CS2209-Assignment 3

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1. Let X be a set with $n \geq 1$ elements. For $0 \leq k \leq n$, let f(k, n) denote the number of subsets of X with k elements.

(i.) For $1 \leq k < n$, prove that $f(k, n) = f(k - 1, n - 1) + f(k, n - 1)$

Case 1:

N belongs to the constructing set of the x elements.

Then we need to find the subsets with k-1 elements from n-1 elements.

The resulting total number of subsets that hold k elements in which n is not in will be f(k-1 ,n-1)

Case 2:

N does not belong to the constructing set

Then we need to find the subsets with k elements from n-1 elements

The resulting total number of subsets that hold k elements in which n is not in f(k, n-1)

Therefore the 2 cases are mutually exclusive proving $f(k, n) = f(k - 1, n - 1) + f(k, n - 1)$

(ii.) Let Pn(x) = Pn k=0 f(k, n)x k . Using (i.)

Prove that (1 + x)Pn(x) = Pn+1(x)

$Pn(n)=f(0,n)+f(1,n)n+\ldots\ldots\ldots\ldots.f(n,n)n^n$

Let (1+n)Pn(n) = f(0,n)+(f(0,n)+f(1,n))n+……….+(f(n-1,n)+f(n,n))n^n +f(n,n)n^n

Since f(0,n) = f(0,n-1) and f(n,n)=f(n+1,n+1) and using part i

(1+n) Pn() = f(0,n+1)+……..+f(n+1,n+1)n^n+1

$=\sum^{N+1}_{k=0}$ f(k,n+1)n^k = Pn+1(n)

(iii.) Use induction to conclude that Pn(x) = (1 + x) n for all $n \geq 1$

When n=1 the statement is true

F(0,1)=1 – number of subsets is 1 with the element 0

F(1,1)=1 – number of subsets is 1 with the element 1

Therefore f1(n)=1+x = (1+x)^1

Let n=m

Pm(n)=(1+)^m

(1+n) Pm(n) = (1+n)^m+1

Pm+1(n) = (1+n)^m+1

Therefore true for n=m+1

Therefore through mathematical induction it is true for all n≥1

(iv.) Using (i.) and induction

Prove that f(k, n) = n!/ k!(n − k)!

True for n=1

F(0,1) = 1!/1!(1-0)! = 1 therefore true

F(1,1)= 1!/1!(1-1)! = 1 therefore true

Let n=m

Therefore f(k,m) = m! / k!(m-k)! proven by 1

Inductive step

F(k,m+1_ = f(k-1,m) + f(k,m) using i

$$= \frac{m!}{(k-1)!(m-k+1)!} + \frac{m!}{k!(m-k)!}$$ By 1

$$= \frac{m!}{(k-1)!\,(m-k)!} * (\frac{1}{m-k+1} + \frac{1}{k})$$

$$= \frac{m!}{(k-1)!\,(m-k)!} * \frac{k+m-k+1}{(m-k+1)k}$$

$$= \frac{(m+1)!}{k!\,(m-k+1)!}$$

Therefore true for n=m+1

Therefore through mathematical induction true for all n≥1

(v.) Conclude that (1 + x)^n = $\sum^{n}_{k=0}$ (n! /k!(n−k)!)x^k

Using iii we have

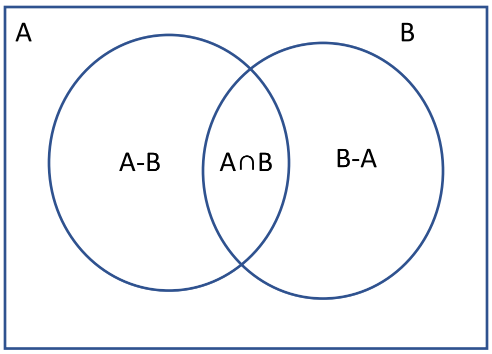
(1+n)^n = Pn(n)

$=\sum_{k=0}^{n} f(k,n)n^k$

Using iv we prove

$=\sum_{k=0}^{n} (n!/(k!(n-k)!))n^k$

2. Let A and B be two finite sets. Prove that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



$n(A \cup B) = n(A) + n(B) + n(A \cap B)$

$=n(A)-n(A\cap B) +n(A\cap B)+n(B)-n(A\cap B)$

$= n(A)+ n(B)-n(A\cap B)$

3. (15 points) Define the sequence $\{hn\}n\in N$ as follows:

