BASIC CALCULUS

STRATEGIC INTERVENTION MATERIAL - 1

INTRODUCTION TO LIMITS

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IN PARTIAL COMPLETION OF THE SUBJECT INVESTIGATION, IMMERSION, AND INQUIRIES

2026

OVERVIEW

Limit of a Function

Imagine you like someone and ***you keep getting closer*** emotionally...

You talk every day, care deeply, and act like more than just friends...

Your feelings are ***approaching*** love...

But the other person has set a boundary: "**Hanggang kaibigan lang kita.**" *(We're just friends)*

This is how ***limits*** work. No matter how close you get, you never actually become a couple...

Joking aside, limits are important in calculus as they are used to ***define the behavior of a given function at a particular point.*** This helps us ***understand continuity, derivatives, and integrals.***

The limit of a function describes the value that the function approaches as the input approaches a certain number.

Limits are written as follows:

$$\lim_{x \to a} = L$$

$f(x)$ - function of the variable x
a - a given constant that x approaches to
L - a value of the limit

This is read as: "The limit of f(x) as x approaches a is equal to L."

Let's try solving a few examples...

EXAMPLE 1 - POLYNOMIAL FUNCTION

When choosing the values for *x* in a table, you can choose any value to start with. However, the values chosen should make it clear what values of *f* they are getting close to.

Evaluate: $\lim_{x \to 2} (1 + 3x)$

Solution:
Here we see that $f(x) = (1 + 3x)$ and $a = 2$.

We must choose ***values of x that get close to 2.*** Keep in mind that ***we can approach any given value of x from two sides.*** We'll start with 1.99 and select additional numbers that get close to 2 but remain less than 2. Next, we choose values greater than 2, starting with 2.01, that gets closer to 2. Finally, we input these values into the function at each choice to get the following tables.

Approaching x from the left	
x	*f(x)*
2.01	7.03
2.001	7.003
2.0001	7.0003

Approaching x from the right	
x	*f(x)*
1.99	6.97
1.999	6.997
1.9999	6.9997

OVERVIEW

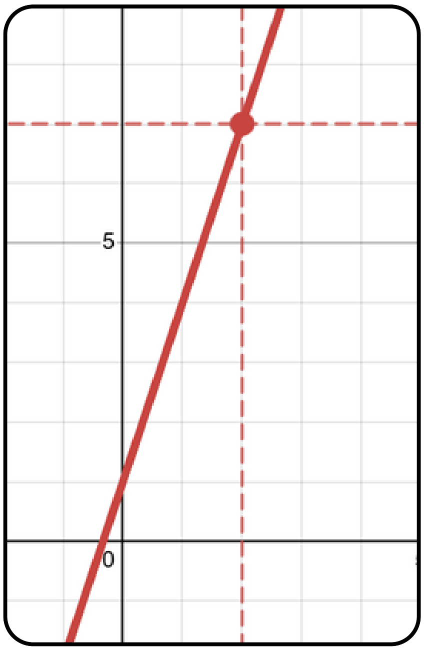
From these tables, we can see that ***as x approaches 3, the value of gets closer to 7.***

This means, $\lim_{x \to 2} (1 + 3x) = 7$

On the image on the right, the graph of $f(x) = (1 + 3x)$ shows that at ***x = 2, the y value is 7.***

Therefore,

$$\lim_{x \to 2} (1 + 3x) = 7$$



EXAMPLE 2 - RATIONAL FUNCTION

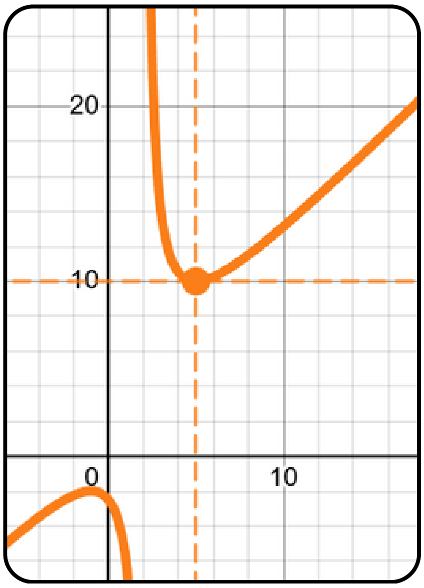
Evaluate: $\lim_{x \to 5} \left(\frac{x^2 + 5}{x - 2} \right)$

Solution:
We first create a table of values by choosing values that slowly approach 1.

