26.10.2021 ->

Seminars 5-7

RE ⇔ RG ⇔ FA

I. RE ⇔ RG

Ex.: $\Sigma = \{0,1\}$

0 -> {0}

01 -> {01}

0+1 -> {0,1}

ϵ -> { ϵ }

$(0+1)^*$ -> { ϵ ,0,1, 00,01,11,10, …}

$(0+1)^+$-> { 0,1, 00,01,11,10, …}

(1+ ϵ)0 -> {10, 0}

1. Given RE $0(0+1)^*1$, find an equivalent RG.

Sol.: *#IW Andrada T.*

S->0A

A->0A| 1A| 1

0 : $G_1 = (\{S_1\}, \{0,1\}, \{S_1 \to 0\}, S_1)$

1 : $G_2 = (\{S_2\}, \{0,1\}, \{S_2 \to 1\}, S_2)$

0+1 : $G_3 = (\{S_1, S_2, S_3\}, \{0,1\}, \{S_1 \to 0, S_2 \to 1, S_3 \to 0|1\ \}, S_3)$, eliminate inaccessible and obtain

$G_3' = (\{S_3\}, \{0,1\}, \{S_3 \to 0|1\ \}, S_3)$

$(0+1)^*$: $G_4 = (\{S_3\}, \{0,1\}, \{S_3 \to \epsilon|0|1|0\ S_3|1\ S_3\ \}, S_3)$

$G_4' = (\{S_3\}, \{0,1\}, \{S_3 \to \epsilon|0\ S_3|1\ S_3\ \}, S_3)$! NOT REGULAR

$0(0+1)^*$: $G_5 = (\{S_1, S_3\}, \{0,1\}, \{S_1 \to 0S_3, S_3 \to \epsilon|0\ S_3|1\ S_3\ \}, S_1)$! NOT REGULAR

$0(0+1)^*1$:

$G_6 = (\{S_1, S_2, S_3\}, \{0,1\}, \{ S_1 \to 0S_3, S_3 \to S_2|0S_3|\ 1S_3, S_2 \to 1\}, S_1)$, !NOT RIGHT LINEAR,

Eliminate single productions & inaccessible symbols and obtain

$\boldsymbol{G_6} = (\{\boldsymbol{S_1}, \boldsymbol{S_3}\}, \{\boldsymbol{0}, \boldsymbol{1}\}, \{ \boldsymbol{S_1} \to \boldsymbol{0S_3}, \boldsymbol{S_3} \to \boldsymbol{1}|\boldsymbol{0S_3}|\ \boldsymbol{1S_3}\}, \boldsymbol{S_1})$

2. Give the RE corresponding to the grammar below.

$G = (\{S, A, B\}, \{a, b\}, P, S)$
$P : S \to aA$
$\quad A \to aA \mid bB \mid b$
$\quad B \to bB \mid b$

Sol.: a^+b^+

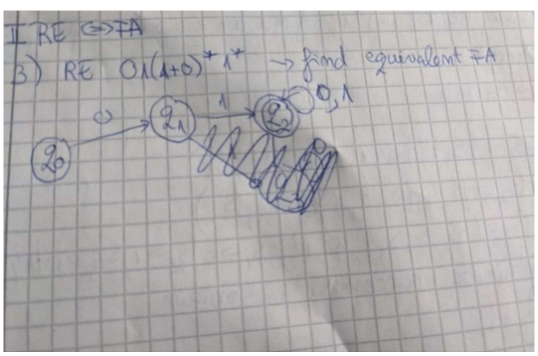
$$\begin{cases} S = aA \\ A = aA + bB + b \\ B = bB + b \end{cases}$$

$X = aX + b \Rightarrow sol. X = a^*b$

- ⇨ $B = b^*b = b^+ \Rightarrow$
- ⇨ $A = aA + b^+ \Rightarrow A = a^*b^+ \Rightarrow$
- ⇨ $S = aa^*b^+ = a^+b^+$

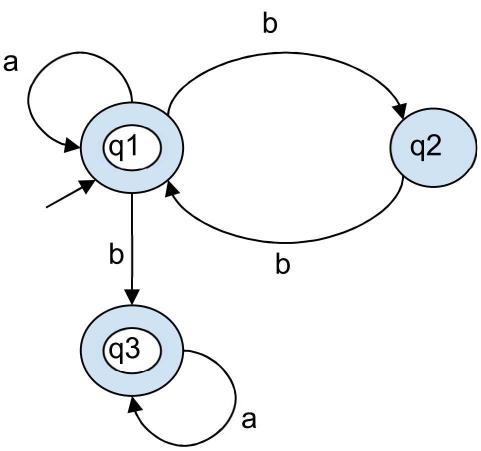
II. RE ⇔ FA

3. Given the RE $01(1 + 0)^*1^*$, find an equivalent FA.



Sol.: @board

4. Given the FA below, build the equivalent RE.



Sol.:

$$\begin{cases} q_1 = \epsilon + q_1 a + q_2 b \\ q_2 = q_1 b \\ q_3 = q_1 b + q_3 a \end{cases}$$

