19.10.2021

Seminar 4 – Finite Automata (FA)

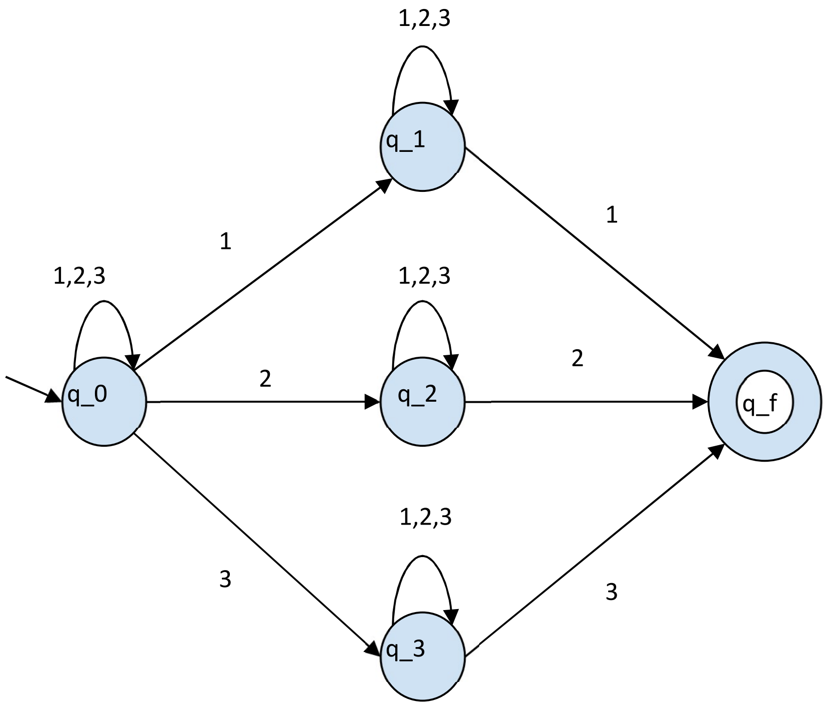
1. Given the FA: $M = (Q, \Sigma, \delta, q_0, F)$, $Q = \{q_0, q_1, q_2, q_3, q_f\}$, $\Sigma = \{1, 2, 3\}$, $F = \{q_f\}$,

δ	1	2	3
$\boldsymbol{q_0}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_3\}$
$\boldsymbol{q_1}$	$\{q_1, q_f\}$	$\{q_1\}$	$\{q_1\}$
$\boldsymbol{q_2}$	$\{q_2\}$	$\{q_2, q_f\}$	$\{q_2\}$
$\boldsymbol{q_3}$	$\{q_3\}$	$\{q_3\}$	$\{q_3, q_f\}$
$\boldsymbol{q_f}$	$\emptyset$	$\emptyset$	$\emptyset$

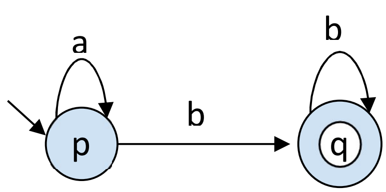
Prove that $w = 12321 \in L(M)$

Sol.: *@B Andrada T.*

$(q_0, 12321) \vdash (q_1, 2321) \overset{3}{\vdash} (q_1, 1) \vdash (q_f, \epsilon)$ => $(q_0, w) \overset{5}{\vdash} (q_f, \epsilon)$ => $w \in L(M)$



2. Find the language accepted by the FA below.



Sol.: *#IW Andrada T., Alexandra T., David T.*

Let $L = \{a^n b^m \mid n \in \mathbb{N}, m \in \mathbb{N}^*\}$ and prove that $L = L(M)$ by double inclusion.

I) ? $L \subseteq L(M)$ (**all** sequences of that shape are accepted by *M*)

? $\forall n \in \mathbb{N},\ \forall m \in \mathbb{N}^*, a^n b^m \in L(M)$

Let $n \in \mathbb{N},\ m \in \mathbb{N}^*$ be fixed.