Approaching x from the left	
x	*f(x)*
4.99	10.0000334...
4.999	10.0000003...
4.9999	10.0000000...

Approaching x from the right	
x	*f(x)*
5.01	10.0000332...
5.001	10.0000003...
5.0001	10.0000000...

As we can see that the ***limit of the function approaches 10.***



Inputting the function into a graphing calculator, we see that ***when x = 5, the y value is 10.***

Therefore,

$$\lim_{x \to 5} \left(\frac{x^2 + 5}{x - 2} \right) = 10$$

EXAMPLE 3 - RADICAL FUNCTION

Evaluate: $\lim_{x \to 5} \sqrt{5x}$

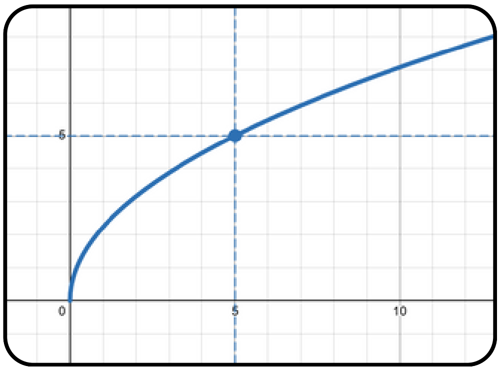
Solution:

Just like the previous examples, we substitute values that approach *a*. In this example, we approach 5.

Approaching x from the left	
x	*f*(*x*)
4.99	4.99499749...
4.999	4.99949997...
4.9999	4.99994999...

Approaching x from the right	
x	*f*(*x*)
5.01	5.00499750...
5.001	5.00049997...
5.0001	5.00004999...

As we can see that the ***limit of the function approaches 5.***



Plugging the function into a graph, we can see that ***when x = 5, the y value is 5.***

$$\lim_{x \to 5} \sqrt{5x} = 5$$

EXAMPLE 4 - THE DIFFERENCE OF $\lim_{x \to a} f(x)$ COMPARED TO $f(a)$

In the previous examples, we see that $\lim_{x \to a} f(x) = f(a)$. ***However, that is not always the case. Here's another example showing just that.***

Evaluate: $\lim_{x \to 1} \left(\frac{x^2 - 5x + 4}{x - 1} \right)$

Solution:

Immediately, we can see that ***if we substitute directly*** into the function, ***it would be indeterminate*** due to ***division by 0.***

But that doesn't matter. Since we are only concerned with ***values close to 1, not exactly at 1.*** We'll elaborate on this later, but for now, let's solve this example using a table of values.

Approaching x from the left	
x	*f*(*x*)
0.99	-3.01
0.999	-3.001
0.9999	-3.0001

Approaching x from the right	
x	*f*(*x*)
1.01	-2.99
1.001	-2.999
1.9999	-2.9999

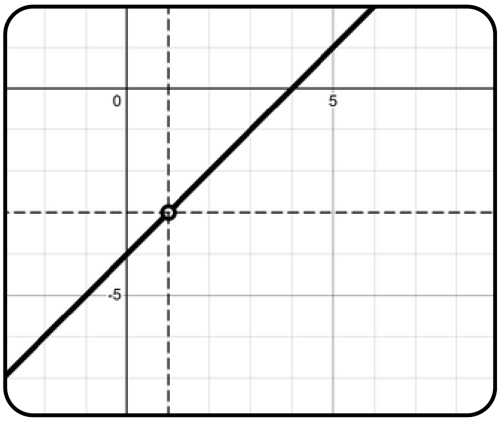
From these tables, we can see that ***as x approaches 1, the value approaches -3.***
This means,

$$\lim_{x\to 1}\left(\frac{x^2-5x+4}{x-1}\right)=-3$$

This is ***not the only way we can approach the problem***; we can also ***check if the function can be factored.***

$$\begin{aligned}\lim_{x\to 1}\left(\frac{x^2-5x+4}{x-1}\right)&=\left(\frac{x^2-5x+4}{x-1}\right)\\&=\left(\frac{(x-1)(x-4)}{(x-1)}\right)\\&=(x-4)\\\lim_{x\to 1}\left(\frac{x^2-5x+4}{x-1}\right)&=\lim_{x\to 1}(x-4)\end{aligned}$$

We see that ***after factoring and cancelling like terms,*** we see that $\left(\frac{x^2-5x+4}{x-1}\right)=(x-4)$.



If we graph the function $\left(\frac{x^2-5x+4}{x-1}\right)$, we see that it is the same as $(x-4)$.

