

Graphing the Mermin-Peres Magic Square

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Joint work with Dr. Samuel Harris and Taylor Smith

Northern Arizona University
Department of Mathematics and Statistics

SUnMaRC
13 April 2025



Department of Mathematics and Statistics

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4. Define Graphs and Coloring

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- built a graph based on the game algebra of the Mermin-Peres Magic Square Game
 - We chose the Mermin-Peres game because it is the smallest synchronous game that has a winning quantum strategy but no classical strategy
- encoded the graph into SageMath and ran a multitude of constructions to identify which vertices could be removed without changing the colorabilities of the graph

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- i.e. $P_A(a|x) = \sum_{b \in \mathcal{O}} P(a, b|x, y)$ is independent of y , and
 $P_B(b|y) = \sum_{a \in \mathcal{O}} P(a, b|x, y)$ is independent of x .

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 - i.e. If they give different answers they lose

Mermin-Peres Magic Square Game

Definition

The Mermin-Peres Magic Square Game is a synchronous non-local game. In this game, Alice and Bob will each receive one equation from the following list:

$$x_1 + x_2 + x_3 \equiv 0 \pmod{2}$$

$$x_4 + x_5 + x_6 \equiv 0 \pmod{2}$$

$$x_7 + x_8 + x_9 \equiv 0 \pmod{2}$$

$$x_1 + x_4 + x_7 \equiv 0 \pmod{2}$$

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Alice and Bob fill out their assigned equations with 0's and 1's simultaneously without knowing how the other fills theirs out. Their goal is to satisfy each of their own equations while producing the same value for any shared variable.

Magic Square Representation

The Magic Square game can be represented by a 3×3 grid by labeling each cell like so:

x_1	x_2	x_3
x_4	x_5	x_6
x_7	x_8	x_9

$$x_1 + x_2 + x_3 = 0$$

$$x_4 + x_5 + x_6 = 0$$

$$x_7 + x_8 + x_9 = 0$$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ + & + & + \\ x_4 & x_5 & x_6 \\ + & + & + \\ x_7 & x_8 & x_9 \\ = & = & = \\ 0 & 0 & 1 \end{array}$$

Magic Square Walkthrough

Alice

x_1	x_2	x_3
x_4	x_5	x_6
x_7	x_8	x_9

Bob

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x_1	x_2	x_3
x_4 1	x_5	x_6
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$$0 = x_4 + x_5 + x_6$$

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x_6	x_9	x_1
0	0	x_2
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$$\begin{matrix} x_3 \\ + \\ x_6 \\ + \\ x_9 \\ = \\ 1 \end{matrix}$$

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Classical Strategy for Magic Square Game Strategy

Definition

A classical strategy for a Non-Local Game is represented by a function $f : \mathcal{I} \rightarrow \mathcal{O}$.

For the Magic Square Game, we can define a classical strategy using the grid below. The best strategy has them agree upon a set table of answers, this allows them to win most of the time. It is impossible to create a solution set that satisfies all nine equations at the same time, otherwise they would be able to win every single game.

x_1	x_2	x_3	$1 + 1 = 0$
1	0	1	
x_4	x_5	x_6	$1 + 1 = 0$
1	1	0	
x_7	x_8	x_9	$1 \neq 0$
0	1	0	