$(p,\ a^n b^m) \underset{(a)}{\overset{n}{\vdash}} (p, b^m) \vdash (q, b^{m-1}) \underset{(b)}{\overset{m-1}{\vdash}} (q, \epsilon) \ \Rightarrow\ (p,\ a^n b^m) \overset{n+m}{\vdash} (q, \epsilon) \ \Rightarrow\ a^n b^m \in L(M)$

(a) $(p,\ a^n) \overset{n}{\vdash} (p, \epsilon),\ \forall n \in \mathbb{N}$ - proven by math. ind. below

(b) $(q,\ b^n) \overset{n}{\vdash} (q, \epsilon),\ \forall n \in \mathbb{N}$ - proven by math. ind. - HW

(a) Take $P(n)$: $(p,\ a^n) \overset{n}{\vdash} (p, \epsilon), n \in \mathbb{N}$ and prove $P(n)$ is true $\forall n \in \mathbb{N}$, by math. induction.

(i) Verification step ⇔ $P(0)$ is true

$(p,\ a^0) \overset{0}{\vdash} (p, \epsilon)$ => $P(0)$ is true

(ii) Proof step ⇔ $P(k) \to P(k+1), k \in \mathbb{N}$

$P(k)$ is true => $(p,\ a^k) \overset{k}{\vdash} (p, \epsilon)$ (induction hypothesis)

$(p,\ a^{k+1}) \vdash (p,\ a^k) \underset{(ind.hyp).}{\overset{k}{\vdash}} (p, \epsilon)$ => $(p,\ a^{k+1}) \overset{k+1}{\vdash} (p, \epsilon)$ => $P(k+1)$ is true

(i) + (ii) => (a)

II) ? $L(M) \subseteq L$ (M accepts **only** sequences of that shape)

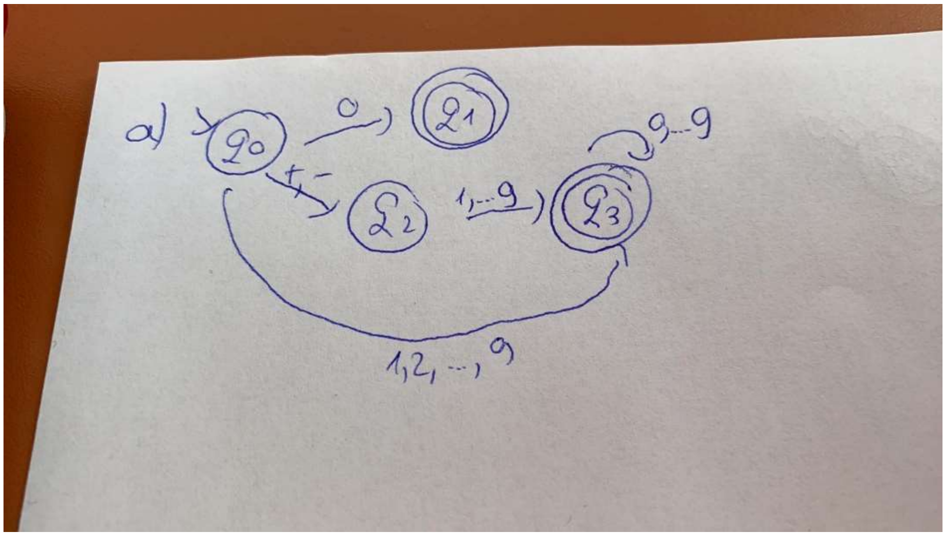
In order to reach the final state q from the initial state p, we should read at least one b. Before the mandatory b, we can read 0 or more a's, while remaining in state p, and after the mandatory b we can read 0 or more b's, while remaining in state q. Therefore, M accepts only sequences of the shape $a^n b b^m,\ n \in \mathbb{N}, m \in \mathbb{N}$ (or $a^n b^m, n \in \mathbb{N}, m \in \mathbb{N}^*$).

Obs.: In order for such a reasoning to count as proof, you should make sure that you have covered **all** paths from initial state to **all** final states.

