Universality and Capacity Metrics in Deep Neural Networks

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Summary

We use our new theory of Implicit Heavy-Tailed Self-Regularization (HT-SR)[a] to develop a Universal capacity control metric, $\hat{\alpha}$, for DNNs.

- We analyze layer weight matrices $\mathbf{W}$ of over 100 pretrained DNNs, from both Computer Vision and NLP (VGG, ResNet, GPT, etc).
- We find that the spectral density $\rho(\lambda)$ of the normalized correlation matrix, $\mathbf{X}=\frac{1}{N}\mathbf{W}^T\mathbf{W}$, can be fit to a power law,

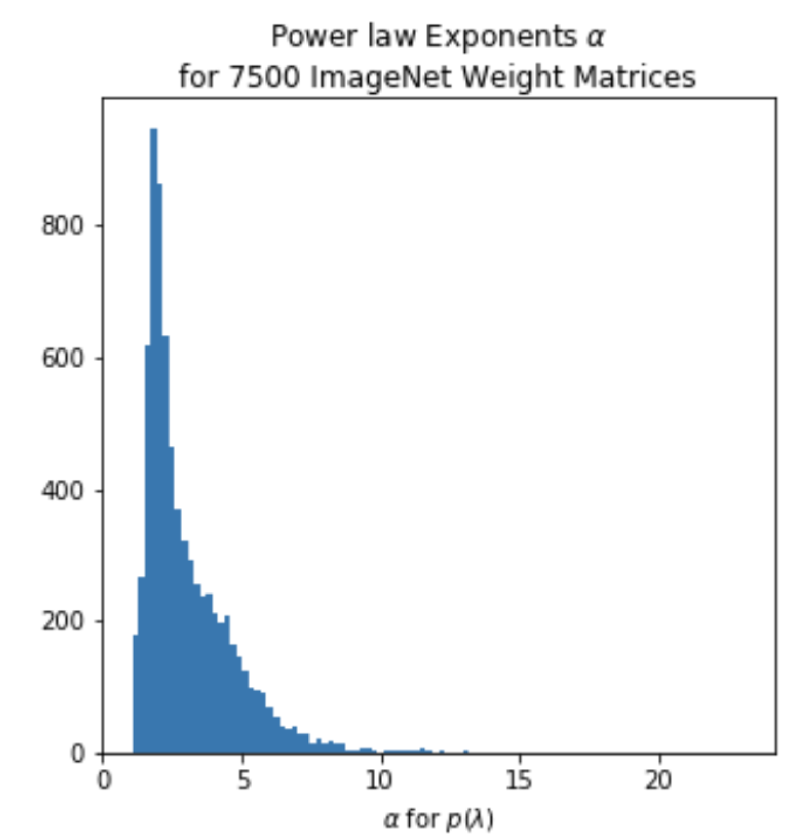
$$\rho(\lambda):=\lambda^{-\alpha},\ \ \lambda<\lambda^{max}$$

  with exponent $\alpha\rightarrow 2$ universally.
- We propose a new Universal capacity metric, $\hat{\alpha}=\sum\alpha_l\log\lambda_l^{max}$, which correlates well with the generalization accuracy across a series of related DNN architectures.

[a] Long (arXiv:1810.01075) and short (ICML 2019) versions.

Universality of α

The power law exponents α for nearly 10,000 layer weight matrices $\mathbf{W}$, and convolutional feature maps, for over 100 CV DNN architectures, empirically approaches *Universal* value of $\alpha\rightarrow 2$.



Theory

Consider the familiar Product Norm Capacity Metric (for say the Spectral or Frobenius norm)

$$\mathcal{C}\sim\|\mathbf{W}_1\|\times\|\mathbf{W}_2\|\cdots\|\mathbf{W}_L\|. \tag{1}$$

Using a standard trick from field theory, we consider the log Product Norm

$$\log\mathcal{C}\sim\log\Big[\|\mathbf{W}_1\|\times\|\mathbf{W}_2\|\cdots\|\mathbf{W}_L\|\Big]\sim\Big[\log\|\mathbf{W}_1\|+\log\|\mathbf{W}_2\|\cdots\log\|\mathbf{W}_L\|\Big],$$

which takes the form of an average Log norm

$$\log\mathcal{C}\rightarrow\langle\log\|\mathbf{W}\|\rangle=\frac{1}{N_L}\sum_l\log\|\mathbf{W}_l\|.$$