$h0 = 5/3$

$h1 = 11/3$

$hn = 3hn-1 + 4hn-2 + 6n$ for $n \geq 2$

Prove that for $n \geq 2$ $hn = 2(4)^n + 3/2 (-1)^n - n - 11/6$

Prove base case n=2

$H2 = 3(h1)+4h0+6(2)$

$= 3(11/3) +4(5/3)+12$

$=11 +(20/3)+ 12$

$=(33+20+36)/3$

$=89/3$

For other

$H2 = 2(4)^2+(3/2)-1^2-2-(11/6)$

$=2*16 +(3/2)-2-(11/6)$

$=32+(3/2)-2-(11/6)$

$=(192+9-12-11)/6$

$=178/6$

$=89/3$

Therefore it is true for the base case

Inductive step

Prove n=k+1

=3hk+4h(k-1)+6(k+1)

=3hk+4h(k-1)+6k+6

Since hk = 2(4)^(k) +((3/2)-1^(k))-k-(11/6)

F(h,k-1)= 2(4)^(k-1) +((3/2)-1^(k-1))-(k-1)-(11/6)

Therefore

3hk+4h(k-1)+6k+6

= 3(2(4)^(k)+(3/2)-1^(k)-k-(11/6))

+4(2(4)^(k-1)+(3/2)-1^(k-1)-(k-1)-(11/6))

+6k+6

=6(4)^(k)+3(3/2)-1^(k)-3k-3(11/6) + ((8^2(4)^(k))/4) +4(3/2)-1^(k-1)-4k+4-4(11/6)+6k6

= 12(4)^(k)+(3-4)(3/2)-1^(k)-k+10-7(11/6)

=3(h)^(k+1)+(3/2)-1^(k+1)-(k+1)+11-7(11/6)

=3(h)^(k+1)+(3/2)-1^(k+1) – (k+1) +(11/6)

Therefore true because n=k+1 and by mathematical induction it is true for n≥2

2 hn = 2(4)^n + 3/2 (−1)^n − n – 11/6

4. Prove that any non-zero Boolean function can be written as a sum of minterms.

X0	X1	……………………..	Xn	F(x0,x1,………,xn-1)
0	0		0	M1
0	0		1	M2
0	0		0	M3
0	0		1	M4
0	0		0	M5
0	0		1	M6

F(x0,x1,…….xn)=c1+c2+…….+ck = x0,x1,x2,…….,xn

A Boolean function of n variables can be written as

F(x1,x2,x2……,xn)

Hence by using Boolean laws and theory

Sum of sum

Product of sum

Canonical form

There are two types of canonical form

Sum of minterms

Product of maxterms

A min term is defined as the product of literals u1,u2,…uk such that each uj is either vj or comp vj

