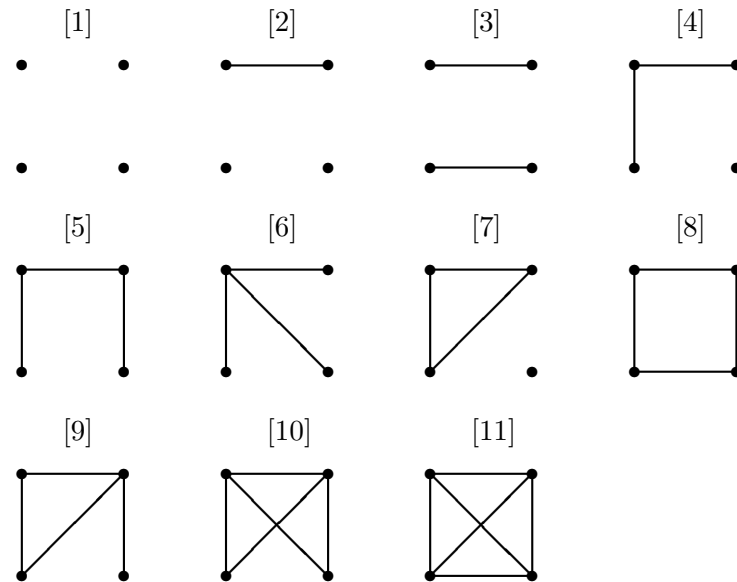# ST102 Class 3 – Additional exercises

1. Consider the patterns of four-person networks below (introduced on page 38 of the course pack).



   Suppose the four people are labelled A, B, C and D. In this exercise we consider the number of different friendship networks for some patterns, when we take into account the identities of who is friends with whom.

   For example, there are 6 different networks of pattern [2]. This is the number of ways in which we can choose the two individuals who are connected to each other. (If you want to list them, the pairs are AB, AC, AD, BC, BD and CD.) In each case the other two are then the two unconnected individuals.

   How many networks of each pattern are there?

2. Two different squares are selected at random on an $8 \times 8$ chessboard. What is the probability that they share a common boundary (i.e. that they have an edge in common, not just a single corner)?

3. A researcher wants to estimate the number of fish, $N$, in a lake and decides to use the following methodology. Initially, the researcher catches 50 fish and tags them, then returns the 50 fish to the lake. A few days later, another 50 fish are caught and it is discovered that three of these are tagged. Assume all fish in the lake have the same probability of being caught.

   (a) For each $N$, i.e. for $N = 50, 51, \ldots$, calculate the probability that in the second catch of 50 fish only three fish are tagged.

   (b) For which value of $N$ is the probability in (a) maximised?

   (c) Is this *most likely* estimate in (b) reasonable? Explain your reasoning.