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Volatility Forecast Based on the Hybrid Artificial Neural Network and GARCH-type Models

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Abstract

This study compares the forecast performance of volatilities between two types of hybrid ANN and GARCH-type models. The findings show that EGARCH-ANN model performs better than other models to forecast the volatilities of log-returns series in Chinese energy market, and there are significant leverage effects in Chinese energy market.

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Keywords: Volatility forecast; ANN; GARCH; Energy index

1. Introduction

Because of the important role played by energy in the world economy, recent increases in energy prices volatility have caused great concern among consumers, corporations and governments. In particular, due to its significant portfolio allocation and risk management, the ability to predict energy prices volatility with greater precision is critical for market participants. Basically, market participants utilize different methods for forecasting the volatility of economic variables. These methods are overall divided into two categories: classic and neural network [1]. Although numerous comparative studies between classic models and ANNs have been conducted in the literatures, findings are mixed with regard to whether the flexible nonlinear approach is better than the time series method in forecasting [2-3]. However, hybrid ANN and generalized autoregressive conditional heteroskedasticity (GARCH-type) [4-5] models are usually found to have advantages in comparison with ANNs or time series models. For example, Hajizadeh, et al. [6] found that the hybrid ANN and Exponential GARCH (EGARCH) models can provides better volatility forecasts for S&P500 index. Bildirici and Ersin [7] discussed the advantage of hybrid ANN and Asymmetric Power GARCH (APGARCH) model in Istanbul Stock Exchange, and found the increase of the forecasting performance of APGARCH model.

In line with Hajizadeh, et al. [6] and Bildirici and Ersin [7], this study focus on the capability of the hybrid ANN and GARCH-type models to forecast volatility in Chinese energy market. Two types of hybrid ANN and

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GARCH-type models are constructed and used to forecast volatilities of Chinese energy index in Shanghai Stock Exchange for from 31 December 2013 to 10 March 2016.

2. Methodology

Due to their successful application to forecast the volatilities of economic and financial variables, GARCH-type models are considered as a part of the proposed hybrid models. Allowing for the leverage effects Ca large financial asset price decline can have a bigger impact on volatility than a large price increase- of financial variables commonly found in the literature, EGARCH [8] and GJR-GARCH [9] models are used to construct the proposed hybrid models. Meanwhile, because of the existence of complex non-linear correlation structure among financial variables, more flexible models are needed to approximate the features of financial variables, such as ANN models. The greatest advantage of ANN models their ability to model complex nonlinear relationship without a priori assumptions of the nature of the relationship. They can relate a set of input variables to one or more output target variables that contain nonlinear latent units to achieve significant flexibility [10]. Therefore, ANNs are employed to construct the proposed hybrid models. There are several training methods for neural networks. The most common method of the model estimation in financial applications is Back-propagation [11]. This method takes inputs only from the previous layer, and sends outputs only to the next layer. The parameters of the model are updated so that the tuning of parameters is in accordance with the quadratic loss function during the process of the model estimation. Therefore, the lowest error can be achieved by estimate iteratively.

2.1. GARCH-type Models

The log-returns series of Chinese energy index are fitted by the autoregressive moving average (ARMA) [12] models for the conditional mean model. Without loss of generality, r_t and P_t denote the log-return and price of a given asset at time t, respectively, and $r_t = 100 \times \log(P_t/P_{t-1})$, $t = 1, \ldots, T$. The ARMA(m, n) can be expressed as follows:

$$r_t = c + \sum_{i=1}^{m} \phi_i r_{t-r} + \sum_{j=1}^{n} \theta_j \varepsilon_{t-j} + \varepsilon_t \tag{1}$$

where ϕ_i and θ_j are unknown parameters, ε_t is the uncorrelated random variable with mean zero and variance σ_ε^2.