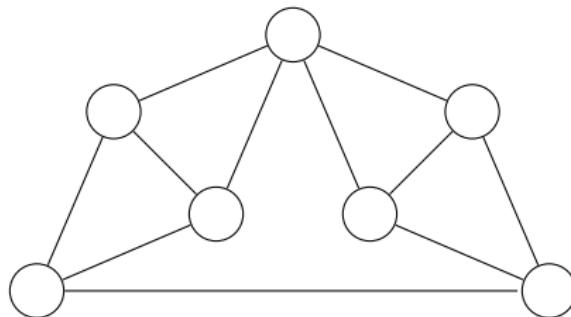
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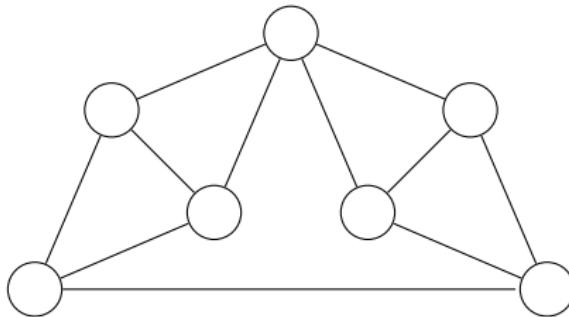
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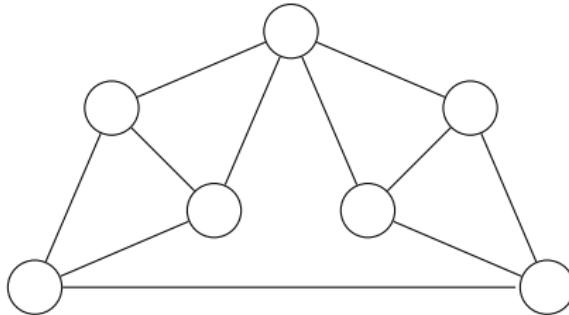
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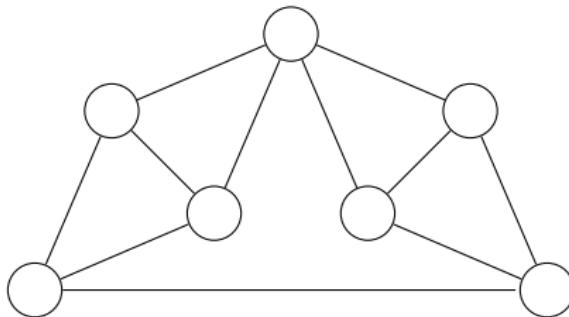
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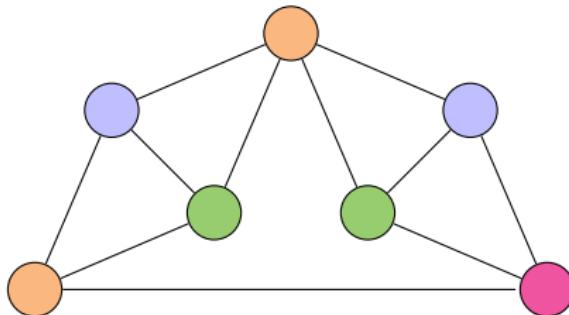
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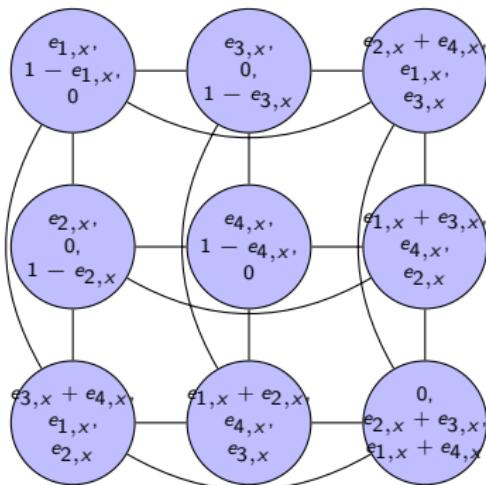
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 - If $1 \leq x, y \leq 6$ and a is a solution to equation x , b is a solution to equation y , and a, b are inconsistent, then $e_{a,x} \cdot e_{b,y} = 0$.