Notice the ***hole in the function that perfectly lines up with (1, -3).***

This is because ***inputting 1 into the function would lead to division by 0***, as mentioned earlier.

This shows that $f(a)$ ***does not exist.***

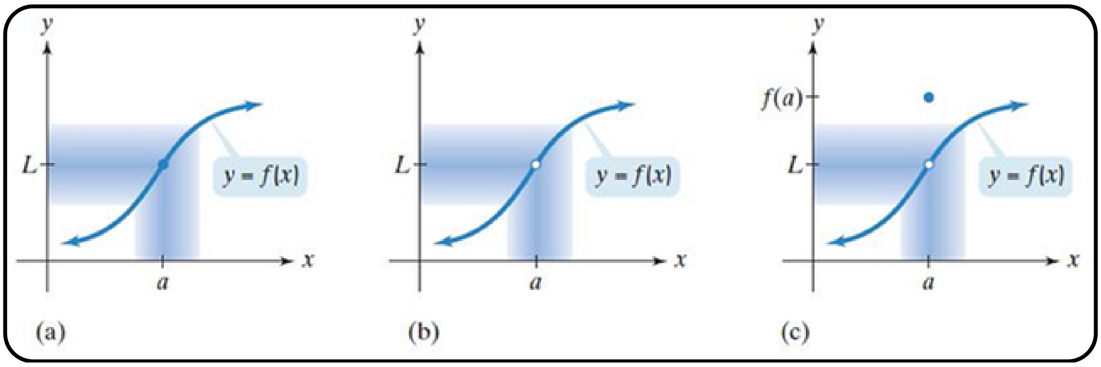
But we see that the $\lim_{x\to a} f(x)$ ***does exist.***

Meaning, $\lim_{x\to a} f(x)\neq f(a)$

Refer to the figure below.
In each graph, as the value of x approaches a, the value of f gets close to L.

From this, we can state that $\lim_{x\to a} f(x)=L$. This is the case **regardless of the value of *f* (*a*).**

In Figure (a), $f(a)=L$, and in Figure (b), $f(a)\neq L$. Figure (c) states that $\lim_{x\to a} f(x)=L$, even though *f* is not defined at *a*.



ACTIVITY I

Introduction to Limits

Let's do some short activities.

Have a go yourself! :D

Instructions:

Evaluate the following limits by filling in the missing values on the following tables. Write down the answer your paper.

1. $\lim_{x \to 0} 2x^2 - 6$

Approaching x from the left	
x	f(x)
-1	**?**
-0.5	**?**
-0.1	**?**
-0.01	**?**
-0.001	**?**

Approaching x from the right	
x	f(x)
1	**?**
0.5	**?**
0.1	**?**
0.01	**?**
0.001	**?**

2. $\lim_{x \to 5} \frac{3x+1}{2}$

Approaching x from the left	
x	f(x)
4	**?**
4.5	**?**
4.9	**?**
4.99	**?**
4.999	**?**

Approaching x from the right	
x	f(x)
6	**?**
5.5	**?**
5.1	**?**
5.01	**?**
5.001	**?**

3. $\lim_{x \to -3} 2\left(x^3 - 5\right)$

Approaching x from the left	
x	f(x)
-4	**?**
-3.5	**?**
-3.1	**?**
-3.01	**?**
-3.001	**?**

Approaching x from the right	
x	f(x)
-2	**?**
-2.5	**?**
-2.9	**?**
-2.99	**?**
-2.999	**?**

OVERVIEW

Limit Laws

As you saw in the previous section, you can *approach* solving limits in two ways. Namely, using tables and a graph. In this section, we will show another method to solving limits.

FINDING A LIMIT USING ALGEBRA

As with all subject matter involving mathematics, there are always a set of rules to follow. Limits are no different as they also come with a set of laws to follow when solving them.

The following formulas are the most basic laws about limits.

For any real number a and any constant c,

Limit of a Constant: $\lim_{x \to a} c = c$ *The limit of a constant is a constant*

Limit of x: $\lim_{x \to a} x = a$ *The limit of x as x approaches c is c*

For the following laws, assume that *f* and *g* are two functions for which both $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ exist. Also assume that L and M are real numbers such that $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$. Let c be a constant.

