CS-2209A

Assignment 2

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1. $P(x)$ = pens
   $Q(x)$ = pencils
   a. $\forall x\ (P(x) \lor Q(x))$
   b. $\exists x\ (\neg(P(x)) \to (Q(x)$
   c. $\exists x\ (\neg P(x) \land \neg Q(x))$
   d. $\neg \forall x(P(x) \land Q(x))$

2.
   a.

1	$\forall z((\exists yP(x,y) \to \forall xQ(x)) \to R(z))$	Question
2	$\forall z(\neg(\exists yP(x,y) \to \forall xQ(x)) \lor R(z))$	Conditional Identity 1
3	$\forall z(\neg(\neg\exists yP(x,y) \lor \forall xQ(x)) \lor R(z))$	Conditional Identity 2
4	$\forall z((\neg\neg\exists yP(x,y) \land \neg\forall xQ(x)) \lor R(z))$	DeMorgans Law 3
5	$\forall z((\exists yP(x,y) \land \neg\forall xQ(x)) \lor R(z))$	Double Negation 4
6	$\forall z((\exists yP(x,y) \land \exists x\neg Q(x)) \lor R(z))$	DeMorgans Law 5
7	$\forall z\exists y\exists x((P(x,y) \land \neg Q(x)) \lor R(z))$	Shifting Quantifiers 6
8	$\forall z\exists y\exists x((R(z) \lor P(x,y)) \land (R(z) \lor \neg Q(x)))$	Distributive Law 7

   b.

1	$((A(y) \to \forall x_1B(x_1)) \to \exists x_2{:}C(x)$	Question
2	$\neg((A(y) \to \forall x_1B(x_1)) \lor \exists x_2\ C(x_2)$	Conditionally Identity 1
3	$\neg(\neg(A(y) \lor \forall x_1B(x_1)) \lor \exists x_2\ C(x_2)$	Conditional Identity 2
4	$(\neg\neg A(y) \land \neg\forall x_1B(x_1)) \lor \exists x_2\ C(x_2)$	DeMorgans Law 3
5	$(A(y) \land \neg\forall x_1B(x)) \lor \exists x_2\ C(x_2)$	Double Negation Law 4
6	$(A(y) \land \exists x_1\neg B(x_1)) \lor \exists x_2\ C(x_2)$	DeMorgans Law 5
7	$(\exists x_2C(x_2) \lor A(y)) \land (\exists x_2C(x_2) \lor \exists x_1\neg B(x_1))$	Distribution Law 6
8	$\exists x_2\exists x_1\ (C(x_2) \lor A(y)) \land (C(x_2) \lor \neg B(x_1))$	Shifting Quantifiers 7

   c.

1	$\exists x_1(P(x_1) \to \forall yQ(x_1,y)) \to \forall x_2B(x_2)$	Question
2	$\exists x_1\neg(P(x_1) \to \forall yQ(x_1,y)) \lor \forall x_2B(x_2)$	Conditional Identity 1
3	$\exists x_1\neg(\neg P(x_1) \lor \forall yQ(x_1,y)) \lor \forall x_2B(x_2)$	Conditional Identity 2
4	$\exists x_1\neg(\neg P(x_1) \lor \forall yQ(x_1,y)) \lor \forall x_2B(x_2)$	DeMorgans Law 3
5	$\exists x_1(\neg\neg P(x_1) \land \exists y\neg Q(x_1,y)) \lor \forall x_2B(x_2)$	DeMorgans Law 4
6	$\exists x_1(P(x_1) \land \neg\exists y\neg Q(x_1,y)) \lor \forall x_2B(x_2)$	Double Negation Law 5
7	$((\forall x_2B(x_2) \lor \exists x_1P(x_1)) \land ((\forall x_2B(x_2) \lor \exists y\neg Q(x_1,y))$	Distributive Law 6
8	$\forall x_2\exists x_1\exists y((B(x_2) \lor P(x_1)) \land ((B(x_2) \lor \neg Q(x_1,y))$	Shifting Quantifiers 7

3.

a.