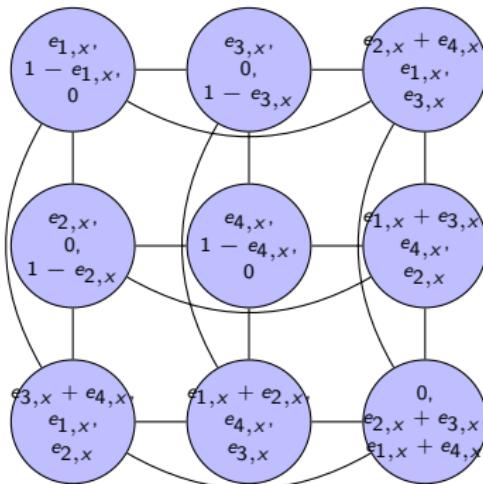
Building a Graph from Mermin-Peres

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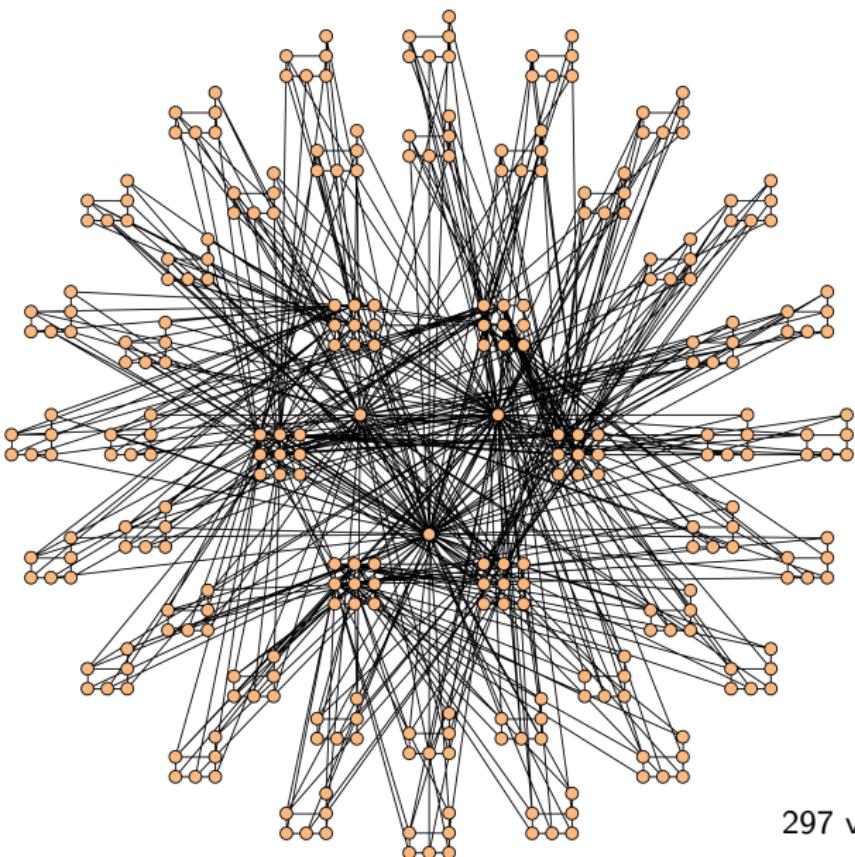
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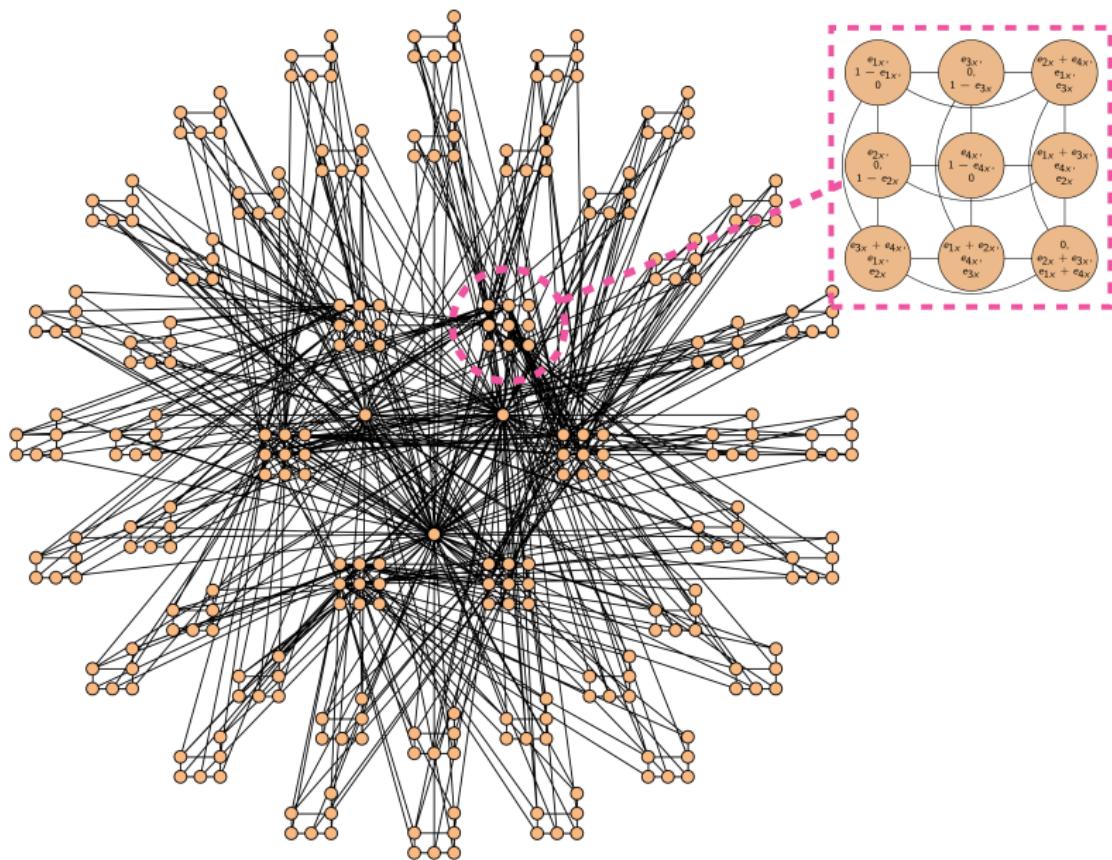


