ST102 Class 15 – Additional exercises

1. A random sample of size 11 was drawn from a normal distribution with unknown mean μ and known standard deviation $\sigma = 1.1$, resulting in the following 11 observations:

   3.26, 1.76, 1.63, 1.79, 2.43, 0.88, 0.99, 1.12, 4.56, 2.11 and 2.73.

   (a) Obtain a 99% confidence interval for μ.

   (b) Plot the data by hand marking the points on the interval $[0, 5]$. Mark the sample mean, the sample median, and the confidence interval obtained in (a).

   (c) If the width of a 99% confidence interval should not exceed 0.4, how many more observations are required in order to achieve this objective?

2. Which of the following two intervals has the greater probability of containing the binomial probability parameter, π, where $X \sim \text{Bin}(n, \pi)$? Assume a large value of n.

$$\left(\frac{X}{n} - 0.67 \times \sqrt{\frac{(X/n)(1 - X/n)}{n}}, \frac{X}{n} + 0.67 \times \sqrt{\frac{(X/n)(1 - X/n)}{n}} \right)$$

   and:

$$\left(\frac{X}{n}, \infty \right).$$

3. Assume that the binomial parameter π is to be estimated with the function X/n, where X is the number of successes in n independent trials. Which demands the larger sample size:

   (a) requiring that X/n has a 96% probability of being within 0.05 of π

   (b) requiring that X/n has a 92% probability of being within 0.04 of π?

4. A coin is tossed n terms to estimate $P(\text{heads}) = \pi$. How large must n be to guarantee that the width of an *approximate* 99% confidence interval for π will be less than 0.01?

5.* A random sample of n independent Bernoulli trials with success probability π results in R successes. Derive an unbiased estimator of $\pi(1 - \pi)$.