Course 6

Problem: Parsing (construct the parsee tree)

if the *source program is sintactically correct*

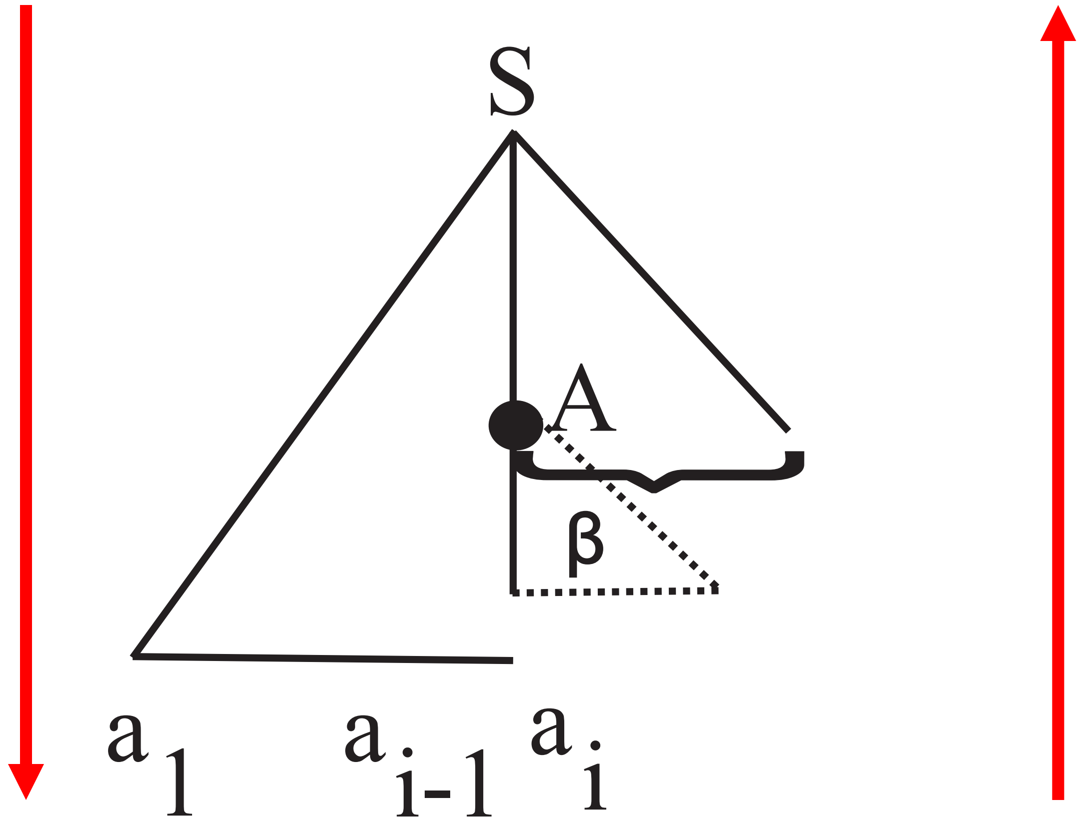
then construct syntax tree

else ”syntax error”

source program is sintactically correct = $w \in L(G) \Leftrightarrow S \overset{*}{=>} w$

Parsing

- How:
  1. Top-down vs. Bottom-up
  2. Recursive vs. linear

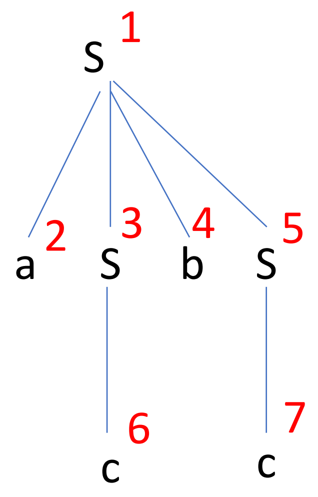


	Descendent	Ascendent
Recursive	Descendent recursive parser	Ascendent recursive parser
Linear	LL(k): LL(1)	LR(k): LR(0), SLR, LR(1), LALR

