BASIC CALCULUS

STRATEGIC INTERVENTION MATERIAL - 3

CONTINUITY OF A FUNCTION

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OVERVIEW

Continuity of a Function

Ang ***Continuity***, parang pag-ibig lang 'yan. Hindi sapat na malapit kayo sa isa't isa (***Limit***), dapat malinaw kung ano ba talaga kayo (***Defined Value***). Kasi kung hindi sila magtutugma, ***Discontinuity*** ang tawag doon—sa madaling salita, hanggang kaibigan lang."

LIMIT AND CONTINUITY AT A POINT

Continuity is a simple concept to understand...

Think of a car driving over a bridge. If the bridge has no holes at any given point, then the car can continue to the other side.

If the bridge does have holes at a given point, then the car cannot continue its journey to the other side.

That's how it works! :D

CONDITIONS OF CONTINUITY

There are three conditions that need to be met to say that a function is continuous when x = a.

1. $f(a)$ exists,
2. $\lim\limits_{x \to a} f(x)$ exists, and;
3. $f(a) = \lim f(x)$

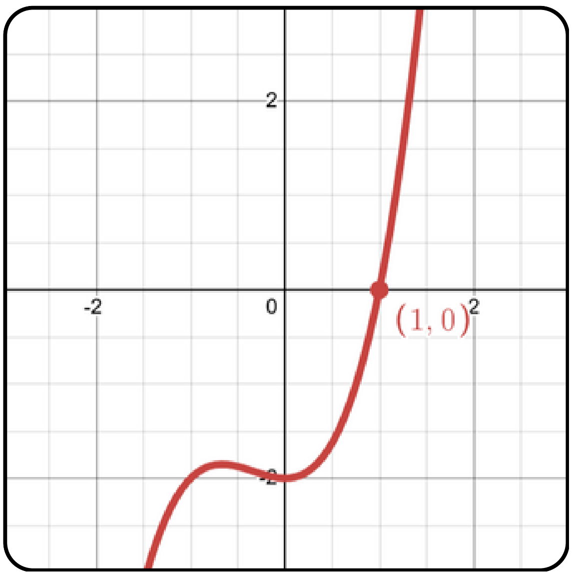
Even if one condition is not met, that means ***f is discontinuous at x = a.***

EXAMPLE 1

Evaluate: Determine if $f(x) = x^3 + x^2 - 2$ is continuous or not at $x = 1$.

Solution: Let's check the function's continuity using the above-mentioned conditions to check.

1. If $x = 1$, then $f(1) = 0$
2. $$\begin{aligned} \lim_{x \to 1} f(x) &= \lim_{x \to 1} x^3 + x^2 - 2 \\ &= (1)^3 + (1)^2 - 2 \\ &= 1 + 1 - 2 \\ &= 2 - 2 \\ &= 0 \end{aligned}$$
3. $f(1) = 0 = \lim\limits_{x \to 1} f(x)$



Therefore, f is ***continuous*** at $x = 1$.

The graph on the right shows that a ***shaded/solid dot where (1, 0) is.***

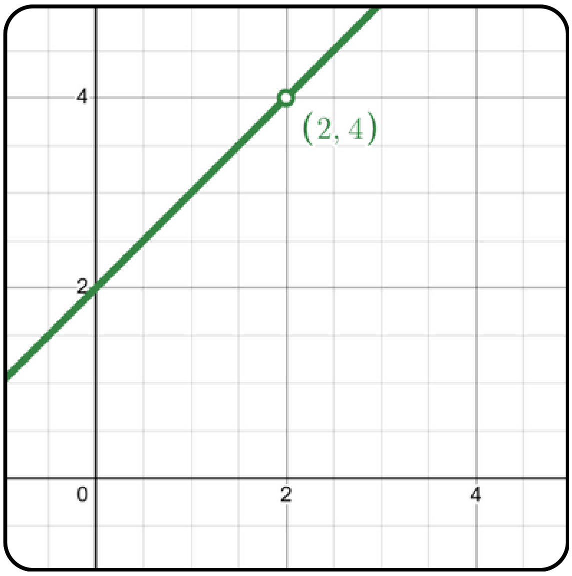
This means that the ***point is defined*** in the function

EXAMPLE 2

Evaluate: Determine if $f(x) = \left(\dfrac{x^2 - 4}{x - 2}\right)$ is continuous or not at $x = 2$.

Solution: Let's check the function's continuity using the above-mentioned conditions to check.

1. If $x = 2$, then $f(2) =$ *indeterminate*
2. $$\begin{aligned}\lim_{x\to 2} f(x) &= \lim_{x\to 2}\left(\frac{x^2-4}{x-2}\right)\\ &= \lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)}\\ &= \lim_{x\to 2}(x+2)\\ &= \lim_{x\to 2} x + \lim_{x\to 2} 2\\ &= 2+2\\ &= 4\end{aligned}$$
3. $f(2) \neq \lim_{x\to 2} f(x)$



Therefore, f is ***discontinuous*** at $x = 2$.

