BASIC CALCULUS

STRATEGIC INTERVENTION MATERIAL - 3

CONTINUITY OF A FUNCTION

GERONIMO, J.E. • IGNACIO, M.A.B. • LOYOLA, A.D.T. • MAGSINO, S.E.D.G.

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OVERVIEW

Continuity of a Function

Ang ***Continuity***, parang pag-ibig lang 'yan. Hindi sapat na malapit kayo sa isa't isa (***Limit***), dapat malinaw kung ano ba talaga kayo (***Defined Value***). Kasi kung hindi sila magtutugma, ***Discontinuity*** ang tawag doon—sa madaling salita, hanggang kaibigan lang."

LIMIT AND CONTINUITY AT A POINT

Continuity is a simple concept to understand...

Think of a car driving over a bridge. If the bridge has no holes at any given point, then the car can continue to the other side.

If the bridge does have holes at a given point, then the car cannot continue its journey to the other side.

That's how it works! :D

CONDITIONS OF CONTINUITY

There are three conditions that need to be met to say that a function is continuous when x = a.

1. $f(a)$ exists,
2. $\lim\limits_{x \to a} f(x)$ exists, and;
3. $f(a) = \lim f(x)$

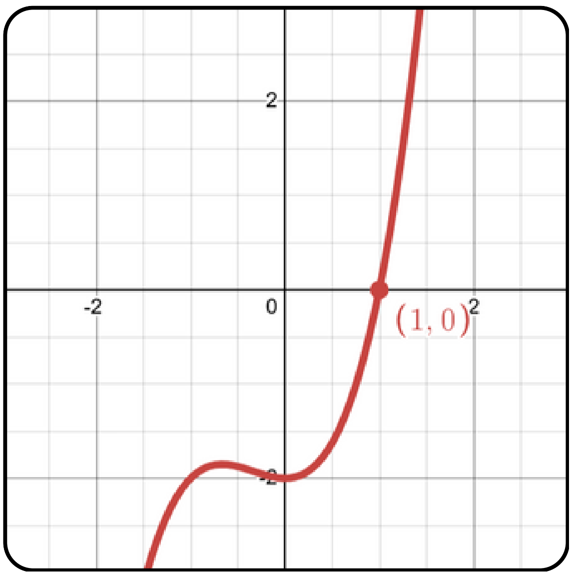
Even if one condition is not met, that means ***f is discontinuous at x = a.***

EXAMPLE 1

Evaluate: Determine if $f(x) = x^3 + x^2 - 2$ is continuous or not at $x = 1$.

Solution: Let's check the function's continuity using the above-mentioned conditions to check.

1. If $x = 1$, then $f(1) = 0$
2. $$\begin{aligned} \lim_{x \to 1} f(x) &= \lim_{x \to 1} x^3 + x^2 - 2 \\ &= (1)^3 + (1)^2 - 2 \\ &= 1 + 1 - 2 \\ &= 2 - 2 \\ &= 0 \end{aligned}$$
3. $f(1) = 0 = \lim\limits_{x \to 1} f(x)$



Therefore, f is ***continuous*** at $x = 1$.

The graph on the right shows that a ***shaded/solid dot where (1, 0) is.***

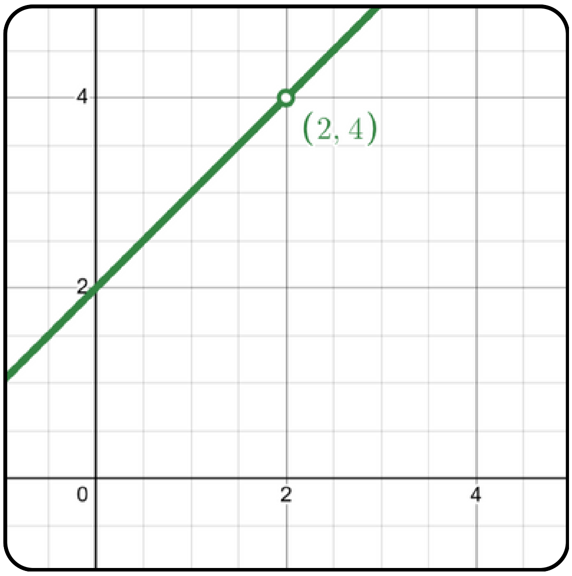
This means that the ***point is defined*** in the function

EXAMPLE 2

Evaluate: Determine if $f(x) = \left(\frac{x^2-4}{x-2}\right)$ is continuous or not at $x = 2$.

