ST102 Class 4 – Additional exercises

1. Three different (i.e. distinct) points, X_1, X_2 and X_3, are chosen at random in the interval $(0, a)$. A second set of distinct points, Y_1, Y_2 and Y_3, is chosen at random in the interval $(0, b)$. Let A be the event that X_2 is *between* X_1 and X_3. Let B be the event that $Y_1 < Y_2 < Y_3$. Find $P(A \cap B)$.

2. Two students, James A and James B, are both registered for a certain course. Assume that James A attends 80% of the time, and James B attends 60% of the time, and the absences of the two students are independent.
   (a) What is the probability that at least one of the two students will be in class on a given day?
   (b) If at least one of the two students is in class on a given day, what is the probability that James A is in class that day?

3. A fair six-sided die (with sides numbered $1, 2, \ldots, 6$) is tossed repeatedly until four '1's have been obtained.
   (a) Define X to be the number of tosses required to obtain four '1's. Show that the probability function is given by:
   $$p(x) = \begin{cases} 5^{x-4}(x-1)(x-2)(x-3)/6^{x+1} & \text{for } x = 4, 5, \ldots \\ 0 & \text{otherwise.} \end{cases}$$
   (b) Given that the probability function in (a) satisfies the conditions for a valid probability function, determine the value of:
   $$\sum_{x=4}^{\infty} (x-1)(x-2)(x-3) \left(\frac{5}{6}\right)^{x-4}.$$

4. A spectator happens to briefly see a poker hand (i.e. of five cards) of one of the players in a game (and sees no hands of any of the other players). The spectator sees an ace, but not which one (i.e. it was not observed whether it was a red ace or a black ace).
   (a) What is the probability that the player's hand had at least two aces?
   (b) The spectator now remembers that the ace seen was black. Remembering this, what is the probability that the hand had at least two aces?
   (c) On further reflection, the spectator remembers that the ace seen was the ace of spades. Remembering this, what is the probability that the hand had at least two aces?
   (d) Discuss how the above conditional probabilities change.