In quantum coloring, each vertex is colored with 3 projections that sum to 1 instead of a single color. These act on a shared entangled state, allowing Alice and Bob to coordinate their responses even when they are separated.

Mermin-Peres Graph



Mermin-Peres Graph



Thank you!

Questions?

Graph Coloring and Representations

Taylor Smith

Joint work with Dr. Samuel Harris and Calder Evans

Northern Arizona University
Department of Mathematics and Statistics

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Department of Mathematics and Statistics

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4. Share progress with whittling down our graph

Game Algebra

- We can represent the rules of the Mermin-Peres Magic Square Game using an algebra $\mathcal{A}(\mathcal{G})$. We use $a \in \mathcal{O}$ and $x \in \mathcal{I}$ to index our elements. $\mathcal{A}(\mathcal{G})$ has elements that satisfy the following.

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- For all $x \in \mathcal{I}$, we have $\sum_{a \in \mathcal{O}} e_{a,x} = 1$.

$$e_{1,2} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{-1}{2} \\ 0 & 0 & \frac{-1}{2} & \frac{1}{2} \end{bmatrix}$$

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- For $x, y \in \mathcal{I}$ and $a, b \in \mathcal{O}$ we have $\lambda(a, b, x, y) = 0$ implies $e_{a,x} e_{b,y} = 0$.
- For all $x \in \mathcal{I}$ and $a \in \mathcal{O}$ we have $e_{a,x}^2 = e_{a,x} = e_{a,x}^*$

$$e_{1,2} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Definition

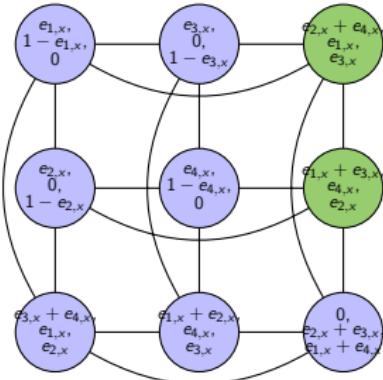
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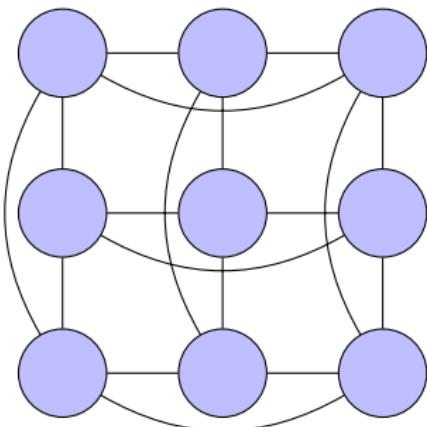
Example

$$\begin{aligned}
 & (e_{2,x} + e_{4,x}, e_{1,x}, e_{3,x}) \cdot (e_{1,x} + e_{3,x}, e_{4,x}, e_{2,x}) \\
 &= ((e_{2,x} + e_{4,x})(e_{3,x} + e_{1,x}), e_{1,x}e_{4,x}, e_{3,x}e_{2,x}) \\
 &= (0, 0, 0).
 \end{aligned}$$



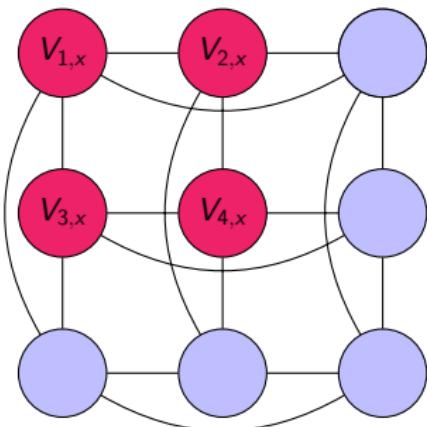
Definitions

- Gadgets are subgraphs that encode the game rules into our graph.



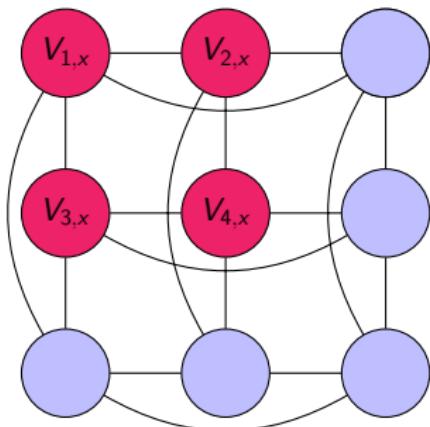
Definitions

- Gadgets are subgraphs that encode the game rules into our graph.
- The vertices in red we denote as $V_{a,x}$, with $a \in \mathcal{O}$ and $x \in \mathcal{I}$. These sets are the same as our input and output sets from the Mermin-Peres Magic Square Game.



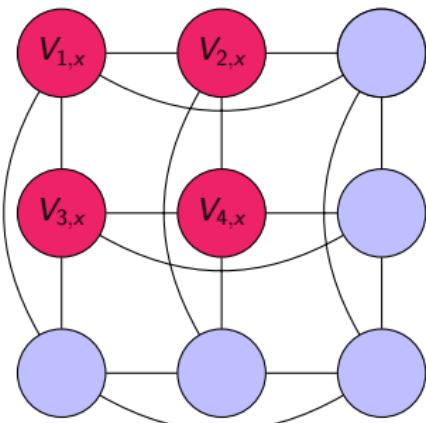
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- Other Gadgets such as E_λ and F_λ subgraphs enforce the graph to not be classically 3-colorable.