As documented in Fatima and Hussain [13], the conditional variances of residuals of financial variables ε_t are not constant throughout, but vary from one period to another. To capture the feature of conditional volatilities, GARCH model was introduced by [4], and future extended by a lot of academics [8-9]. Allowing for the leverage effects of financial variables, EGARCH and GJR-GARCH models are used in this study. Both models can be represented as follows:

$$EGARCH(p, r, q) : \log(h_t) = \omega + \sum_{j=1}^{q} \beta_j \log(h_{t-j}) + \sum_{i=1}^{p} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right| + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{\sqrt{h_{t-k}}} \tag{2}$$

$$GJR - GARCH(p, q) : h_t = \omega + \sum_{m=1}^{p} \alpha_m \varepsilon_{t-m}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1} + \sum_{n=1}^{q} \beta_n h_{t-n} \tag{3}$$

$$\varepsilon_t = \sqrt{h_t} \eta_t \tag{4}$$

where $\eta_t \sim i.i.d.(0, 1)$. For EGARCH, no restrictions are placed on the parameters α_i, β_j, and γ_k. For GJR-GARCH, $\omega \geq 0$, $\alpha_i \geq 0$, $\alpha_i + \gamma \geq 0$, $\sum\limits_{m=1}^{p} \alpha_m + \gamma + \sum\limits_{n=1}^{q} \beta_n < 1$. I_t equals 1 if $\varepsilon_t < 0$, and 0 otherwise.

2.2. ANN

On account of advantages of nonlinearity and flexibility, artificial neural network is employed to forecast the volatilities of Chinese energy index. To overcome the possibility of overfitting the training data and failing to capture the true statistical process generating the log-returns series of Chinese energy index, back propagation training algorithm is used to minimize the quadratic error by descent maximum gradient. That is to say, the so-called back propagation neural network (BPNN) is utilized in this study. The model is the multilayer perceptron, with 1 input layer, 1 hidden layer, and 1 output layer. The input layer is represented by a vector $d = (x_1, x_2, \ldots, x_d)'$. The hidden layer is a vector $m = (h_1, h_2, \ldots, h_m)'$. The output layer is a vector $c = (y_1, y_2, \ldots, y_c)'$. The multilayer perceptron model is obtained by a weighted linear combination of the d input values in the form.

$$a_j = \sum_{i=1}^{d} w_{ji}^{(1)} x_i \tag{5}$$

The activation of hidden unit j can be achieved by transforming the linear sum using a logistic activation function $g(a) = \exp(a)/(1 + \exp(a))$:

$$h_j = g(a) = g\left(\sum_{i=1}^{d} w_{ji}^{(1)} x_i\right) \tag{6}$$

The node of output layer is defined as:

$$y_k = \tilde{g}\left(\sum_{j=1}^{m} w_{jk}^{(2)} g\left(\sum_{i=1}^{d} w_{ji}^{(1)} x_i\right)\right) \tag{7}$$

If the output function is taken linear, $\tilde{g}(a) = a$, the output model reduces to:

$$y_k = \sum_{j=1}^{m} w_{jk}^{(2)} g\left(\sum_{i=1}^{d} w_{ji}^{(1)} x_i\right) \tag{8}$$

The multilayer feed-forward BPNN is shown as Fig. 1.

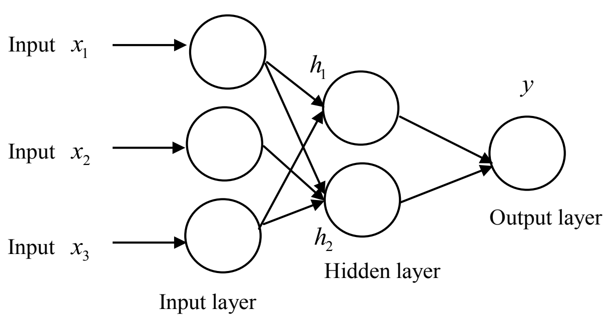

Fig. 1. Multilayer feed-forward back propagation neural network

To establish an effective neural network in application to real financial data, the dataset used is divided into three subsets: the training set is 80% of the dataset, the validation set is 10%, and the remaining 10% is for test set.

