BASIC CALCULUS

STRATEGIC INTERVENTION MATERIAL - 4

DISCONTINUITY OF A FUNCTION

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IN PARTIAL COMPLETION OF THE SUBJECT INVESTIGATION, IMMERSION, AND INQUIRIES

2026

OVERVIEW

Discontinuity of a Function

You talk to someone every day...

The replies are fast...

The conversations are sweet...

Everything feels smooth, like nothing will change...

Then suddenly, no reply...

The next day, still nothing.

Akala mo tuloy-tuloy na, pero biglang naputol :((

That sudden break? That's **discontinuity**.

In calculus, a function can seem okay as you get closer to a certain point from the left and from the right. But when you finally reach that point, something feels off. The value might be *missing*. It might *suddenly jump*. Or it might not match what you were expecting.

No matter how close you get, if the behavior doesn't stay consistent, the function is not continuous.

CONDITIONS OF CONTINUITY

Recall from the previous SIM that there are three conditions that need to be met to say that a function is continuous when x = a.

1. $f(a)$ exists,
2. $\lim f(x)$ exists, and;
3. $f(a) = \lim f(x)$

Even if one condition is not met, that means f is discontinuous at x = a.

TYPES OF DISCONTINUITY

EXAMPLE 1: HOLE/REMOVABLE DISCONTINUITY

For any given function f, if $\lim\limits_{x \to a} f(x)$ exists (where $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x)$) but is not equal to $f(a)$. It is called a removable discontinuity or hole discontinuity.

Evaluate: $\lim\limits_{x \to 6} \dfrac{x^2 - 4x - 12}{x - 6}$

Solution:
We can see that if we substitute $x = 6$ into $f(x) = \dfrac{x^2 - 4x - 12}{x - 6}$ we can see that division by 0 will will occur which would be indeterminate.

But first, let's see if we can factor out the polynomial in the numerator.

$$f(x) = \frac{x^2 - 4x - 12}{x - 6} = \frac{(x-6)(x+2)}{x-6} = \frac{(x+2)}{1} = (x+2)$$

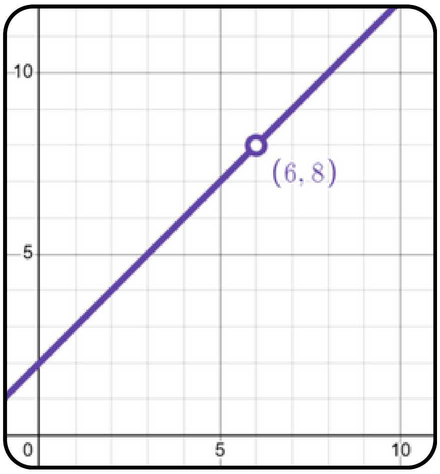
OVERVIEW

After factoring, we can apply basic limit laws.

$$\lim_{x \to 6} \frac{x^2 - 4x - 12}{x - 6} = x + 2$$
$$= (6) + 2$$
$$= 8$$

As you can see, $\lim_{x \to a} f(x)$ exists but $\lim_{x \to a} f(x) \neq f(a)$.

If you look at the graph to the right, we can see the graph of $f(x) = \dfrac{x^2 - 4x - 12}{x - 6}$.



The hole you see in the graph aligns with $\lim_{x \to 6} f(x)$ aligns with our previously calculated value.

From here, we say that ***the function is discontinuous at x = 6, but the discontinuity can be removed.***

EXAMPLE 2: JUMP/NON-REMOVABLE DISCONTINUITY

This happens when the **left-hand behavior and right-hand behavior are not the same.**
Meaning, $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$.

Evaluate: $\lim_{x \to 3} f(x)$,where $\begin{cases} \frac{x}{2} & \text{,if } x < 3 \\ -\frac{x^2}{3} + 4x - 6 & \text{,if } x \geq 3 \end{cases}$

Solution:
Here we can see that the limit of a piecewise function is used as an example.
Let's create a table of values while applying the rules given by the piecewise function.