Derive as a generalized weighted average, which resembles a (weighted) average log Spectral Norm

$$\hat{\alpha}=\sum\alpha_l\log\lambda_l^{max}$$

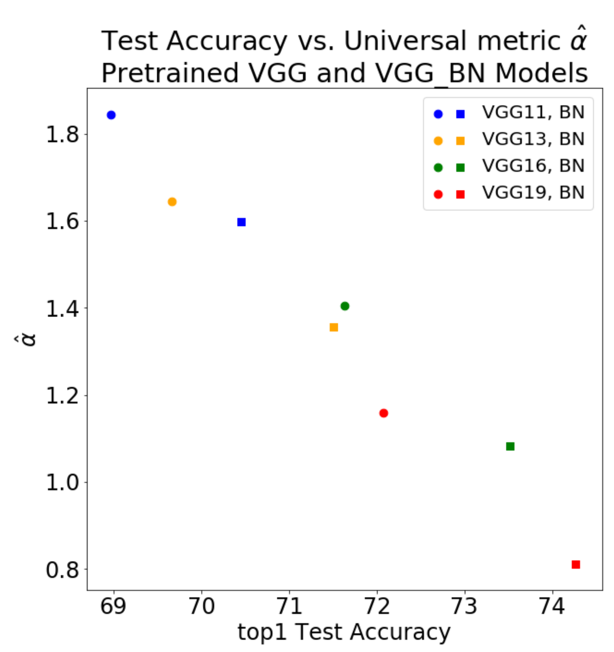
and/or which looks like the Soft Rank $\mathcal{R}_s^{log}$ in log units (from EVT, for small α)

$$\mathcal{R}_s^{log}:=\frac{\log\|\mathbf{W}\|_F^2}{\log\lambda^{max}}\approx\alpha,\ \alpha\rightarrow 1.$$

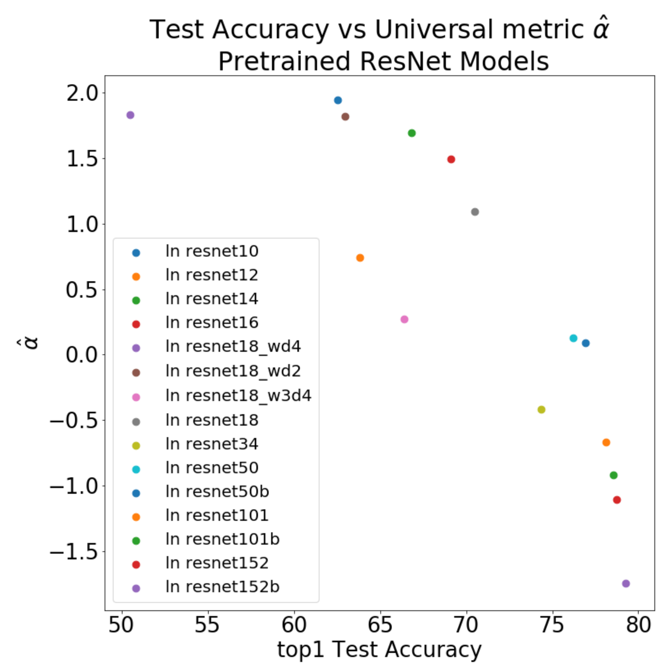
Capacity vs Test Accuracies

Norm metrics actually correlate with test accuracies across series of pretrained DNNs

VGG Series:



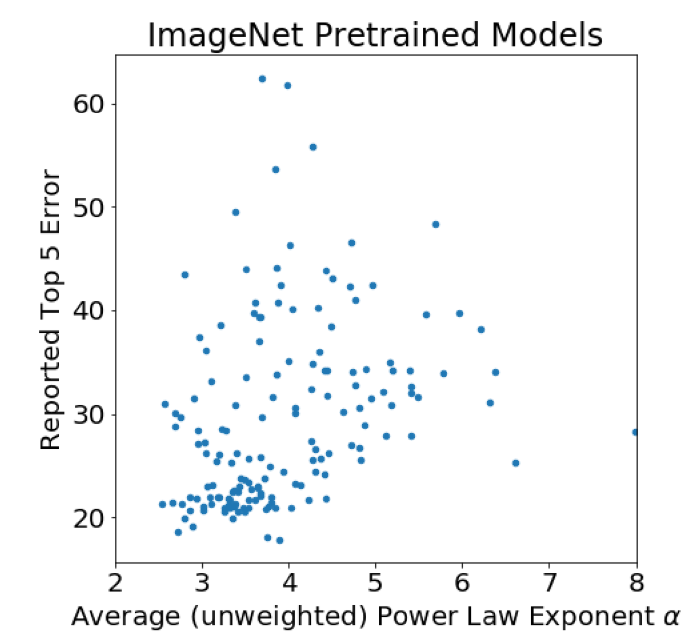
ResNet Series:



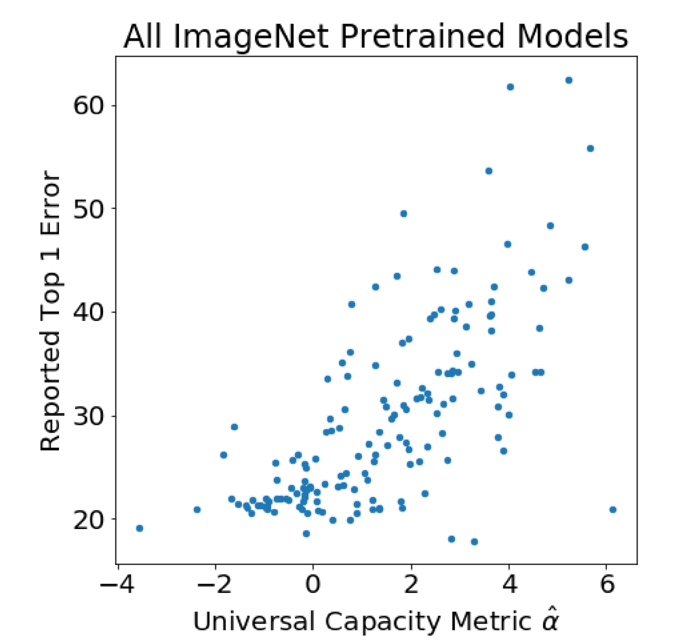
More Test Accuracies

Capacity versus Test Accuracy for over 100 pretrained ImageNet models

Average (unweighted) $avg(\alpha)$



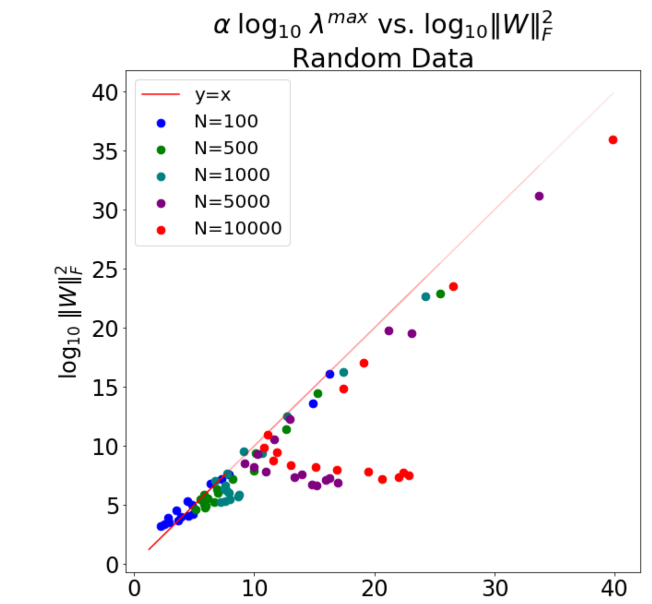
Universal Capacity Metric $\hat{\alpha}$



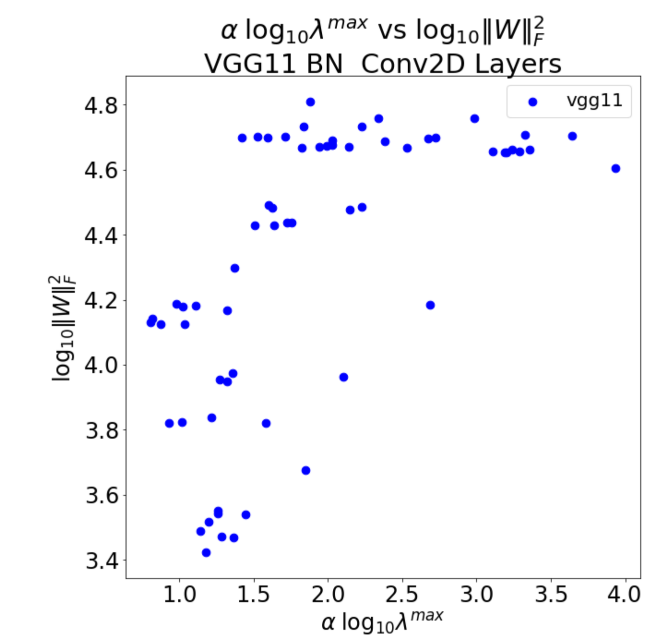
Power Law - Norm Relation

EVT provides the relation between the Frobenius norm and the Power law exponent $\alpha\sim 1$

Random Pareto Matrices



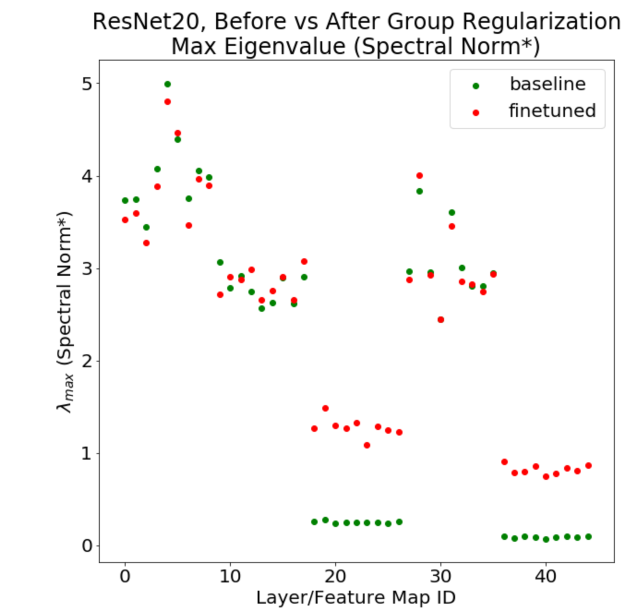
VGG11 Weight Matrices



Analysis of Distilled ResNet

Distillation sometimes induces anomalous jumps in the scale of the weight matrices

Spectral Norms (max eigenvalues λ^{max})



Power law exponents α are not correlated w/ λ^{max}