CS 2209A
Assignment-3
Due on December 1st, 11:55PM

November 28, 2019

Instructions: Upload a single pdf to OWL. Corrupted files are student's responsibility.

1. (25 points) Let X be a set with $n \geq 1$ elements. For $0 \leq k \leq n$, let $f(k, n)$ denote the number of subsets of X with k elements.

   (i.) For $1 \leq k < n$, prove that
   $$f(k, n) = f(k-1, n-1) + f(k, n-1)$$

   (ii.) Let $P_n(x) = \sum_{k=0}^{n} f(k, n) x^k$. Using (i.) prove that
   $$(1 + x)P_n(x) = P_{n+1}(x)$$

   (iii.) Use induction to conclude that
   $$P_n(x) = (1 + x)^n$$
   for all $n \geq 1$

   (iv.) Using (i.) and induction prove that
   $$f(k, n) = \frac{n!}{k!(n-k)!}$$

   (v.) Conclude that $(1 + x)^n = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} x^k$

2. (10 points) Let A and B be two finite sets. Prove that
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

3. (15 points) Define the sequence $\{h_n\}_{n \in \mathbb{N}}$ as follows:

   $h_0 = 5/3$
   $h_1 = 11/3$
   $h_n = 3h_{n-1} + 4h_{n-2} + 6n$ for $n \geq 2$

   Prove that for $n \geq 2$
   $$h_n = 2(4)^n + \frac{3}{2}(-1)^n - n - \frac{11}{6}$$

4. (25 points) Prove that any non-zero Boolean function can be written as a sum of minterms.

   Hint: See section 6.2 for definitions. Draw a 'truth table' with n variables and identify the 'truth table' with $n-1$ variables. Use induction.

5. (25 points) Construct a Boolean circuit for the Boolean function $f(x, y, z) = (x + \bar{y})(y + \bar{z})$

   i. ONLY using NAND gates.

   ii. ONLY using NOR gates.

6 (Bonus question: will not be graded but useful to solve it) Prove that every formula in predicate logic can be expressed in prenex normal form.

   Hint: Structural induction.