-\lambda x}$, $x \in (0,\infty)$
    - Normal $N(\mu,\sigma)$ $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{(x-\mu)^2}{2\sigma^2}\}$, $x \in (-\infty,\infty)$
  - properties
    - $E[X] = \begin{cases} \sum_{i=1}^{n} x_i p(x_i) \\ \int_{-\infty}^{\infty} x f(x)\,dx \end{cases}$, $E[g(X)] = \begin{cases} \sum_{i=1}^{n} g(x_i)p(x_i) \\ \int_{-\infty}^{\infty} g(x)f(x)\,dx \end{cases}$
    - $Var(X) = E[X^2] - (E[X])^2$
    - $M_X(t) = E[e^{tX}]$, $\begin{cases} M_X^{(k)}(0) = E[X^k] \\ M_X(0) = 1 \end{cases}$