1	∀x(P(x)→Q(x))	Hypothesis
2	∀x(Q(x)→R(x))	Hypothesis
3	∀x(P(x)→R(x))	Hypothetical Syllogism 1,2

b.

1	∃x(P(x)∧R(x))	Hypothesis
2	(c is a particular element) ∧(P(c)∧R(c))	Existential instantiation 1
3	(c is a particular element)	Simplification 2
4	(P(c)∧R(c))∧(c is a particular element)	Commutative law 2
5	P(c)∧R(c)	Simplification 4
6	∀x(P(x)→(Q(x)∧S(x))	Hypothesis
7	P(c)→(Q(c)∧S(c))	Universal instantiation 4,5
8	¬P(c)∨(Q(c)∧S(c))	Conditional Identity 7
9	¬P(c)∨(S(c)∧Q(c))	Commutative Law 8
10	¬P(c)∨S(c)	Simplification 9
11	S(c)∨¬P(c)	Commutative Law 10
12	(S(c)∨¬P(c))∧(P(c)∧R(c))	Conjunction 5,11
13	S(c)∧F∧R(c)	Complement Law 12
14	S(c)∧R(c)	Domination Law
15	R(c)∧S(c)	Commutative Law 14
16	(c is a particular element)∧(R(c)∧S(c))	Conjunction 3,15
17	∃x(R(x)∧S(c))	Existential Generalization 16

c.

1	∀x(P(x)∨Q(x))	Hypothesis
2	∀x(¬Q(x)∨S(x))	Hypothesis
3	∀x(R(x)→¬S(x))	Hypothesis
4	∃x(¬P(x))	Hypothesis
5	(c is a particular element)∧(¬P(c))	Existential Instantiation 4
6	(¬P(c))∧(c is a particular element)	Commutative Law 5
7	C is a particular element	Simplification 5
8	P(C)∨Q(c)	Universal Instantiation 1, 7
9	¬Q(c)∨S(c)	Universal Instantiation 2,7
10	R(c)→¬S(c)	Universal Instantiation 3,7
11	¬P(c)	Simplification 6
12	Q(c)∨P(c)	Commutative Law 8
13	P(c)∨S(c)	Resolution 9,12
14	S(c)	Disjunctive Syllogism 11,13
15	¬R(c)	Modus Tollens 10,14
16	(c is a particular element) ∧ ¬R(x)	Conjunction 7,16
17	∃x(¬R(x))	Existential Generalization 16

4.

a. $P(x)$: some pizza store open in London
$H(x)$: there is a party
s: there is a party at Sally's house
d: there is a party at Dan's house
$\exists x P(x) \rightarrow H(d)$
$\neg H(d) \wedge \neg H(s)$

$\forall x \neg P(x)$

1	$\exists x P(x) \rightarrow H(d)$	Hypothesis
2	$\neg H(d) \wedge \neg H(s)$	Hypothesis
3	$\neg H(d)$	Simplification 2
4	$\neg \exists x P(x)$	Modus Tollens 1,3
5	$\forall x \neg P(x)$	De Morgans Law 4

Therefore the statement is valid

b. P: Darryl is happy
Q: it is raining outside
R: pigs have wings
$p \vee q$
$\neg q \vee r$
R

$\neg p$
To prove validity, we check
$(p \vee q) \wedge (\neg q \vee r) \wedge (r) \rightarrow \neg p$: C

P	Q	R	$\neg p$	$\neg Q$	$p \vee q$	$\neg q \vee r$	$(p \vee q) \wedge (\neg q \vee r)$	$(p \vee q) \wedge (\neg q \vee r) \wedge (r)$	C
T	T	T	F	F	T	T	T	T	F
T	T	F	F	F	T	F	F	F	T
T	F	T	F	T	T	T	T	T	F
T	F	F	F	T	T	T	T	F	T
F	T	T	T	F	T	T	T	T	T
F	T	F	T	F	T	F	F	F	T
F	F	T	T	T	F	T	F	F	T
F	F	F	T	T	F	T	F	F	T

