BASIC CALCULUS

STRATEGIC INTERVENTION MATERIAL - 2

LIMIT OF TRANSCENDENTAL FUNCTIONS

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IN PARTIAL COMPLETION OF THE SUBJECT INVESTIGATION, IMMERSION, AND INQUIRIES

2026

OVERVIEW

Limit of Transcendental Functions

Struggling with limits? Stop overthinking! ✋
Calculus is hard, but finding "the one" shouldn't be. If you're tired of x approaching zero but never getting anywhere, it's time to shoot your shot. Walk up to them and say:

"Kung ang English ng ilog ay river, pwede ba kitang maging forever? Kasi sa math, okay lang na may limit... basta pagdating sa dulo, ikaw at ako pa rin ang mag-ma-match."

EXAMPLE 1 - EXPONENTIAL FUNCTIONS

Let's try solving an example involving the natural exponential function $f(x) = e^x$, where e is called the Euler's number, and its value is 2.718281828...

Evaluate: $\lim_{x \to 0} e^x$

Solution:
Let's construct table of values for the given function where the value of x approaches 0.

Approaching x from the left	
x	f(x)
-1	0.36787944
-0.5	0.60653066
-0.1	0.90483742
-0.01	0.99004983
-0.001	0.99900049

Approaching x from the right	
x	f(x)
1	2.71828183
0.5	1.64872127
0.1	1.10517092
0.01	1.01005017
0.001	1.0010005

What we just did there is called a one-sided limit. It can either come from the left or the right side of any given value. Such limits are written like this.

$$\lim_{x \to 0^-} e^x = 1$$

Left-Hand Limit

$$\lim_{x \to 0^+} e^x = 1$$

Right-Hand Limit

Take note of the symbols next the constant that x is approaching...

The left-hand limit has a negative sign since the values approaching the constant are less than it. While the right-hand limit has a positive sign, since the values used are larger than the given constant.

As you can see, both limits are equal to each other. When that happens, we can write a two-sided limit.

$$\lim_{x \to 0} e^x = 1$$

Keep these in mind when dealing with limits:

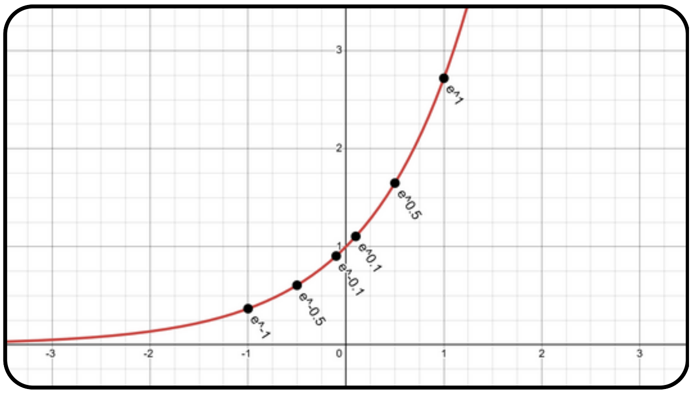
If $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$,then $\lim_{x \to a} f(x)$ exists.

If $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$,then $\lim_{x \to a} f(x)$ does not exist (DNE).

OVERVIEW

We can also use a graph to determine the limit of $f(x) = e^x$ as *x* approaches 0.

As you can see, the values we used on the tables indeed do approach the value we computed from them.



EXAMPLE 2 - LOGARITHMIC FUNCTIONS

Let's try out a different function, this being the natural logarithm $f(x) = ln\ x$.

Recall that $ln\ x = log_e x$

Evaluate: $\lim_{x \to 1} ln\ x$

Solution:

Once again, we'll use a table of values. We'll use values that slowly approach 1.

Approaching x from the left	
x	f(x)
0	*undefined*
0.5	-0.69314718
0.9	-0.10536052
0.99	-0.01005034
0.999	-0.0010005

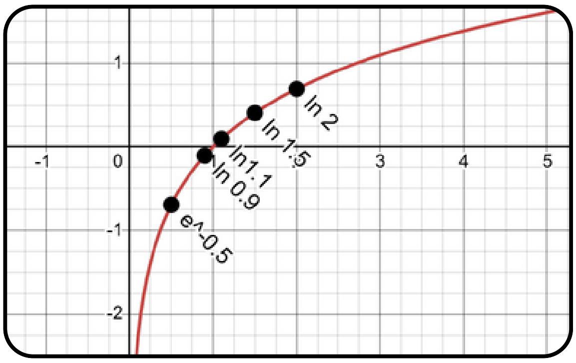
Approaching x from the right	
x	f(x)
2	0.69314718
1.5	0.40546511
1.1	0.09531018
1.01	0.00995033
1.001	0.0009995

After taking the one-sided limits of the function, we can see that:

$$\lim_{x \to 1^-} ln\ x = 0 \qquad \lim_{x \to 1^+} ln\ x = 0$$

From here, we can say that: Since $\lim_{x \to 1^-} ln\ x = \lim_{x \to 1^+} ln\ x$,therefore $\lim_{x \to 1} ln\ x$ exists.

