ST102 Class 2 – Solutions to Additional exercises

1. (a) These are the points not in A, hence they must be either below 1 or above 5, so:

$$A^c = \{x \,|\, x < 1 \text{ or } x > 5\}.$$

   (b) These are the points in either A or B or both, so they must be between 1 and 5 or between 3 and 7. Hence:

$$A \cup B = \{x \,|\, 1 \leq x \leq 7\}.$$

   (c) These are the points in B but not in C. Noting that $B \subset C^c$, then:

$$B \cap C^c = B = \{x \,|\, 3 < x \leq 7\}.$$

   (d) These are the points in none of the three sets, hence:

$$A^c \cap B^c \cap C^c = \{x \,|\, 0 < x < 1 \text{ or } x > 7\}.$$

   (e) These are the points common to $A \cup B$ and C. There are no such values, hence:

$$(A \cup B) \cap C = \emptyset.$$

2. We have the following.

   (a) $Z \subseteq X$.

   (b) $S \subseteq (A \cap X)$.

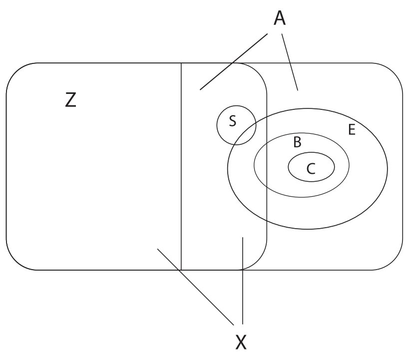
   (c) $E \cap Z = \emptyset$.

   (d) $B \subseteq E \cap S^c$.

   (e) $C \subseteq B \cap X^c$.

   Note that there are alternative ways of depicting the above statements in set notation.

   The Venn diagram, taking into account all of the above relationships, might look something like the one shown below.



   Finally, we have that $n(S) = 150 = 10\%$ implies that $n(\text{travellers}) = 1{,}500$.

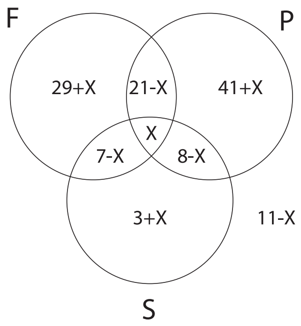
Also $n(C) = 160$ and, therefore, $n(C)/n(B) = 0.8$ which implies that $n(B) = 200$.

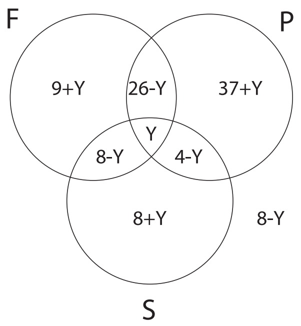
Hence $n(E) = 250$, $n(Z) = 600$ and $n(A) = 900$.

If $S \cap E = \emptyset$, then $n(S^c \cap E^c \cap A) = 900 - (250 + 150) = 500$ (the minimum).

If $n(S \cap E) = 50$ (the most, since $B \cap S = \emptyset$) then $n(S^c \cap E^c \cap A) = 550$ (the maximum).

3. (a) We have the following Venn diagrams for workforce and % of total salary, respectively.





(b) Since each subset must have a positive integer order then $\{X \,|\, X = 1, 2, \ldots, 6\}$ and $\{Y \,|\, Y = 1, 2, 3\}$. Hence the minimum value of X is 1, and the maximum value of Y is 3.

(c) Using the above values for X and Y and evaluating the percentage of salaries per person for each of the eight subsets we can construct the following table:

Subset	Workforce	% total salary	$\frac{\text{\% total salary}}{\text{workforce}}$
$F \cap P^c \cap S^c$	30	12	0.40
$P \cap F^c \cap S^c$	42	40	0.95
$S \cap F^c \cap P^c$	4	11	2.75
$F \cap P \cap S^c$	20	23	1.15
$F \cap S \cap P^c$	6	5	0.83
$S \cap P \cap F^c$	7	1	**0.14**
$F \cap P \cap S$	1	3	3.00
$F^c \cap P^c \cap S^c$	10	5	0.50

Hence the lowest salary per person (in bold) for subsets is $S \cap P \cap F^c$, skilled males employed on the production line (perhaps surprisingly).