From the truth table we can see that the argument is not valid as not all truth values of C are true

5.

a. 2.5.8.
   i. $\forall x \forall y\ W(x) \wedge O(y) \rightarrow N(x,y)$
   ii. $\exists x \forall y\ O(x) \rightarrow \neg N(x,y)$
   iii. $\forall x \exists y\ N(x,y)$
   iv. $\exists x \forall y\ N(x,y)$
   v. $\forall x (\exists y\ N(x,y))$
   vi. $\exists x\ N(x,sam)$

b. 2.4.2

i. E) ∀x∃y Q(x,y) False, when x=1 there is no value of y that is true
ii. F) ∀x∃y P(x,y) True, for all x there is at least one y value that is true
iii. G) ∀x∀y P(x,y), false P(2,1), P(2,2),P(3,3) are all false
iv. I) ∀x∀y ¬S(x,y), true all values that exist are true

6.

a. Theorem: $\frac{1}{x}+\frac{1}{y}=1$, x,y are positive integers

Proof: $\frac{1}{y}=1-\frac{1}{x}$

$$\frac{1}{y}=\frac{x-1}{x}$$

x-1 and x are two consecutive positive integers
Let p divide x-1 and x where p>= 1 is a positive integer
Then x -1 = pt1
X=pt2 and t2>t1, such that t2 and t1 are positive integers
So x-(x-1)=pt2-pt1
1= p(t2-t1)
And p is positive in positive integers t2-t1>0 is a positive integer
So p=1, t2-t1=1
So if px and p(x-1) then p =1
So x, x-1 = 1
Hence 1/y and x-1/x are rational numbers with 1/y ae co-prime and x-1, x are coprime
So that 1/y=x-1/x
1= x-1, y=x
Y=x=2

b. Theorem: For any positive integer a, if a^3 is an even number then a is an even number.
Proof: If a is an odd number, then a^3 is odd
Then a = 2m+1 for some integer
And $a^3 = (2m+1)^3$
$= (2m)^3 + 1^3 + 3(2m)^3 +3*2m*1^2$
$= 8m^3+12m^2+6m +1$
$= 2(4m^3 +12m^2+3m)+1$
Let $n=(4m^3 +12m^2+3m)$
$= 2n+1$
So a^3 is an odd number hence if a is not even then a^3 is not even or if a^3 is even then a is an even number

c. Theorem: $\sqrt[3]{2}$ is not a rational number
Proof: $\sqrt[3]{2}$=a/b (we assume that a and b have no common factors and are therefore irreducible)
$2=a^3/b^3$
$2b^3=a^3$
Let b^3 =n
2n=always even

=>a^3 must be even

=a*a*a is even

= even*even*even => even

- ⇨ A is even =>a=2k

$2b^3=(2k)^3$

$2b^3=8k^3$

$B^3=4k^3$

- ⇨ B^3 must be even
    - $=4k^3$ must be even
    - B is even

If a and b are even then they both have common factors of 2 and therefore the original assumption of the a/b being irreducible is not true, this is a contradiction of the original statement therefore $\sqrt[3]{2}$ is not a rational number

d. Theorem: if (a, b >0) then a/b if there exists an integer k such that b=ka
If a>1 then a does not divide a^2+a+1
Proof:

a = a>1

$b=a/a^2+a+1$

$a \rightarrow b$

contrapositive $\neg b \rightarrow \neg a$

$(a/a^2+a+1) \rightarrow 1 \geq a$

$1 \geq a$

$Ka = a^2+a+1$ where k is an integer

Therefore, the argument is invalid and by contrapositive the original theorem is true