2.3. Hybrid Models

The purpose of this study is to propose two types of hybrid ANN and GARCH-type models and to compare the forecast performance of volatility for both hybrid models. Type I model is established by inputting the outcome of the preferred GARCH-type models into ANN, called ANN-GARCH model. That is, the outcome of the preferred GARCH-type models is considered as the input variable so as to augment the forecasting performance of volatility in Chinese energy market. On the other hand, Type II model is built by considering the output layer of ANN as a variable of GARCH-type models, called GARCH-ANN model, so that the augmented GARCH-type models can behavior better on forecasting volatility. Both types of models are expressed as follows:

Type I: ANN-GARCH model

Based on the preferred EGARCH model, $\beta_1 \log(h_{t-1})$, $\alpha_1 \left|\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right|$, and $\gamma_1 \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}$ are chosen as the endogenous explanatory variables, which are regarded as the input variables in the ANN. Similar to EGARCH, $\beta_1 h_{t-1}$, $\alpha_1 \varepsilon_{t-1}^2$, and $\gamma \varepsilon_{t-1}^2 I_{t-1}$ of the preferred GJR-GARCH model are chosen as the input variables in the ANN.

Type II: GARCH-ANN model

$$EGARCH-ANN : \log(h_t) = \omega + \sum_{j=1}^{q} \beta_j \log(h_{t-j}) + \sum_{i=1}^{p} \alpha_i \left|\frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}}\right| + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{\sqrt{h_{t-k}}} + \sum_{h}^{s} \xi_h \psi(z_t \lambda_h) \tag{9}$$

$$GJR-GRACH-ANN : h_t = \omega + \sum_{m=1}^{p} \alpha_m \varepsilon_{t-m}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1} + \sum_{n=1}^{q} \beta_n h_{t-n} + \sum_{h}^{s} \xi_h \psi(z_t \lambda_h) \tag{10}$$

$$\psi(z_t \lambda_h) = \left[1 + \exp\left(\lambda_{h,d,w} + \sum_{d=1}^{\upsilon}\left[\sum_{w=1}^{m}\left(\lambda_{h,d,w} z_{t-d}^{w}\right)\right]\right)\right]^{-1} \tag{11}$$

where, $z_{t-d} = [\varepsilon_{t-d} - E(\varepsilon)] \big/ \sqrt{E(\varepsilon^2)}$. $\frac{1}{2}\lambda_{h,d,w} \sim uniform[-1, +1]$.

3. Data Analysis

3.1. ARMA and GARCH models

The dataset investigated in this study is the Chinese energy index in Shanghai Stock Exchange from 31 December 2013 to 10 March 2016 for a total of 534 observations. The log-returns series of the dataset are stationary according to Augmented Dickey-Fuller (ADF) test. Therefore, ARMA(m, n) model is used to capture the mean features of log-returns series. According to the AIC and BIC criteria, the model is estimated as follows:

$$r_t = -0.000117 + 0.091924^* r_{t-1} - 0.957180^* r_{t-2} - 0.092390^{**} \varepsilon_{t-1} + 0.906037^* \varepsilon_{t-2} + \varepsilon_t \tag{12}$$

The $*$ means that the parameter is significant at 0.1 significance level, and the $**$ means that the parameter is significant at 0.05 significance level. The results show that the log-returns of energy index at time t are positively influenced by its log-returns of energy index at time $t-1$ and negatively influenced by its log-returns at time $t-2$. The residuals of the model are tested by serial correlation LM and ARCH LM tests. The results show that the serial correlation has been captured by ARMA model, and the conditional heteroskedasticity of the residuals is confirmed because the F-statistic of ARCH LM test significantly rejects the null hypothesis. Thus, GARCH models are employed to fit the conditional heteroskedasticity of the residuals. According to the AIC and BIC criteria, the preferred GARCH models are illustrated as follows:

Table 1. Parameters estimation of GARCH-type models

Parameter	ω	α	β	γ	AIC	BIC	ARCH LM
EGARCH(1,1,1)	-0.229640*	0.186406*	0.987980*	0.033339*	-4.944347	-4.912146	1.385152
GJR-GARCH(1,1,1)	3.16E-06*	0.114989*	0.914467*	-0.049867*	-4.938990	-4.906788	0.529583

The $*$ means that the parameter is significant at 0.05 significance level. ARCH LM test shows that there are not the conditional heteroskedasticity in the residuals of both GARCH models. AIC and BIC criteria show that EGARCH perform better than GJR-GARCH on fitting the dataset. The leverage effect is not found in both GARCH models in Chinese energy market. This means that the fluctuation of energy price in Chinese market dont have an asymmetric impact on volatility of energy index. The result is contrary to the findings that financial assets usually have an asymmetric impact on their volatility. This is because the investors are able to make relatively rational decisions in the Chinese energy market.

3.2. Hybrid models

First, the ANN-GARCH model is estimated by using the outcome of preliminary GARCH-type models. According to the multilayer feed-forward back propagation neural network, three subsets are chosen to fit the parameters of hidden layer and output layer, and then the forecasting volatilities based on the ANN can be obtained. The outcome is exhibited in Fig.2.

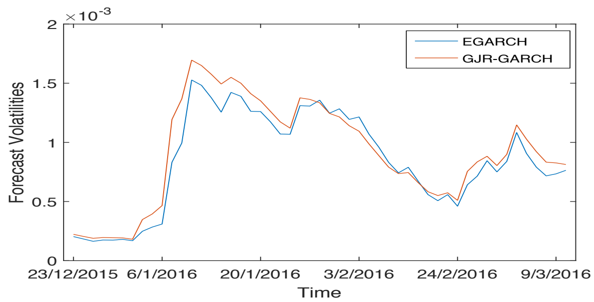

Fig. 2. Forecasting volatilities of ANN-GARCH model

Second, the GARCH-ANN model is estimated by considering the output layer of ANN as a variable of GARCH-type models. The outcome of ANN is based on residuals of Equation (12). The forecast volatilities of the preferred GARCH-ANN model are exhibited in Fig.3.

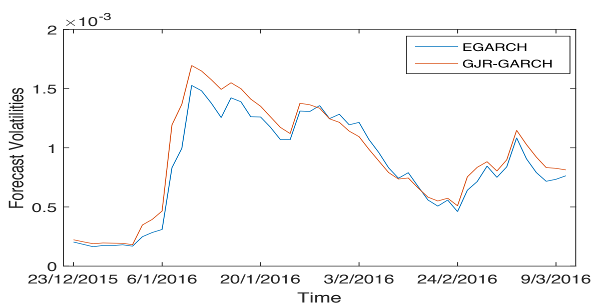

Fig. 3. Forecasting volatilities of GARCH-ANN model

3.3. Comparison of forecasting performance between two types of hybrid ANN and GARCH-type models

To estimate forecast accuracy, this study is in line with the work by [6] and compares the forecasting performance of volatility of both proposed hybrid ANN and GARCH-type models with realized volatility (RV). The RV on day t is computed by

$$RV_t = \sqrt{\frac{1}{n} \sum_{i=t-n+1}^{t} (r_i - \bar{r})^2} \qquad (13)$$

where $\bar{r} = n^{-1} \sum_{i=t-n+1}^{t} r_i$, and n is the number of days prior to t. According to Kristjanpoller et al. [14] and Prokopczuk et al. [15], n equals to 22. That means there are about 22 transactions in every month. In order to evaluate the performance of models in forecasting volatility, a loss function are considered: Root mean square error (RMSE). It is defined as follows:

$$RMSE = \left(\frac{1}{n} \sum_{i=1}^{n} (\sigma_i - RV_i)^2 \right)^{1/2} \qquad (14)$$

Allowing for the testing set in ANN, 51 out-of-sample observations are used to forecast the volatilities and to examine the performance of these hybrid models. The results are reported in Table 2.

