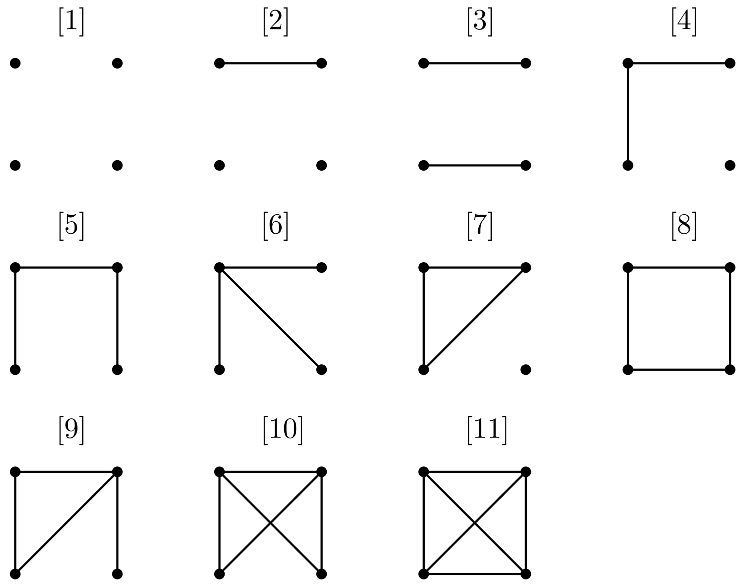
ST102 Class 5 – Additional exercises

1. Consider again the four-person friendship networks:



   Suppose that you select one person at random from a network of a given pattern, and define the following two random variables:

   (a) X: the number of friends the selected person has

   (b) Y: the average number of friends that the selected person's friends have.

   For example, reading clockwise from the bottom-left corner, for pattern [5] the values of X are 1, 2, 2 and 1, each with probability 1/4, so $P(X = 1) = 1/2$ and $P(X = 2) = 1/2$. Similarly, the values of Y for pattern [5] are 2, 1.5, 1.5 and 2, each with probability 1/4, so $P(Y = 1.5) = 1/2$ and $P(Y = 2) = 1/2$.

   Calculate $\mathrm{E}(X)$ and $\mathrm{E}(Y)$ for patterns [6], [8] and [9] (i.e. 6 expected values in total). Comparing $\mathrm{E}(X)$ and $\mathrm{E}(Y)$ in each case, what do you observe?

2. An examination consists of four multiple choice questions, each with a choice of three answers. Let X be the number of questions answered correctly when a student resorts to pure guesswork for each answer.

   (a) Draw the probability distribution of X, and find its mean and variance.

   (b) An examiner calculates a rescaled mark using the formula $Y = 10 + 22.5X$. Find the mean and variance of Y.

3. A discrete random variable X has possible values $0, 1, 2, \ldots, n$, where n is a known integer. The probability function of X is:

$$p(x) = \begin{cases} \binom{n}{x} \pi^x (1-\pi)^{n-x} & \text{for } x = 0, 1, 2, \ldots, n \\ 0 & \text{otherwise} \end{cases}$$

where:

$$\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$$

denotes the binomial coefficient, and π is a probability parameter such that $0 \leq \pi \leq 1$, i.e. the binomial distribution.

Consider this distribution in the case where $n = 4$ and $\pi = 0.8$.

(a) Calculate the values $p(x) = P(X = x)$ for each of $x = 0$, 1, 2, 3 and 4.

(b) Sketch the cumulative distribution function of X.

4. Suppose we have a biased coin which comes up heads with probability π. An experiment is carried out so that X is the number of independent flips of the coin required until r heads show up, where $r \geq 1$ is known. Determine the probability function of X.