That means: $\lim_{x \to 1^-} ln\ x = 0$



Once again using a graph and substituting our chosen values into the function $f(x) = ln\ x$, we can see that they all converge to 0.

Additionally, you can see why inputting 0 into the function would be undefined because the function's vertical asymptote is 0.

Keep the word asymptote in mind, that would come again in following material.

EXAMPLE 3 - TRIGONOMETRIC FUNCTIONS

Before we continue with solving another example, make sure that your scientific calculator is set to Rad mode.

> **Note:**
> To solve this properly on a scientific calculator, you must first set its mode to Rad.

Evaluate: $\lim_{x\to0}\sin x$

Solution:

Yet again, we'll use another set of tables to illustrate what the limit of $f(x)=\sin x$ could be.

Approaching x from the left	
x	f(x)
-1	-0.84147098
-0.5	-0.47942554
-0.1	-0.09983342
-0.01	-0.00999983
-0.001	-0.00099999

Approaching x from the right	
x	f(x)
1	0.84147098
0.5	0.47942554
0.1	0.09983342
0.01	0.00999983
0.001	0.00099999

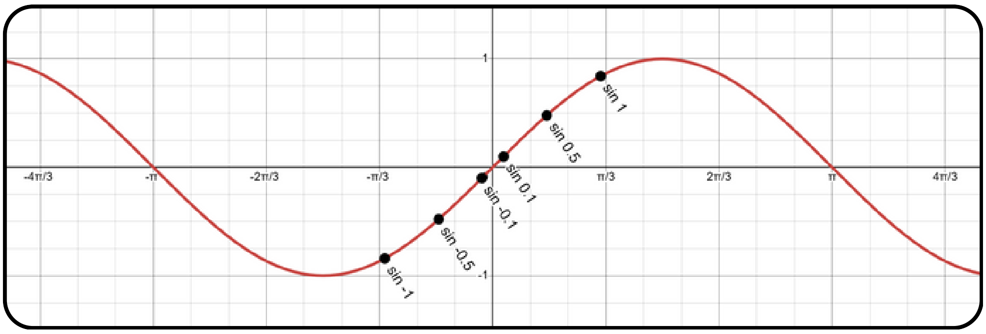
After taking the one-sided limits of the function, we can see that:

$$\lim_{x\to0^-}\sin x=0 \qquad \lim_{x\to0^+}\sin x-0$$

From here, we can say that: Since $\lim_{x\to0^-}\sin x=\lim_{x\to0^+}\sin x$,therefore $\lim_{x\to0}\sin x$ exists.

That means: $\lim_{x\to0}\sin x=0$

As always, we can also use a graph to find the limit of $f(x)=\sin x$



We can see that substituting our chosen values into $\lim_{x\to0}\sin x$ does indeed approach 0.

Sometimes, you may encounter trigonometric functions that may not be present on your scientific calculator.

Luckily, they can be expressed in terms of the functions that are available

OVERVIEW

TRIGONOMETRIC IDENTITIES

Does this sound familiar? It should, hopefully you have discussed this in Pre-Calculus.

Here are the six trigonometric functions

$\sin\theta$	***sine theta***
$\cos\theta$	***cosine theta***
$\tan\theta$	***tangent theta***
$\sec\theta$	***secant theta***
$\csc\theta$	***cosecant theta***
$\cot\theta$	***cotangent theta***

If you look at your scientific calculator, you may notice that the first three functions are the only ones available.

Sometimes, you may encounter the last three functions. Fortunately, they can be expressed in terms of each other.

$$\frac{\sin\theta}{\cos\theta} = \tan\theta \qquad \frac{\cos\theta}{\sin\theta} = \cot\theta$$

$$\frac{1}{\sin\theta} = \csc\theta \qquad \frac{1}{\csc\theta} = \sin\theta$$

$$\frac{1}{\cos\theta} = \sec\theta \qquad \frac{1}{\sec\theta} = \cos\theta$$

$$\frac{1}{\tan\theta} = \cot\theta \qquad \frac{1}{\cot\theta} = \tan\theta$$

ACTIVITY I

Limits of Transcendental Functions

Let's do some practice activities.

Have a go yourself! :D

Instructions:

Evaluate the following limits using a table of values. Round to 5 decimal places for the table and round to 3 decimal places for the limit.