Result – parse tree -representation

- Arbitrary tree – child sybling representation
- Sequence of derivations S => $\boldsymbol{\alpha}_1$ => $\boldsymbol{\alpha}_2$ =>… => $\boldsymbol{\alpha}_n$ = w
- String of production – index associated to prod – which prod is used at each derivation step: 1,4,3,…

index	Info	Parent	Right sibling
1	S	0	0
2	a	1	0
3	S	1	2
4	b	1	3
5	S	1	4
6	c	3	0
7	c	5	0



Descendent recursive parser

- Example

Formal model

- Configuration

$$(\mathbf{s}, \mathbf{i}, \boldsymbol{\alpha}, \boldsymbol{\beta})$$

where:

- s = state of the parsing, can be:
  - q = normal state
  - b = back state
  - f = final state - corresponding to success: $w \in L(G)$
  - e = error state – corresponding to insuccess: $w \notin L(G)$
- i – position of current symbol in input sequence
  $w = a_1 a_2 \ldots a_n$, $i \in \{1, \ldots, n+1\}$
- α = working stack, stores the way the parse is built
- β = input stack, part of the tree to be built

Initial configuration:
$(q, 1, \varepsilon, S)$

Define moves between configurations

Final configuration:
$(f, n+1, \alpha, \varepsilon)$

Expand

WHEN: head of input stack is a nonterminal

$(q,i, \boldsymbol{\alpha}, A\boldsymbol{\beta}) \vdash (q,i, \boldsymbol{\alpha}A_1, \boldsymbol{\gamma}_1\boldsymbol{\beta})$

where:

$A \rightarrow \boldsymbol{\gamma}_1 \mid \boldsymbol{\gamma}_2 \mid \ldots$ represents the productions corresponding to A

1 = first prod of A

Advance

WHEN: head of input stack is a terminal = current symbol from input

$(q,i, \boldsymbol{\alpha}, a_i\boldsymbol{\beta}) \vdash (q,i+1, \boldsymbol{\alpha}a_i, \boldsymbol{\beta})$

Momentary insuccess

WHEN: head of input stack is a terminal ≠ current symbol from input

$(q,i, \boldsymbol{\alpha}, a_i\boldsymbol{\beta}) \vdash (b,i, \boldsymbol{\alpha}, a_i\boldsymbol{\beta})$

Back

WHEN: head of working stack is a terminal

$(b, i, \boldsymbol{\alpha}a, \boldsymbol{\beta}) \vdash (b, i-1, \boldsymbol{\alpha}, a\boldsymbol{\beta})$

Another try

WHEN: head of working stack is a nonterminal

$(b,i,\boldsymbol{\alpha} A_j, \boldsymbol{\gamma}_j \boldsymbol{\beta}) \vdash$ $(q,i,\boldsymbol{\alpha} A_{j+1}, \boldsymbol{\gamma}_{j+1} \boldsymbol{\beta})$, if $\exists\ A \rightarrow \boldsymbol{\gamma}_{j+1}$

$(b,i,\boldsymbol{\alpha}, A\,\boldsymbol{\beta})$, otherwise with the exception

$(e,i,\boldsymbol{\alpha}, \boldsymbol{\beta})$, if $i=1$, $A = S$, **ERROR**

Success

$(q, n+1, \boldsymbol{\alpha}, \varepsilon) \vdash (f, n+1, \boldsymbol{\alpha}, \varepsilon)$

Algorithm

Algorithm Descendent Recursive

INPUT: G, w =$a_1a_2\ldots a_n$
OUTPUT: string of productions and message

```
config = (q,1, ε,S);                                    //initial configuration (s,i,α,β)
while (s ≠ f) and (s ≠ e) do
   if s = q
     then if (i=n+1) and IsEmpty(β)
             then Success(config)
             else
                  if Head(β) = A
                     then Expand(config)
                     else
                        if  Head(β) = a_i
                            then Advance(config)
                            else MomentaryInsuccess(config)
     else
         if s = b
            then
                 if Head(α) = a
                    then Back(config)
                    else AnotherTry(config)
endWhile
if s = e  then message"Error"
         else message "Sequence accepted";
               BuildStringOfProd(α)
```