Sum Law: $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = L + M$

Difference Law: $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = L - M$

Constant Multiple Law: $\lim_{x \to a} [c \times f(x)] = c \times \lim_{x \to a} f(x) = c \times L$

Product Law: $\lim_{x \to a} [f(x) \times g(x)] = \lim_{x \to a} f(x) \times \lim_{x \to a} g(x) = L \times M$

Quotient Law: $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L}{M}$
for $M \neq 0$

Power Law: $\lim_{x \to a} (f(x))^n = \left(\lim_{x \to a} f(x)\right)^n = L^n$
for every positive integer

Root Law: $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{L}$
for all L *if* n *is odd and for* $L \geq 0$ *if* n *is even*

It looks like a lot, but it's ***no different from applying the basic order of operations*** to the function! :D

Recall PEMDAS? Yep, ***the same principle applies*** when solving limits using algebra. :)

Let's ***use these laws on the examples we showed earlier.***

EXAMPLE 5

Evaluate: $\lim_{x\to 2}(1+3x)$

Solution:
Let's apply limits laws step by step.

$$\begin{aligned}
? &= \lim_{x\to 2}(1+3x) \\
&= \lim_{x\to 2}(1) + \lim_{x\to 2}(3x) && \textit{Sum Law} \\
&= \lim_{x\to 2}(1) + 3\lim_{x\to 2}(x) && \textit{Constant Multiple Law} \\
&= 1 + 3(2) && \textit{Limit of a Constant \& Limit of x} \\
&= 1 + 6 && \textit{Multiplication} \\
&= 7 && \textit{Addition \& Final Answer}
\end{aligned}$$

EXAMPLE 6

Evaluate: $\lim_{x\to 5}\left(\frac{x^2+5}{x-2}\right)$

Solution:
Apply limit laws

$$\begin{aligned}
? &= \lim_{x\to 5}\left(\frac{x^2+5}{x-2}\right) \\
&= \frac{\lim_{x\to 5}\left(x^2+5\right)}{\lim_{x\to 5}(x-2)} && \textit{Quotient Law} \\
&= \frac{\lim_{x\to 5}x^2 + \lim_{x\to 5}5}{\lim_{x\to 5}x - \lim_{x\to 5}2} && \textit{Sum \& Difference Law} \\
&= \frac{\left(\lim_{x\to 5}x\right)^2 + \lim_{x\to 5}5}{\lim_{x\to 5}x - \lim_{x\to 5}2} && \textit{Power Law} \\
&= \frac{(5)^2+5}{5-2} && \textit{Limit of x \& Limit of a Constant} \\
&= \frac{25+5}{5-2} && \textit{Exponent} \\
&= \frac{30}{3} && \textit{Addition \& Subtraction} \\
&= 10 && \textit{Division \& Final Answer}
\end{aligned}$$

OVERVIEW

EXAMPLE 7

Evaluate: $\lim_{x\to 5}\sqrt{5x}$

Solution:
Apply limit laws

$$
\begin{aligned}
? &= \lim_{x\to 5}\sqrt{5x} \\
&= \sqrt{\lim_{x\to 5} 5x} && \textit{Root Law} \\
&= \sqrt{5\lim_{x\to 5} x} && \textit{Constant Multiple Law} \\
&= \sqrt{5\,(5)} && \textit{Limit of x} \\
&= \sqrt{25} && \textit{Multiplication} \\
&= 5 && \textit{Square Root \& Final Answer}
\end{aligned}
$$

EXAMPLE 8

Evaluate: $\lim_{x\to 1}\left(\frac{x^2-5x+4}{x-1}\right)$

Solution:
Apply limit laws

$$
\begin{aligned}
? &= \lim_{x\to 1}\left(\frac{x^2-5x+4}{x-1}\right) \\
&= \lim_{x\to 1}\left(\frac{(x-1)\,(x-4)}{x-1}\right) && \textit{Factor the function (Only do so if possible)} \\
&= \lim_{x\to 1}(x-4) && \textit{Cancel like terms} \\
&= \lim_{x\to 1} x - \lim_{x\to 1} 4 && \textit{Difference Law} \\
&= (1)-4 && \textit{Limit of x \& Limit of a Constant} \\
&= -3 && \textit{Subtraction \& Final Answer}
\end{aligned}
$$

As you can see, we arrive at the same answers as before! :D

Now it’s time to test your skills with another activity.

ACTIVITY II

Limit Laws

May mga bagay palang kahit anong lapit mo...

Hindi talaga umaabot.

Hindi dahil kulang ka, kundi dahil hanggang doon lang talaga...

Ganun ang limit. Ganun din minsan ang pagibig, kayo'y pinagtagpo pero hindi itinadhana...