Solution: Let's check the function's continuity using the above-mentioned conditions to check.

1. If $x = 2$, then $f(2) =$ *indeterminate*
2. $$\begin{aligned}\lim_{x\to 2} f(x) &= \lim_{x\to 2}\left(\frac{x^2-4}{x-2}\right)\\ &= \lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)}\\ &= \lim_{x\to 2}(x+2)\\ &= \lim_{x\to 2} x + \lim_{x\to 2} 2\\ &= 2+2\\ &= 4\end{aligned}$$
3. $f(2) \neq \lim_{x\to 2} f(x)$



Therefore, f is ***discontinuous*** at $x = 2$.

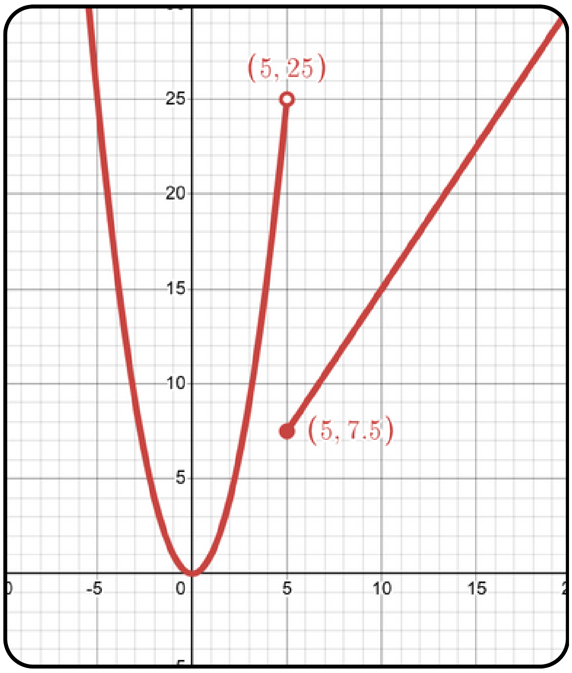
Looking at the graph above, ***the point (2, 4) is unshaded/hollow.*** This means ***the point is not defined.***

EXAMPLE 3

Evaluate: Determine the continuity of the piecewise function $f(x) = \begin{cases} x^2, x < 5 \\ \frac{3x}{2}, x \geq 5 \end{cases}$ at x = 5.

Solution: Let's check the function's continuity using the above-mentioned conditions to check.

1. If $x = 5$, then $f(5) = 7.5$
2. $\lim_{x\to 5} f(x) = DNE$
3. $f(5) \neq \lim_{x\to 5} f(x)$



The graph on the right shows ***why the function exists while the limit doesn't.***

If we approach ***x = 5*** from both sides, ***it would not converge on the same value.***

The ***left would approach 25***, while the ***right would approach 7.5***

Hence, f is ***discontinuous*** at $x = 5$.

ACTIVITY I

Continuity of a Function

Instructions:

Evaluate the following limits and determine if they are ***continuous or discontinuous.***