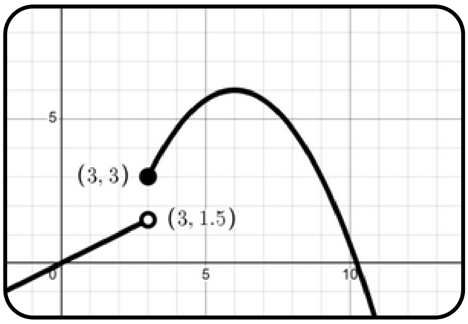
Approaching x from the left	
x	f(x)
2	1
2.5	1.25
2.9	1.45
2.99	1.495
2.999	1.4995

Approaching x from the right	
x	f(x)
4	4.66666667
3.5	3.91666667
3.1	3.16666667
3.01	3.01996667
3.001	3.00199967

After taking the one-sided limits of the function, we can see that:

$$\lim_{x \to 3^-} f(x) = 1.5 \qquad \lim_{x \to 3^+} f(x) = 3$$

Since both one-sided limits are not equal to each other, we say that the **function is discontinuous at x = 3.**



Looking at the graph of the function, we see that our one-sided limits align with where each part of the function ends.

EXAMPLE 3: ESSENTIAL/INFINITE/ASYMPTOTIC DISCONTINUITY

This one is similar to example 2. But instead of constant values, the one-sided limits approach infinity.

Evaluate: $\lim\limits_{x \to 5} \dfrac{2x-9}{x-5}$

Solution:
This looks similar to example 1, however the numerator cannot be factored no matter how hard you try. So, we will have to bring back the table of values to see the function's behavior.

Approaching x from the left	
x	f(x)
4	1
4.5	0
4.9	-8
4.99	-98
4.999	-998

Approaching x from the right	
x	f(x)
6	3
5.5	4
5.1	12
5.01	102
5.001	1002

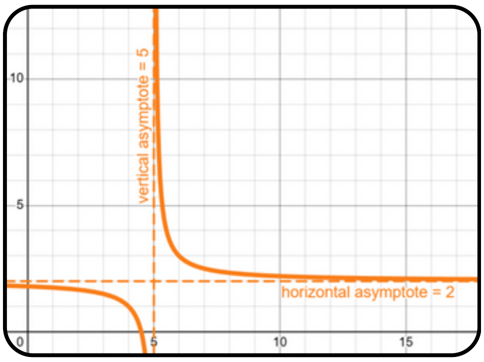
After taking the one-sided limits of the function, we can see that:

$$\lim_{x \to 5^-} \frac{2x-9}{x-5} = -\infty \qquad \lim_{x \to 5^+} \frac{2x-9}{x-5} = \infty$$

Since both one-sided limits are not equal to each other, we say that the **function is discontinuous at x = 5.**

The key distinction that this example has from example 2 is that instead each one-sided limit approaching a single fixed value, they each approach infinity (either positive or negative infinity).

Looking at the graph to the right, you can see that the function asymptotes are x = 5.



ACTIVITY I

Discontinuity of a Function

Instructions:

Evaluate the following limits. ***If the function is discontinuous, state what kind of discontinuity is present (Hole, Jump, Infinite). If the function is continuous, write "continuous".*** Do this on a separate sheet of paper.

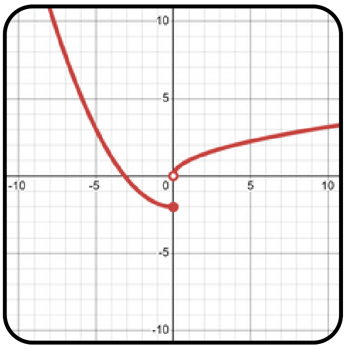
1. $\lim_{x\to 4}\left(\frac{x^2-16}{x-4}\right)$
2. $\lim_{x\to 1}\left(\frac{x^3-2}{x-1}\right)$
3. $\lim_{x\to 3}\left(\frac{4x}{x-2}\right)$
4. $\lim_{x\to -7} f(x)\,,\, f(x) = \begin{cases} x-5, x<-6 \\ x^2 \geq -6 \end{cases}$
5. $\lim_{x\to 2}\left(\frac{x^2}{x-2}\right)$
6. $\lim_{x\to 1}\left(\frac{1}{x+1}\right)$
7. $\lim_{x\to 1} f(x), f(x) = \begin{cases} 2x+1, x<1 \\ x^2, x\geq 1 \end{cases}$
8. $\lim_{x\to -3}\left(\frac{x+3}{x+3}\right)$
9. $\lim_{x\to 0}\left(\frac{5x}{|x|}\right)$
10. $\lim_{x\to 4}\left(\frac{1}{x-4^2}\right)$

ACTIVITY II

Discontinuity of a Function

Instructions:

Examine the following graphs. ***Determine the type of discontinuity shown*** (***Hole, Jump, Infinite***). Do this on a separate sheet of paper.