w ∈ L(G) - HOW

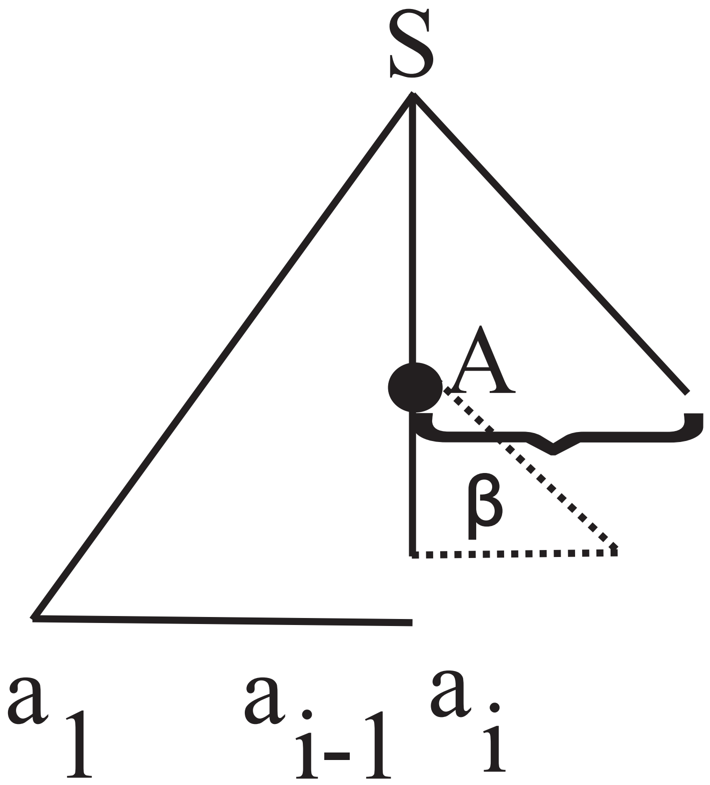
- Process $\boldsymbol{\alpha}$:
  - From left to right (reverse if stored as stack)
  - Skip terminal symbols
  - Nonterminals – index of prod

- Example: $\boldsymbol{\alpha} = S_1$ a S_2 a S_3 c b S_3 c

When the algorithm never stops?

- S->S**α** – expand infinitely (left recursive)

LL(1) Parser



Linear algorithm

FIRST_k

- ≈ first k terminal symbols that can be generated from α
- **Definition**:

$$FIRST_k : (N \cup \Sigma)^* \rightarrow \mathcal{P}(\Sigma^k)$$
$$FIRST_k(\alpha) = \{u|u \in \Sigma^k, \alpha \overset{*}{\Rightarrow} ux, |u| = k \text{ sau } \alpha \overset{*}{\Rightarrow} u, |u| \leq k\}$$

Construct FIRST

Concatenation of length 1

- $FIRST_1$ denoted FIRST
- Remarks:
  - If L_1, L_2 are 2 languages over alphabet Σ, then : $L_1 \oplus L_2 = \{w|x \in L_1, y \in L_2, xy = w, |w| \leq 1 \text{ sau } xy = wz, |w| = 1\}$ and
  - $FIRST(\alpha\beta) = FIRST(\alpha) \oplus FIRST(\beta)$

    $FIRST(X_1 \ldots X_n) = FIRST(X_1) \oplus \ldots \oplus FIRST(X_n)$

L1 = {aa,ab,ba}

L2 = {00,01}

L1L2 ={}

L1$\bigoplus$L2 = {}

L1 ={a,b}

L2 ={0,1}

L1$\bigoplus$L2 ={}

L1={a, $\boldsymbol{\varepsilon}$}

L2={0,1}

L1$\bigoplus$L2 ={}

Algoritmul 3.3 FIRST

INPUT: G

OUTPUT: $FIRST(X), \forall X \in N \cup \Sigma$

for $\forall a \in \Sigma$ **do**

  $F_i(a) = \{a\}, \forall i \geq 0$

end for

i := 0;

