Additional Exercise 4

Q1.

You may not know the continuous random variable at this point. Here what you need to know is that $P(X_1 = X_2 = X_3) = 0$('distinct' in question), that is, these 3 independent random variables will not take the same value.

You can think as follow, we can treat them as a fair die that only take finite numbers of values (1,2,3,4,5,6). Each X can only take one of those values with equal probability and any of two X_i will not have the same values (there can't be $X_i = X_j, i \neq j$) . Then no matter how you assign the value to each of the X_i, they always follow some ordering, which is equivalent to ordering X_1 X_2 X_3 and the total number of ways is 3! = 6.

Now you can treat the interval (0,a) as the sample space that takes many values such that (0, 0.1a, 0.2a, 0.3a,..., a) or (0,0.01a, 0.02a, ... , 0.99a, a) or even thinner. Each X can only take one of those values with equal probability and any of two X_i will not have the same values. Use the same reasoning as above, no matter how you assign the values to each of the X, they always follow some ordering. The total number of ways is the same as above 3! = 6.

Q4.

(a) We have no information about the which kind of Ace. Like choosing the lightbulbs in the EX3. You can simply classify the cards into 2 set.

One set is all Aces (4 cards); The other one can be any other cards (48 cards)

$n(A_2)$ choose at least 2 cards (can be 2,3,4) from set of Aces; and choose rest of the cards (3,2,1) from the other set of cards

$n(A_1)$ choose at least 1 card (can be 1,2,3,4) from set of Aces; and choose rest of the cards (4,3,2,1) from the other set of cards. Or using the complement argument

(b) Similarly, we can classify the cards into 2 set, one is 2 black Aces (2) and other cards (50)

$n(B)$ choose at least one card (can be 1, 2) from set of black Aces (2) and choose rest of the cards from the other sets (4,3)

$B \cap A_2$ is the event that there are at least 2 Aces and one of them must be black

Classify the card into 3 sets, 2 black Aces (2), 2 red Aces (2), other cards (48)

$n(B \cap A_2)$

1. choose 1 from black, choose 1 from red, 3 from other cards

choose 1 from black, choose 2 from red, 2 from other cards

2. choose 2 from black, 0 from red, 3 from other cards

   choose 2 from black, 1 from red, 2 from other cards

   choose 2 from black, 2 from red, 1 from other cards

The last three lines is equivalent to classify the cards into 2 sets, one is Black Aces (2) and other cards (50)

choose 2 from black, 3 from other cards

(c) Classify the cards into 2 sets, one is black spades Ace (1) and other cards (51)

$n(S)$ choose 1 from spades Ace, and 4 from other cards

Classify the cards into 3 sets, one is black spades Ace (1), one is other Aces (3) and other cards (48)

$n(S \cap A_2)$ choose 1 from spades Ace, at least one from other Aces (can be 1,2,3), and choose (3,2,1) from other card