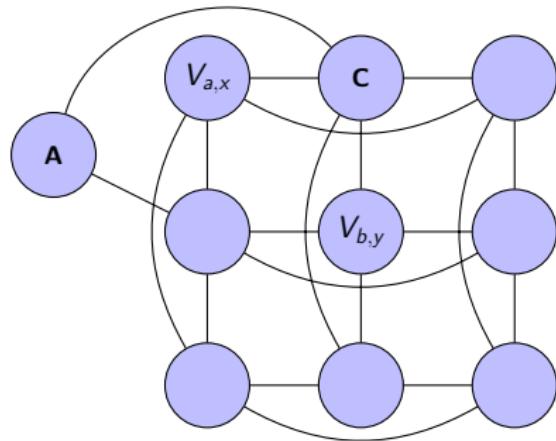
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- Other Gadgets such as E_λ and F_λ subgraphs enforce the graph to not be classically 3-colorable.
- We call $V_{a,x}$ and $V_{b,y}$ incompatible whenever $\lambda(a, b, x, y) = 0$.

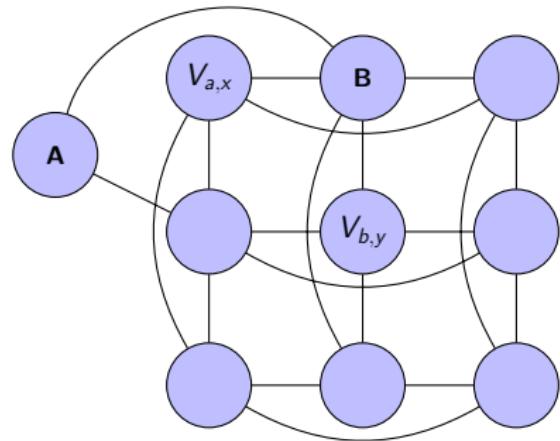


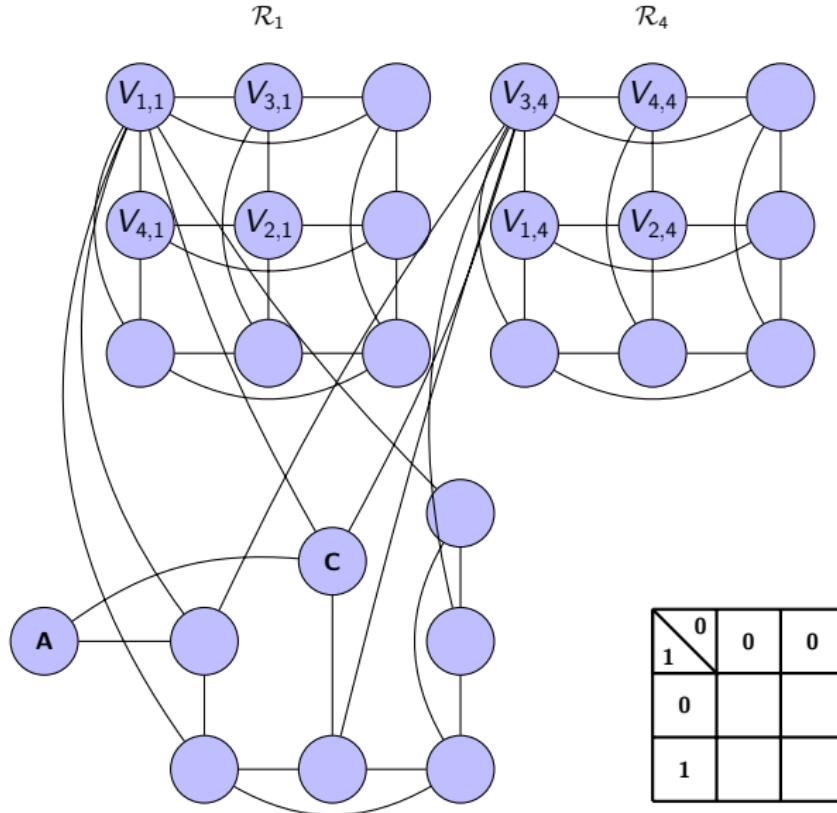
For vertices $V_{a,x}$ and $V_{b,y}$ who are incompatible, $\lambda(a, b, x, y) = 0$, and are not orthogonal to one another we create one of the subgraphs,