Instructions:

Evaluate the following limits using the previously mentioned limit laws.

1. $\lim_{x \to -3} (4x + 2)$
2. $\lim_{x \to 4} \left(x^2 - 5x + 6\right)$
3. $\lim_{x \to 3} \frac{\sqrt[2]{6 + x}}{4x}$
4. $\lim_{x \to 2} (x + 5)\left(3x^2\right)$
5. $\lim_{x \to 3} \frac{7x}{\sqrt[2]{x + 1}}$
6. $\lim_{x \to -2} \left(\frac{x^2}{x + 6}\right)$
7. $\lim_{x \to 8} \left(\sqrt[3]{x^2}\right) - 2$
8. $\lim_{x \to 5} \left(\frac{1}{x^2 - x}\right)$
9. $\lim_{x \to 2} \sqrt[2]{\frac{x + 2}{(x + 1)^2}}$
10. $\lim_{x \to 3} 5^{(x-1)} + 14x$

REFERENCES

Limits of Transcendental Functions

FRONT COVER

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CONTENT

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BACK COVER

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ANSWER KEY

1. $\lim_{x \to 0} 2x^2 - 6$

Approaching x from the left	
x	f(x)
-1	***-4***
-0.5	***-5.5***
-0.1	***-5.98***
-0.01	***-5.9998***
-0.001	***-5.999998***

Approaching x from the right	
x	f(x)
1	***-4***
0.5	***-5.5***
0.1	***-5.98***
0.01	***-5.9998***
0.001	***-5.999998***

$$\lim_{x \to 0} 2x^2 - 6 = -6$$

2. $\lim_{x \to 5} \frac{3x+1}{2}$

Approaching x from the left	
x	f(x)
4	***6.5***
4.5	***7.25***
4.9	***7.85***
4.99	***7.985***
4.999	***7.9985***

Approaching x from the right	
x	f(x)
6	***9.5***
5.5	***8.75***
5.1	***8.15***
5.01	***8.015***
5.001	***8.0015***

$$\lim_{x \to 5} \frac{3x+1}{2} = 8$$

3. $\lim_{x \to -3} 2\left(x^3 - 5\right)$

Approaching x from the left	
x	f(x)
-4	***-138***
-3.5	***-95.75***
-3.1	***-69.582***
-3.01	***-64.54180***
-3.001	***-64.05402***

Approaching x from the right	
x	f(x)
-2	***-26***
-2.5	***-41.25***
-2.9	***-58.778***
-2.99	***-63.46180***
-2.999	***-63.94602***

$$\lim_{x \to -3} 2\left(x^3 - 5\right) = -64$$

ANSWER KEY

1. $\lim_{x\to -3}(4x+2) = (4(-3)+2) = -12+2 = -10$

2. $\lim_{x\to 4}\left(x^2-5x+6\right) = \left((4)^2-5(4)+6\right) = (16-20+6) = 2$

3. $\lim_{x\to 3}\frac{\sqrt{6+x}}{4x} = \frac{\sqrt{6+3}}{4(3)} = \frac{\sqrt{9}}{12} = \frac{3}{12} = \frac{1}{4}$

4. $\lim_{x\to 2}(x+5)\left(3x^2\right) = (2+5)\left(3(2)^2\right) = (7)(3(4)) = 7(12) = 84$

5. $\lim_{x\to 3}\frac{7x}{\sqrt{x+1}} = \frac{7(3)}{\sqrt{3+1}} = \frac{21}{\sqrt{4}} = \frac{21}{2}$

6. $\lim_{x\to -2}\left(\frac{x^2}{x+6}\right) = \left(\frac{(-2)^2}{-2+6}\right) = \left(\frac{4}{-4}\right) = -1$

7. $\lim_{x\to 8}\left(\sqrt[3]{x^2}\right)-2 = \left(\sqrt[3]{8^2}\right)-2 = 4-2 = 2$

8. $\lim_{x\to 5}\left(\frac{1}{x^2-x}\right) = \left(\frac{1}{(5)^2-5}\right) = \left(\frac{1}{25-5}\right) = \frac{1}{20}$

9. $\lim_{x\to 2}\sqrt{\frac{x+2}{(x+1)^2}} = \sqrt{\frac{(2)+2}{(2+1)^2}} = \sqrt{\frac{4}{(3)^2}} = \frac{2}{3}$

10. $\lim_{x\to 3}5^{(x-1)}+14x = 5^{(3-1)}+14(3) = 5^2+42 = 25+42 = 67$