3. Build FAs that accept the following languages

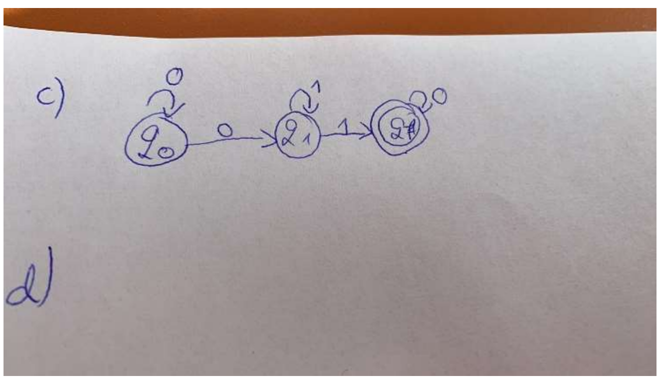
a. Integer numbers
b. Variable declarations (Pascal, C, ...)
c. $L = \{0^n 1^m 0^q \mid n, m \in N^*, q \in N\}$
d. $L = \{0(01)^n \mid n \in N\}$
e. $L = \{c^{3n} \mid n \in N^*\}$
f. The language over $\Sigma = \{0,1\}$ having the property that all sequences have at least two consecutive 0's.

a. *#IW Andrada T.*

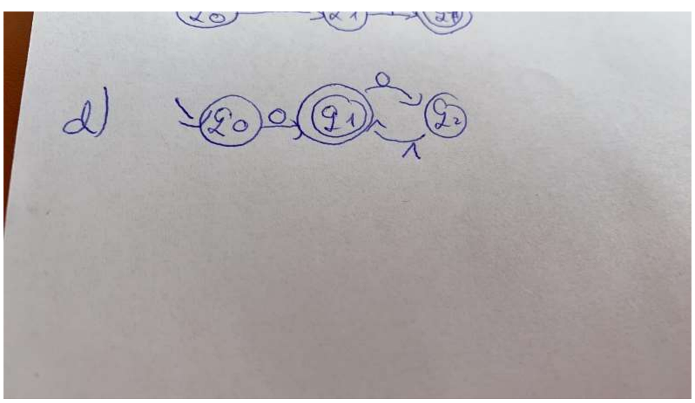


b. HW

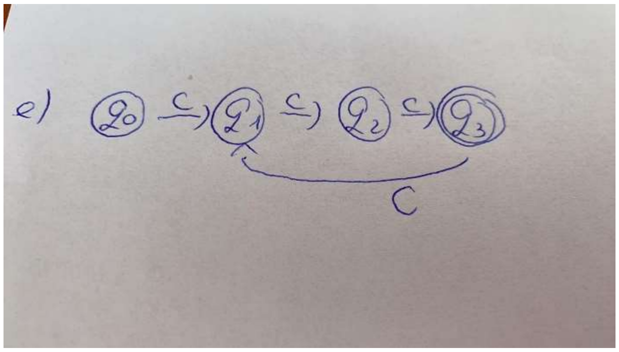
c. *#IW Andrada T.*



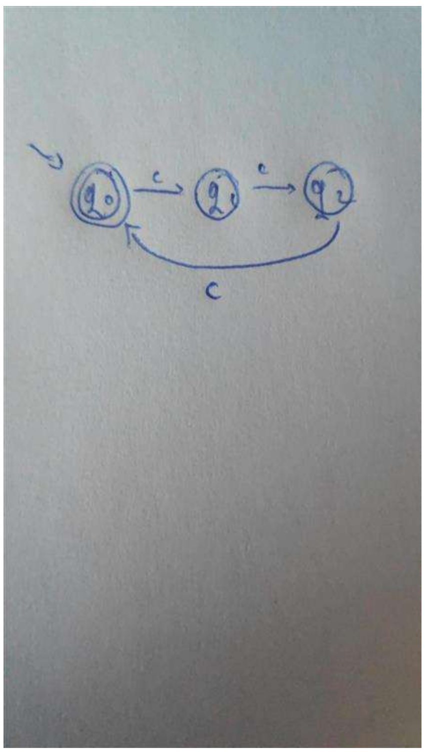
d. *#IW Andrada T.*



e. *#IW Andrada T.*



#IW David T. (for $n \in \mathbb{N}$)



f. #IW Andrada T.