Table 2. The results of forecasting volatilities

Loss function	ANN-EGARCH	ANN-GJR-GARCH	EGARCH-ANN	GJR-GARCH-ANN
RMSE	0.003626	0.004146	0.003151	0.003630

The results show that EGARCH-ANN model performs better than other models to forecast the volatilities of log-returns series in Chinese energy market according to RMSE. This means that the EGARCH model is augmented by considering the outcome of ANN as a variable in variance equation. Although the results of GARCH-type models are regarded as input variables of ANN, the combination of ANN and GARCH-type models has a less performance than the hybrid GARCH-ANN model.

4. Conclusions

This study has two main contributions. First, two types of hybrid ANN and GARCH-type models are demonstration to forecast the volatilities of log-returns series in Chinese energy market. The results show that EGARCH-ANN model performs better than other models to forecast the volatilities of log-returns series in Chinese energy market according to RMSE. That means the augmented EGARCH model behavior better when it is used to fit the conditional heteroskedasticity of log-returns of financial assets. Second, we examine the leverage effect of energy index, the results show that there are significant leverage effect in Chinese energy market. That means the fluctuation of energy price in Chinese market have an asymmetric impact on volatility of energy index. The result is consistent with the findings in the literature.

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References

[1] Nazarian R, Alikhani NG, Naderi E, Amiri A. Forecasting stock market volatility: A forecast combination approach. *MPRA Paper* 2013.
[2] Adya M, Collopy F. How effective are neural networks at forecasting and prediction? A review and evaluation.
[3] Wang JH, Leu JY. Stock market trend prediction using ARIMA-based neural networks. *IEEE International Conference on Neural Networks* 1996; **4**: 2160-2165.
[4] Engle RF. Autogressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflations. *Econometrica* 1982; **50**: 987-1007.
[5] Bollerslev T. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics* 1986; **31**: 307-327.
[6] Hajizadeh E, Seifi A, Zarandi MHF, Turksen IB. A hybrid modeling approach for forecasting the volatility of S&P 500 index return. *Expert Systems with Applications* 2012; **39**: 431-436.
[7] Bildirici M, Ersin ÖÖ. Improving forecasts of GARCH family models with the artificial neural networks: An application to the daily returns in Istanbul Stock Exchange. *Expert Systems with Applications* 2009; **36**: 7355-7362.
[8] Nelson DB. Conditional heteroskedasticity in asset returns: A new approach. *Econometrica* 1991; **59**: 347-370.
[9] Glosten LR, Jagannathan R, Runkle D. On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance* 1993; **48**: 1779-1801.
[10] Kay JW, Titterington DM. *Statistics and Neural Networks: Advances at the Interface*. 1st ed. Oxford: Oxford University Press; 1999.
[11] Rumelhart DE, Rubin R, Golden R, Chauvin Y. Backpropagation: The basic theory. In: Chauvin Y, Rumelhart DE, editors. *Backpropagation: Theory, architectures, and applications*, Hillsdale NJ: Lawrence Erlbaum; 1995, p. 1-34.
[12] Box GEP, Jenkins GM. *Time Series Analysis, Forecasting and Control*. Holden-Day; 1970. *Journal of Forecasting* 1998; **17**: 481-495. *Econometrica* 1982; **50**: 987-1007.
[13] Fatima S, Hussain G. Statistical models of KSE100 index using hybrid financial systems. *Neurocomputing* 2008; **71**: 2742-2746.
[14] Kristjanpoller W, Fadic A, Minutolo MC. Volatility forecast using hybrid artificial neural network models. *Expert Systems with Applications* 2014; **41**: 2437-2442.
[15] Prokopczuk M, Symeonidis L, Simen CW. Do jumps matter for volatility forecasting? Evidence from energy markets. *Journal of Futures Markets* 2015; forthcoming.