1. $\lim_{x \to -1} e^{3x}$

Approaching x from the left	
x	f(x)
-2	**?**
-1.5	**?**
-1.1	**?**
-1.01	**?**
-1.001	**?**

Approaching x from the right	
x	f(x)
0	**?**
-0.5	**?**
-0.9	**?**
-0.99	**?**
-0.999	**?**

2. $\lim_{\theta \to 12} \frac{3 \sin \theta}{2\theta}$

Approaching x from the left	
x	f(x)
11	**?**
11.5	**?**
11.9	**?**
11.99	**?**
11.999	**?**

Approaching x from the right	
x	f(x)
13	**?**
12.5	**?**
12.1	**?**
12.01	**?**
12.001	**?**

3. $\lim_{x \to -25} \ln\left(x^2 - 5\right)$

Approaching x from the left	
x	f(x)
-26	**?**
-25.5	**?**
-25.1	**?**
-25.01	**?**
-25.001	**?**

Approaching x from the right	
x	f(x)
-24	**?**
-24.5	**?**
-24.9	**?**
-24.99	**?**
-24.999	**?**

ACTIVITY II

Limits of Transcendental Functions

Let's do some practice activities.

Have a go yourself! :D

Instructions:

Evaluate the following limits. Round to 3 decimal places. Use a scientific calculator if needed.

1. $\lim_{x \to -3} \ln(2x+7)$
2. $\lim_{x \to \pi} (\cos x + \sin x)$
3. $\lim_{x \to -2} -x + e^{(x+2)}$
4. $\lim_{x \to e} (\ln(x))^3$
5. $\lim_{x \to \frac{\pi}{2}} \csc\left(\frac{x}{\sin x}\right)$
6. $\lim_{x \to \frac{5}{3}} \ln(3x-4)$
7. $\lim_{x \to 1} \left(e \left| \frac{x}{2e} \right|^x\right)$
8. $\lim_{x \to 1} (\ln(x_1) + 5x_1)^2$
9. $\lim_{x \to 2\pi} \cos x - \left(\frac{\tan x}{\cot x}\right)$
10. $\lim_{x \to 5} (\cos x)^2 + (\sin x)^2$

REFERENCES

Limits of Transcendental Functions

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ANSWER KEY

1. $\lim_{x \to -1} e^{3x}$

Approaching x from the left	
x	f(x)
-2	***0.00248***
-1.5	***0.01111***
-1.1	***0.03688***
-1.01	***0.04832***
-1.001	***0.04964***

Approaching x from the right	
x	f(x)
0	***1***
-0.5	***0.22313***
-0.9	***0.06721***
-0.99	***0.05130***
-0.999	***0.04994***

$$\lim_{x \to -1} e^{3x} \approx 0.050$$

2. $\lim_{\theta \to 12} \frac{3 \sin \theta}{2\theta}$

Approaching x from the left	
x	f(x)
11	***-0.13636***
11.5	***-0.11419***
11.9	***-0.07792***
11.99	***-0.06818***
11.999	***-0.06718***

Approaching x from the right	
x	f(x)
13	***0.04848***
12.5	***-0.00796***
12.1	***-0.05574***
12.01	***-0.06596***
12.001	***-0.06696***

$$\lim_{\theta \to 12} \frac{3 \sin \theta}{2\theta} \approx -0.067$$

3. $\lim_{x \to -25} \ln \left(x^2 - 5\right)$

Approaching x from the left	
x	f(x)
-26	***6.50877***
-25.5	***6.46964***
-25.1	***6.43777***
-25.01	***6.43053***
-25.001	***6.42980***

Approaching x from the right	
x	f(x)
-24	***6.34739***
-24.5	***6.38898***
-24.9	***6.42164***
-24.99	***6.42891***
-24.999	***6.42964***

$$\lim_{x \to -25} \ln \left(x^2 - 5\right) \approx 6.430$$

ANSWER KEY

1. $\lim_{x \to -3} \ln(2x + 7) = 0$
2. $\lim_{x \to \pi} (\cos x + \sin x) = -1$
3. $\lim_{x \to -2} -x + e^{(x+2)} = 3$
4. $\lim_{x \to e} (\ln(x))^3 = 1$
5. $\lim_{x \to \frac{\pi}{2}} \csc\left(\frac{x}{\sin x}\right) = 1$
6. $\lim_{x \to \frac{5}{3}} \ln(3x - 4) = 1.667$
7. $\lim_{x \to 1} \left(e\left|\frac{x}{2e}\right|^x\right) = 0.5$
8. $\lim_{x \to 1} (\ln(x) + 5x)^2 = 25$
9. $\lim_{x \to 2\pi} \cos x - \left(\frac{\tan x}{\cot x}\right) = -1$
10. $\lim_{x \to 5} (\cos x)^2 + (\sin x)^2 = 1$