A variable is in complement form if its value is assigned to 0

A variable is not in complement form if its value is assigned to 1

Ex) xy can be x'y', x'y, xy', x,y

0=minterm for which f=0

1= minterm for which f=1

Any Boolean function can be expressed as the sum (0,0) of its 1-minterms

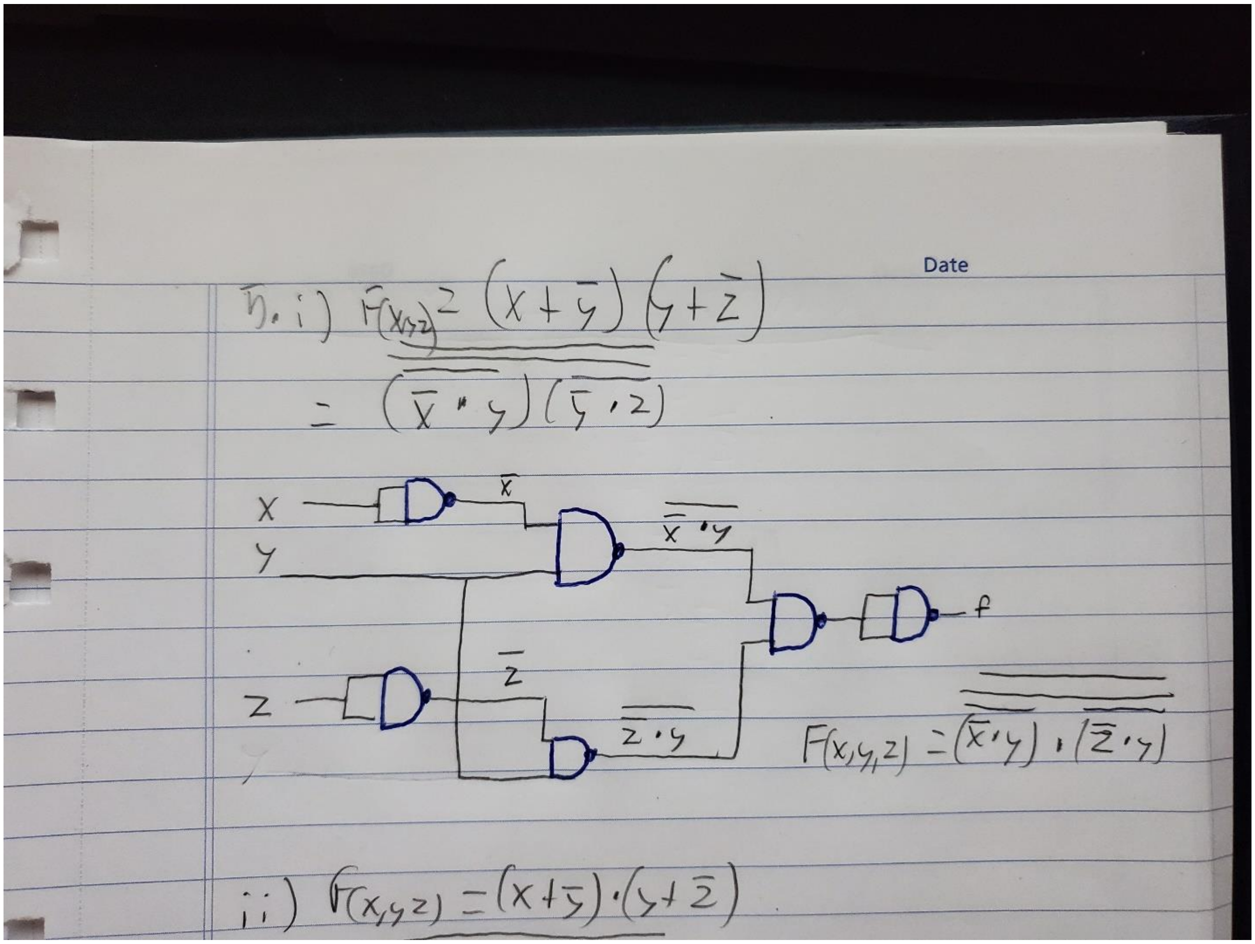
F(variables) = ∑ 1-miterm index

Ex

X	Y	Z	Minterms	
0	0	0	1	x'y'z'
0	0	1	0	X'y'z
0	1	0	0	X'yz'
0	1	1	1	X'yz
1	0	0	1	Xy'z'
1	0	1	0	Xy'z
1	1	0	1	Xyz'
1	1	1	0	xyz

5. Construct a Boolean circuit for the Boolean function $f(x, y, z) = (x + \bar{y})(y + \bar{z})$

i. ONLY using NAND gates.

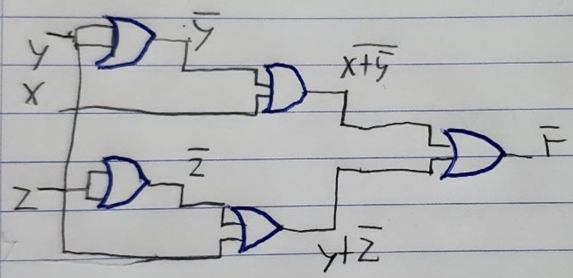


ii. ONLY using NOR gates

$$F(x,y,z) = \overline{\overline{(\bar{x}\cdot y)}\cdot\overline{(\bar{z}\cdot y)}}$$

ii) $F(x,y,z) = (x+\bar{y})\cdot(y+\bar{z})$

$$= \overline{\overline{(x+\bar{y})}+\overline{(y+\bar{z})}}$$



$$F(x,y,z) = \overline{\overline{(x+\bar{y})}+\overline{(y+\bar{z})}}$$

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