$F_0(A) = \{x | x \in \Sigma, A \to x\alpha \text{ sau } A \to x \in P\}$; $\{inițializare\}$

repeat

  i := i+1;

A **for** $\forall \cancel{X} \in N$ **do**

    if F_{i-1} au fost calculate $\forall X \in N \cup \Sigma$ **then**

      $\{dacă\ \exists Y_j, F_{i-1}(Y_j) = \emptyset\ atunci\ nu\ se\ poate\ aplica\}$

      $F_i(A) = F_{i-1}(A) \cup$

      $\{x | A \to Y_1 \ldots Y_n \in P, x \in F_{i-1}(Y_1) \oplus \ldots \oplus F_{i-1}(Y_n)\}$

    end if

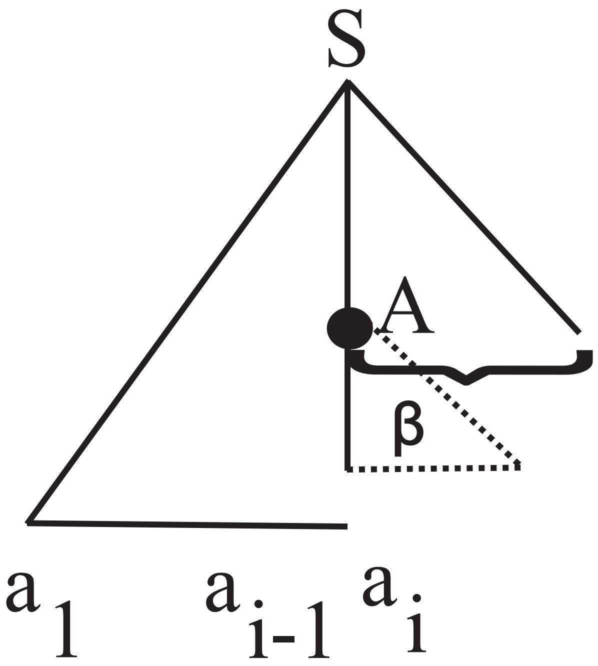
  end for

until $F_{i-1}(A) = F_i(A)$

$FIRST(X) := F_i(X), \forall X \in N \cup \Sigma$

FOLLOW

$A \to \boldsymbol{\varepsilon}$



- $FOLLOW_k(A) \approx$ next k symbols generated after/ following A

$$FOLLOW : (N \cup \Sigma)^* \to \mathcal{P}(\Sigma)$$
$$FOLLOW(\beta) = \{w \in \Sigma | S \overset{*}{\Rightarrow} \alpha\beta\gamma, w \in FIRST(\gamma)\}$$

Algorithm FOLLOW

INPUT: G, FIRST(X), $\forall X \in N \cup \Sigma$

OUTPUT: FOLLOW(A), $\forall A \in N$

for $A \in N - \{S\}$ **do** {init}

$\quad L_0(A) = \Phi;$

endFor;

$L_0(S) = \{\varepsilon\};$ {init}

$i = 0;$

repeat

$\quad i = i+1;$

$\quad$ **for** $B \in N$ **do**

$\quad\quad$ **for** $A \rightarrow \alpha B y \in P$ **do**

$\quad\quad\quad$ **for** $\forall a \in FIRST(y)$ **do**

$\quad\quad\quad\quad$ **if** $a = \varepsilon$ **then** $F_i(B) = F_i(B) \cup F_{i-1}(A)$

$\quad\quad\quad\quad\quad$ **else** $F_i(B) = F_{i-1}(B) \cup First(y)$

$\quad\quad\quad\quad$ **endif**

$\quad\quad\quad$ **endFor**

$\quad\quad$ **endFor**

$\quad$ **endfor**

until $Fi(X) = Fi-1(X), \forall X \in N$

$FOLLOW(X) = Fi(X), \forall X \in N$

$S \Rightarrow^0 S$ // ε after S

$S \Rightarrow aAc \Rightarrow abBc$

$A \rightarrow bB$