

ST102 Class 2 – Additional exercises

1. Suppose that a number x is to be selected from the real line, \mathbb{R} , and let A , B and C be the events represented by the following subsets of \mathbb{R} :

$$A = \{x \mid 1 \leq x \leq 5\}$$

$$B = \{x \mid 3 < x \leq 7\}$$

$$C = \{x \mid x \leq 0\}.$$

Describe each of the following events as a set of real numbers:

- (a) A^c
 - (b) $A \cup B$
 - (c) $B \cap C^c$
 - (d) $A^c \cap B^c \cap C^c$
 - (e) $(A \cup B) \cap C$.
2. An airline keeps information about its passengers and has noted the following facts about the services it supplied between London and Singapore during a particular week. The airline only operated two different types of aircraft, which they denote by A and Z .
- (a) Travellers on Z always have excess baggage, X , whereas travellers on A sometimes do not.
 - (b) Smokers, S , always travel on A and always have excess baggage.
 - (c) There is no Executive Class, E , travel on Z .
 - (d) Businessmen, B , always travel Executive Class and never smoke.
 - (e) Passengers requesting champagne, C , are always businessmen and never have excess baggage.

Interpret each of the above statements (a) to (e) in set notation, and hence construct a single Venn diagram to illustrate the relationships between A , B , C , E , S , X and Z .

The following additional quantitative data are available for the week and route in question.

- 40% of all travellers on the airline used type Z aircraft.
- Only 20% of all travelling businessmen did not request champagne.
- Businessmen make up 80% of all the Executive Class travellers.
- 150 smokers travelled with the airline. This represents 10% of all the travellers.
- There were 160 passengers who requested champagne.

What is the minimum and maximum number of non-smoking, non-Executive Class travellers on A aircraft?

(The number of elements in a set is called the *order* of the set and is denoted by $n(\cdot)$. For example, we have $n(C) = 160$ and $n(S) = 150$.)

3. The 120 employees of a company are classified according to whether or not they are:
- skilled, S , or unskilled
 - female, F , or male
 - employed on the production line, P , or not.

| Group | Number of workforce | % of total salary bill |
|-------------------|---------------------|------------------------|
| F | 57 | 43 |
| P | 70 | 67 |
| S | 18 | 20 |
| $F \cap P$ | 21 | 26 |
| $F \cap S$ | 7 | 8 |
| $S \cap P$ | 8 | 4 |
| $F \cap P \cap S$ | X (unknown) | Y (unknown) |

The table gives the number of employees which fall into each group identified, and also the percentage of the total salary bill paid to each group.

- (a) From this table calculate the number of people (as a function of X) in each of the eight disjoint subsets which can be logically identified and produce an appropriate Venn diagram. Similarly, produce a fully annotated Venn diagram for each group's % of the total salary bill with subset orders as a function of Y .
- (b) Assuming that each subset of the above Venn diagrams has a strictly positive integer order, determine the smallest possible value for X and the largest possible value for Y .
- (c) Assuming the values of X and Y determined in (b), which one of the eight subsets has the lowest % salary per person?