$X = Xa + b => sol. X = ba^*$

$$q_1 = \epsilon + q_1 a + q_1 bb = q_1(a + bb) + \epsilon => q_1 = \epsilon(a + bb)^* = (a + bb)^*$$
$$q_3 = q_1 b + q_3 a = q_3 a + q_1 b => q_3 = q_1 ba^* = (a + bb)^* ba^*$$

Required RE: $q_1 + q_3 = (a + bb)^* + (a + bb)^* ba^* = (a + bb)^*(\epsilon + ba^*)$

III. RG ⇔ FA

5. Given the RG G=({S,A},{a,b}, P, S)

P: S→ϵ| aA
A→aA|bA|a|b

build the equivalent FA.

Sol.:

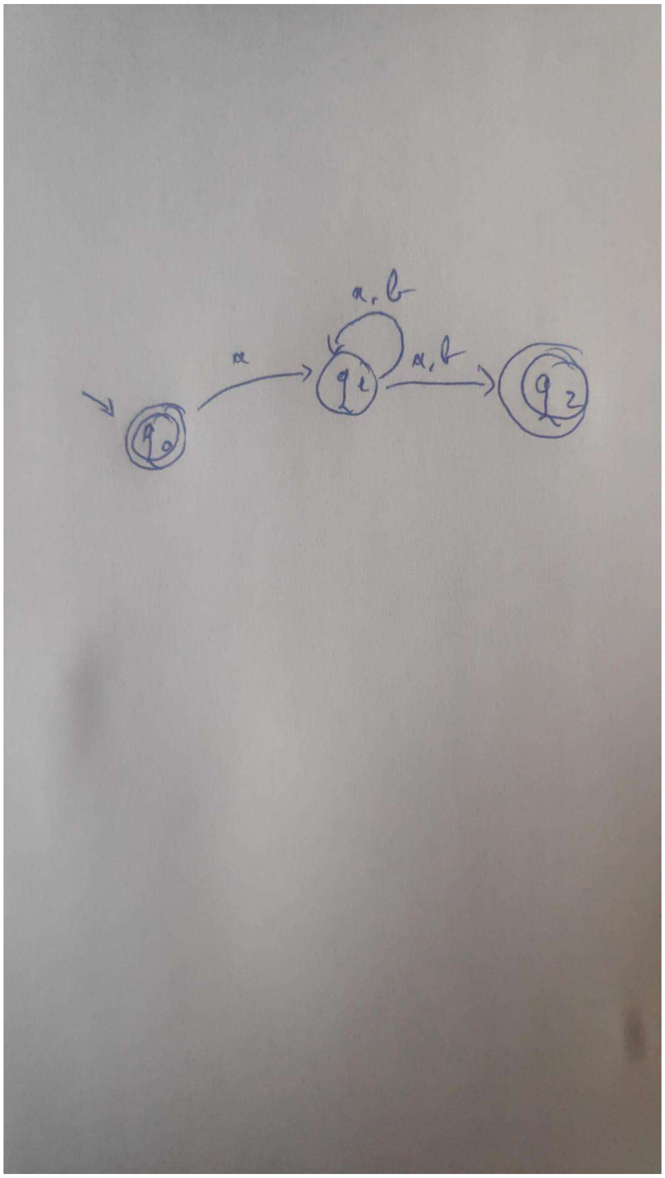
$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{S, A, K\}$, $q_0 = S$, $F = \{K, S\}$, $\Sigma = \{\text{a, b}\}$

$\delta(S, a) = \{A\}$

$\delta(A, a) = \{A, K\}$

$\delta(A, b) = \{A, K\}$



6. Given the following FA M=({p,q,r}, {0,1},δ, p, {p,r})

δ	0	1
p	q	p
q	r	p
r	r	r

build the equivalent RLG.

Sol.:

$G = (N.\Sigma, P, S)$

N={p,q,r}
S=p
P: $p \to \epsilon|0q|1p|1$
$q \to 0r|0|1p|1$
$r \to 0r|0|1r|1$

Test:

1. $\boldsymbol{a}^*\boldsymbol{b}^*$
2. $\boldsymbol{a}^*\boldsymbol{b}^+$
3. $\boldsymbol{a}^+\boldsymbol{b}^*$
4. $\boldsymbol{a}^+\boldsymbol{b}^+$

? ⇔ RLG