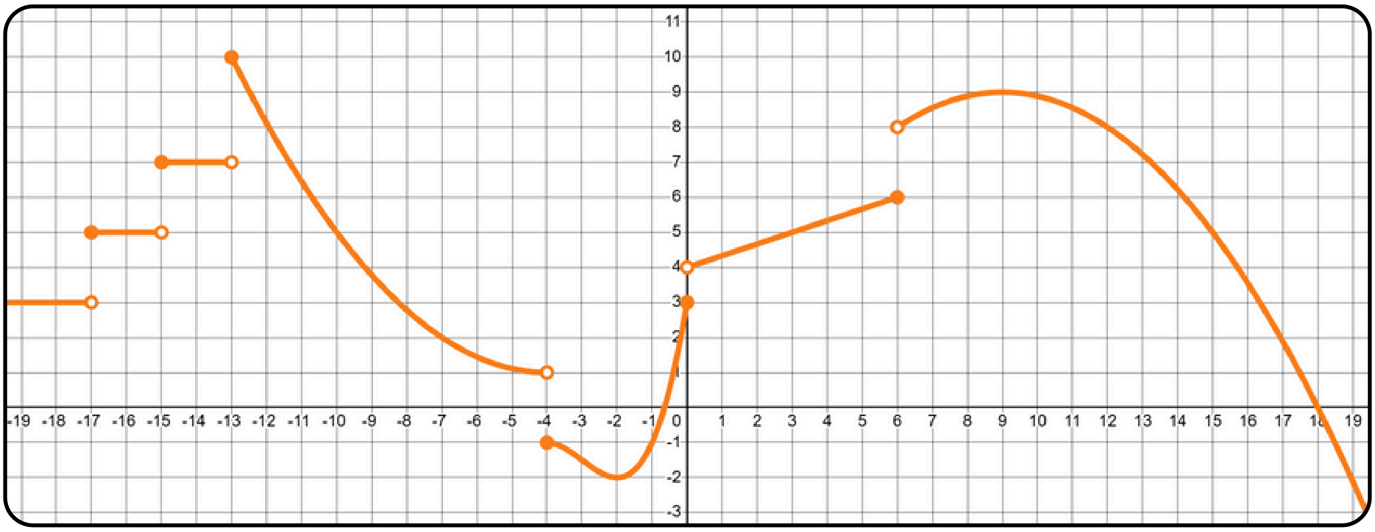
1. $\lim\limits_{x \to 3} \frac{x^2 - 9}{x - 3}$
2. $\lim\limits_{x \to 4} \frac{x^2 - 5}{x - 4}$
3. $\lim\limits_{x \to 5} f(x), \quad f(x) = \begin{cases} \sin x, x < 5 \\ \cos x, x > 5 \end{cases}$
4. $\lim\limits_{x \to -2} \frac{x^3}{x - 2}$
5. $\lim\limits_{x \to 5} \frac{x^2 - 25}{x}$
6. $\lim\limits_{x \to -2} f(x), \ f(x) = \begin{cases} x^2, x < 0 \\ x - 5, x \geq 0 \end{cases}$
7. $\lim\limits_{x \to -7} \frac{x^2 - 5}{x + 8}$
8. $\lim\limits_{x \to 3} f(x), \quad f(x) = \begin{cases} \sqrt{x}, x \leq 3 \\ x^2, 3 < x \leq 6 \\ x - 5, x > 6 \end{cases}$
9. $\lim\limits_{x \to 5} \frac{x^3 - 6}{x - 5}$
10. $\lim\limits_{x \to 4} \frac{2x^2 - 3x - 2}{x - 2}$

ACTIVITY II

Continuity of a Function

Instructions:

Evaluate the following values of x and determine if they are ***continuous or discontinuous*** at that point.



1. $x = -16$
2. $x = -4$
3. $x = 3$
4. $x = 6$
5. $x = 10$
6. $x = 1$
7. $x = 5$
8. $x = -2$
9. $x = 12$
10. $x = -11$
11. $x = -17$
12. $x = 13$
13. $x = -15$
14. $x = -13$
15. $x = 0$

REFERENCES

Limits of Transcendental Functions

FRONT COVER

Webb, S. (2018, April 7). Low Angle View Photography of Gray Building. Pexels. https://www.pexels.com/photo/low-angle-view-photography-of-gray-building-992987/

K, J. (2024, November 6). Graph Paper with Transparent Rulers and Protractor. Pexels. https://www.pexels.com/photo/graph-paper-with-transparent-rulers-and-protractor-29279367/

CONTENT

Alegre, J. B. (2022). BASIC CALCULUS QUARTER 3 - Module 1 Lessons1-9 (2nd ed.) [PDF]. Department of Education. https://studylib.net/doc/26179556/

BACK COVER

Lioncat, O. (2021, March 24). Black and White Photo of a Structure. Pexels. https://www.pexels.com/photo/black-and-white-photo-of-a-structure-7245550/

ANSWER KEY

ACTIVITY I

1. ***Discontinuous***
2. ***Discontinuous***
3. ***Discontinuous***
4. ***Continuous***
5. ***Continuous***
6. ***Continuous***
7. ***Continuous***
8. ***Discontinuous***
9. ***Discontinuous***
10. ***Continuous***

ACTIVITY II

1. ***Continuous***
2. ***Discontinuous***
3. ***Continuous***
4. ***Discontinuous***
5. ***Continuous***
6. ***Continuous***
7. ***Continuous***
8. ***Continuous***
9. ***Continuous***
10. ***Continuous***
11. ***Discontinuous***
12. ***Continuous***
13. ***Discontinuous***
14. ***Discontinuous***
15. ***Discontinuous***