if $(a, b, x, y) \in \mathcal{E}_\lambda$

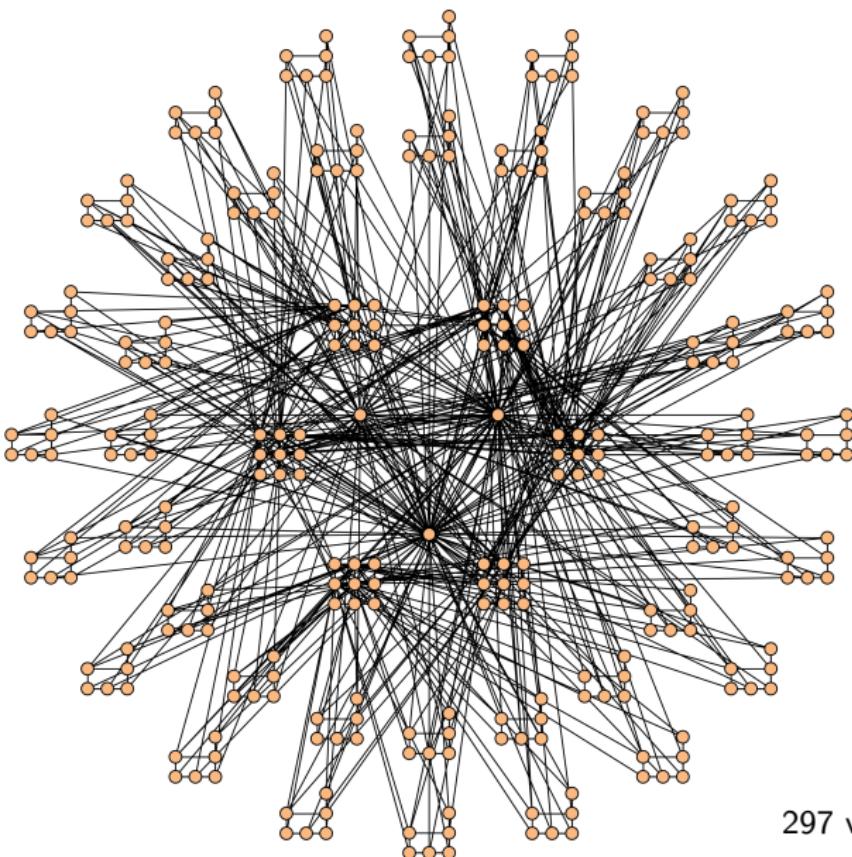


if $(a, b, x, y) \in \mathcal{F}_\lambda$