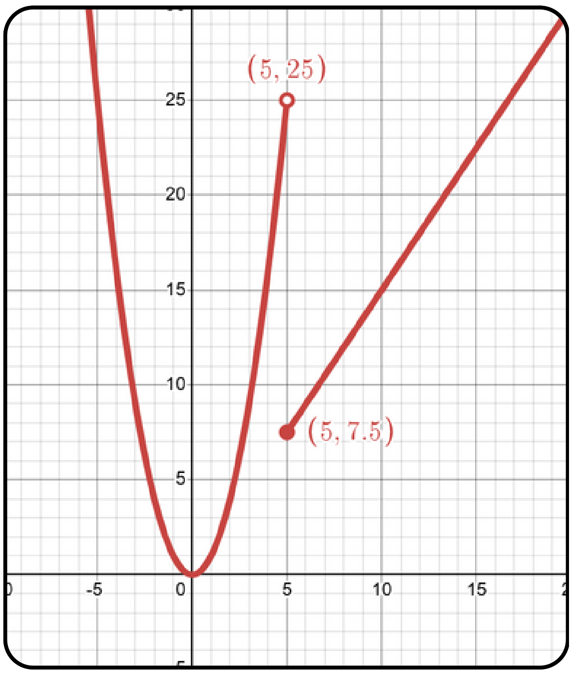
Looking at the graph above, ***the point (2, 4) is unshaded/hollow.*** This means ***the point is not defined.***

EXAMPLE 3

Evaluate: Determine the continuity of the piecewise function $f(x) = \begin{cases} x^2, x < 5 \\ \frac{3x}{2}, x \geq 5 \end{cases}$ at x = 5.

Solution: Let's check the function's continuity using the above-mentioned conditions to check.

1. If $x = 5$, then $f(5) = 7.5$
2. $\lim_{x\to 5} f(x) = DNE$
3. $f(5) \neq \lim_{x\to 5} f(x)$



The graph on the right shows ***why the function exists while the limit doesn't.***

If we approach ***x = 5*** from both sides, ***it would not converge on the same value.***

The ***left would approach 25***, while the ***right would approach 7.5***

Hence, f is ***discontinuous*** at $x = 5$.

ACTIVITY I

Continuity of a Function

Instructions:

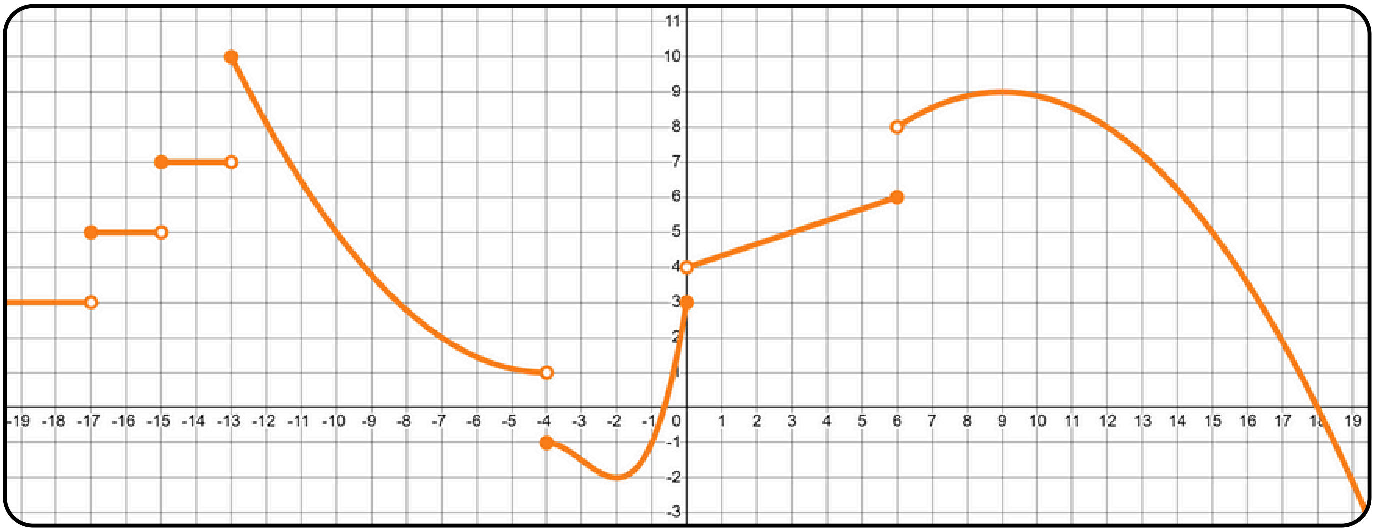
Evaluate the following limits and determine if they are ***continuous or discontinuous.***

1. $\lim_{x\to 3} \frac{x^2-9}{x-3}$
2. $\lim_{x\to 4} \frac{x^2-5}{x-4}$
3. $\lim_{x\to 5} f(x), \quad f(x)=\begin{cases} \sin x, x<5 \\ \cos x, x>5 \end{cases}$
4. $\lim_{x\to -2} \frac{x^3}{x-2}$
5. $\lim_{x\to 5} \frac{x^2-25}{x}$
6. $\lim_{x\to -2} f(x), \ f(x)=\begin{cases} x^2, x<0 \\ x-5, x\geq 0 \end{cases}$
7. $\lim_{x\to -7} \frac{x^2-5}{x+8}$
8. $\lim_{x\to 3} f(x), \quad f(x)=\begin{cases} \sqrt{x}, x\leq 3 \\ x^2, 3<x\leq 6 \\ x-5, x>6 \end{cases}$
9. $\lim_{x\to 5} \frac{x^3-6}{x-5}$
10. $\lim_{x\to 4} \frac{2x^2-3x-2}{x-2}$

ACTIVITY II

Continuity of a Function

Instructions:
Evaluate the following values of x and determine if they are ***continuous or discontinuous*** at that point.



1. $x = -16$
2. $x = -4$
3. $x = 3$
4. $x = 6$
5. $x = 10$
6. $x = 1$
7. $x = 5$
8. $x = -2$
9. $x = 12$
10. $x = -11$
11. $x = -17$
12. $x = 13$
13. $x = -15$
14. $x = -13$
15. $x = 0$

REFERENCES

Limits of Transcendental Functions

FRONT COVER

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CONTENT

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BACK COVER

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ANSWER KEY

To receive the answer key, first submit your answer sheet to the website link provided to you.

Properly fill out the corresponding fields in the "Submit" page.

See you there :D