1. 

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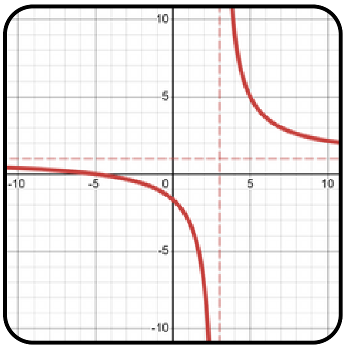
2. 

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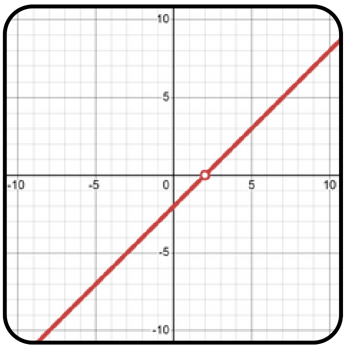
3. 

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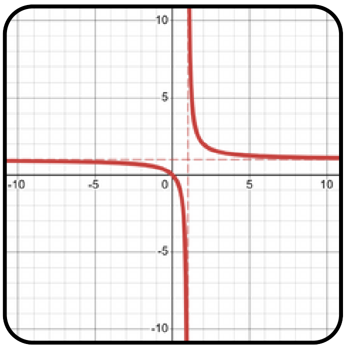
4. 

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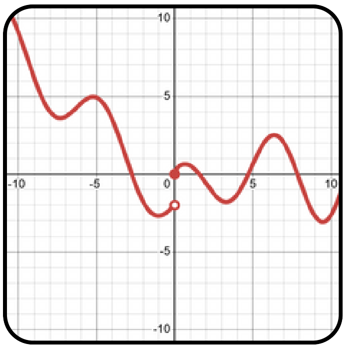
5. 

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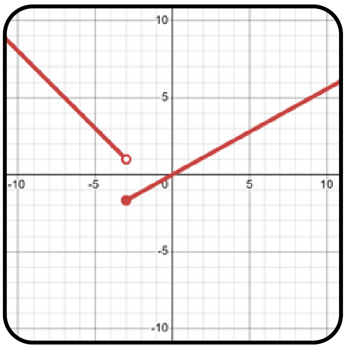
6. 

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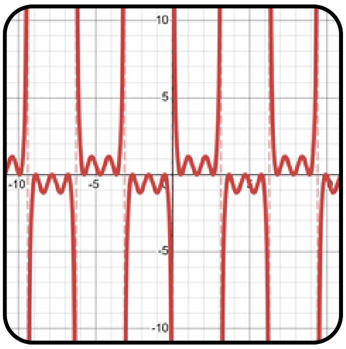
7. 

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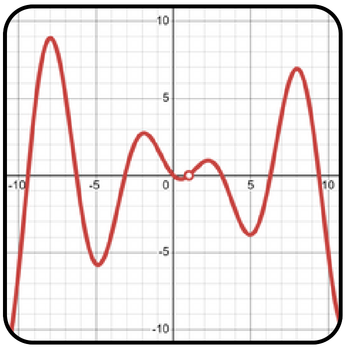
8. 

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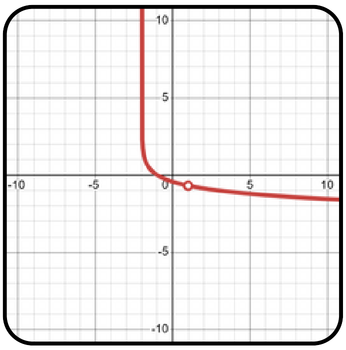
9. 

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FRONT COVER

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CONTENT

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BACK COVER

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ANSWER KEY

ACTIVITY I

1. ***Hole***
2. ***Infinite***
3. ***Continuous***
4. ***Continuous***
5. ***Infinite***
6. ***Continuous***
7. ***Jump***
8. ***Hole***
9. ***Jump***
10. ***Continuous***

ACTIVITY II

1. ***Jump***
2. ***Infinite***
3. ***Hole***
4. ***Infinite***
5. ***Jump***
6. ***Jump***
7. ***Infinite***
8. ***Hole***
9. ***Hole***