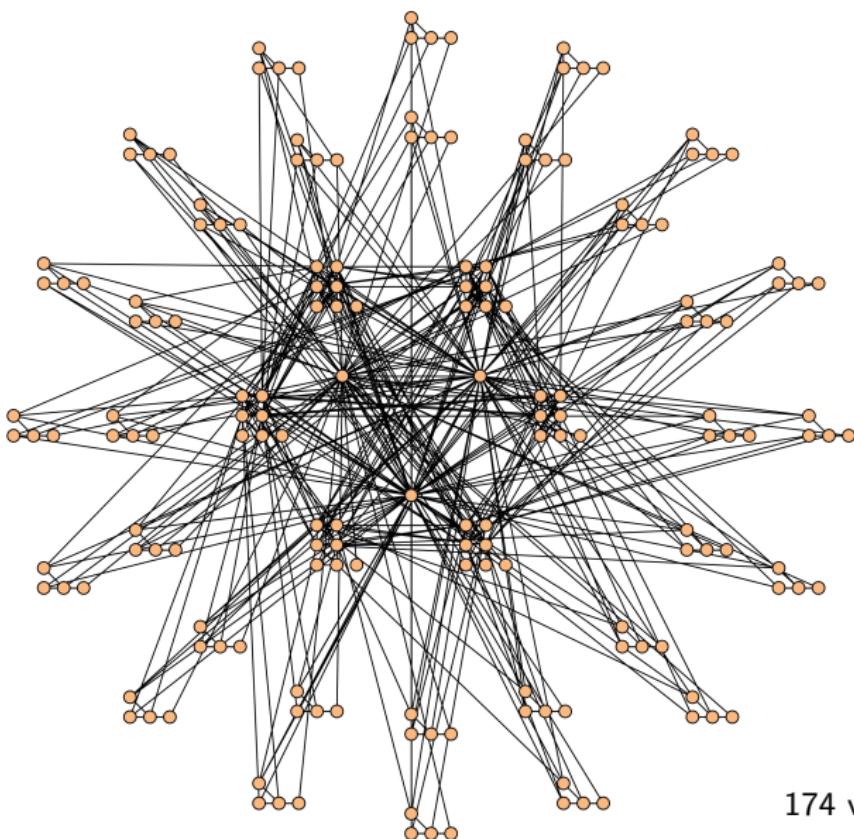




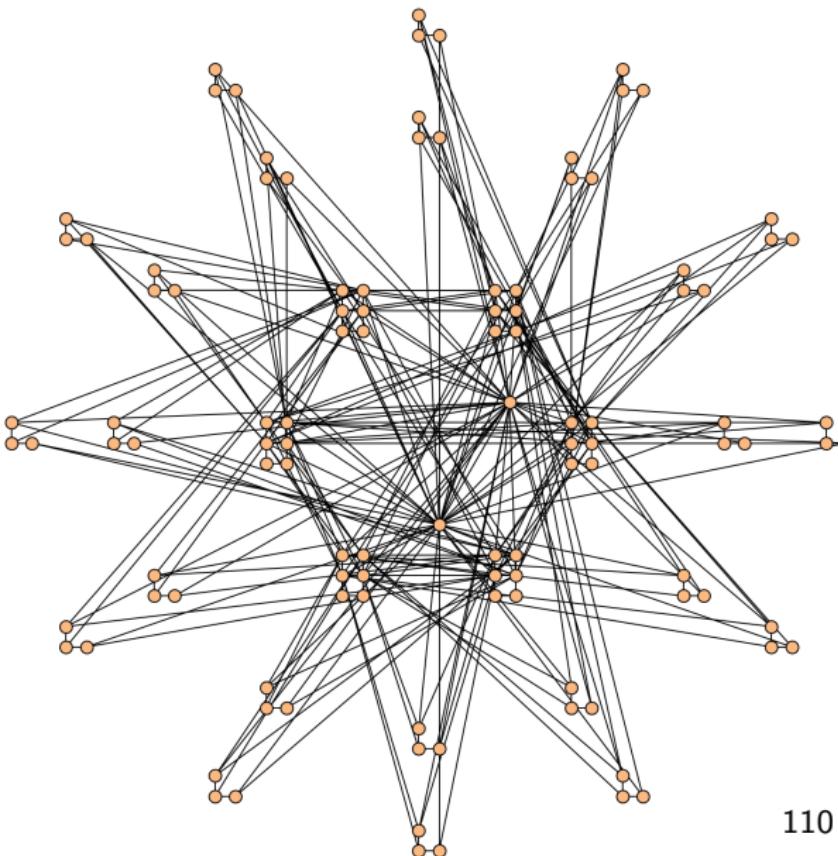
Original Graph



Reduced Graph



Final Reduced Graph



Thank you!

Questions?