CS 2209 Assignment 1
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1.

a. prove that $p \rightarrow (q\rightarrow r)$ and $(p\rightarrow q)\rightarrow r$ is not equivalent

	$p \rightarrow (q\rightarrow r)$		$(p\rightarrow q)\rightarrow r$	
Cond Identity	$\neg p\vee(q\rightarrow r)$		$\neg(p\rightarrow q)\vee r$	Cond Identity
Cond Identity	$\neg p\vee(\neg q\vee r)$		$\neg(\neg p\vee q)\vee r$	Cond Identity
Associative	$(\neg p\vee\neg q)\vee r$		$\neg\neg p\wedge\neg q\vee r$	DeMorgans
DeMorgans	$\neg(p\wedge q)\vee r$		$p\wedge\neg q\vee r$	Double Negation

P	Q	R	$\neg p$	$\neg q$	$(p\vee q)$	$\neg(p\wedge q)$	$\neg(p\wedge q)\vee r$	$p\wedge\neg q$	$p\wedge\neg q\vee r$
T	T	T	F	F	T	F	T	F	T
T	T	F	F	F	T	F	F	F	F
T	F	T	F	T	T	F	T	T	T
T	F	F	F	T	T	F	F	t	T
F	T	T	T	F	T	F	T	F	T
F	T	F	T	F	T	F	F	F	F
F	F	T	T	T	F	T	T	F	T
f	f	f	t	t	f	t	t	f	f

The two columns are not the exact same and as such the statements are not equivalents
Not equivalent when $p=q=r=F$

2.

a. $(p\wedge q)\vee(\neg p\vee\neg q)$

P	Q	$\neg p$	$\neg q$	$(P\wedge q)$	$(\neg p\vee\neg q)$	$(p\wedge q)\vee(\neg p\vee\neg q)$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
f	f	t	t	f	t	T

The truth table shows it is always true therefore it is a tautology

b. $(p\rightarrow\neg q)\rightarrow(p\rightarrow q)$

P	Q	$\neg q$	$p\rightarrow\neg q$	$p\rightarrow q$	$(p\rightarrow\neg q)\rightarrow(p\rightarrow q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	T	T
f	f	t	t	t	T

The truth table shows that is it neither a tautology or a contradiction as there is both true and false values

c. $(p\rightarrow q)\rightarrow(\neg q\rightarrow\neg p)$

P	Q	$\neg p$	$\neg q$	$p\rightarrow q$	$\neg q\rightarrow\neg p$	$(p\rightarrow q)\rightarrow(\neg q\rightarrow\neg p)$
T	T	F	F	T	T	T
T	F	F	T	F	f	T
F	T	T	F	T	T	T
f	f	t	t	t	t	T

The truth table shows it is a tautology as the expression is true for all values of p and q

d. $(p\rightarrow q)\wedge(q\rightarrow r)\rightarrow(p\rightarrow r)$

P	Q	R	$p\rightarrow q$	$q\rightarrow r$	$(p\rightarrow q)\wedge(q\rightarrow r)$	$p\rightarrow r$	$(p\rightarrow q)\wedge(q\rightarrow r)\rightarrow(p\rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
f	f	f	t	t	t	t	T

This is a tautology as the expression evaluated to true for all values of p, q and r

e. $(p\rightarrow q)\wedge(p\wedge\neg q)$

P	Q	$p\rightarrow q$	$\neg q$	$p\wedge\neg q$	$(p\rightarrow q)\wedge(p\wedge\neg q)$
T	T	T	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
f	f	t	t	f	F

This is a contradiction as the truth table shows that for all values of p and q the expression is false

3.

a. $(p\vee p)\wedge p\equiv p$

1.	$(p\vee p)\wedge p$	
2.	$p\wedge p$	Idempotent Law 1
3.	p	Idempotent Law 2

b. $(p\rightarrow(q\vee r))\equiv p\wedge\neg q\rightarrow r$

1.	$p\wedge\neg q\rightarrow r$	
2.	$\neg(p\wedge\neg q)\vee r$	Conditional Identity 1
3.	$\neg p\vee\neg\neg q\vee r$	DeMorgans Law 2
4.	$\neg p\vee q\vee r$	Double Negation Law 3
5.	$p\rightarrow(q\vee r)$	Conditional Identity 4

c. $p \equiv \neg p \rightarrow (p \wedge q)$

1.	$\neg p \rightarrow (p \wedge q)$	
2.	$\neg\neg p \vee (p \wedge q)$	Conditional Identity 1
3.	$p \vee (p \wedge q)$	Double Negation law 2
4.	P	Absorption Law 3

d. $(\neg p \wedge \neg q) \rightarrow r \equiv \neg p \rightarrow (q \vee r)$

1.	$(\neg p \wedge \neg q) \rightarrow r$	
2.	$\neg(\neg p \wedge \neg q) \vee r$	Conditional Identity 1
3.	$\neg\neg p \vee \neg\neg q \vee r$	DeMorgans Law 2
4.	$\neg\neg p \vee q \vee r$	Double Negation Law 3
5.	$\neg p \rightarrow (q \vee r)$	Conditional Identity 4

4.

a.

1	$p \rightarrow (q \wedge r)$	hypothesis
2	$\neg p \vee (q \wedge r)$	Conditional Identity 1
3	$(\neg p \vee q) \wedge (\neg p \vee r)$	Distributive law 2
4	$(p \rightarrow q) \wedge (p \rightarrow r)$	Conditional Identity 3
5	$p \rightarrow r$	Hypothetical Syllogism 4
6	$\neg r$	Hypothesis
7	$\neg p$	Modus Tollens 5,6

b.

1	$(\neg p) \wedge q \rightarrow \neg r$	Hypothesis
2	$\neg(\neg p \wedge q) \vee \neg r$	Conditional Identity 1
3	$(\neg\neg p \vee \neg q) \vee \neg r$	DeMorgans Law 2
4	$(p \vee \neg q) \vee \neg r$	Double Negation Law 3
5	$\neg r \vee (p \vee \neg q)$	Associative law 4
6	r	Hypothesis
7	$\neg\neg r$	Double Negation Law 6
8	$(p \vee \neg q)$	Disjunctive Syllogism 5, 7
9	$\neg q \vee p$	Associative Law, 8
10	q	Hypothesis
11	$\neg\neg q$	Double Negation 10
12	p	Disjunctive Syllogism 9, 11

c.

1	$p \vee q$	Hypothesis
2	$\neg p \vee r$	Hypothesis

3	$q\vee r$	Resolution 1,2
4	$r\vee q$	Commutative Law 3
5	$\neg q$	Hypothesis
6	r	Disjunctive Syllogism 3,4
7	$\neg r\vee s$	Hypothesis
8	S	Disjunctive Syllogism 5,6

5.

a. $p\oplus q \equiv q\oplus p$

1	$p\oplus q$	
2	$(p\vee q)\wedge\neg(p\wedge q)$	Definition 1
3	$(q\vee p)\wedge\neg(p\wedge q)$	Commutative Law 2
4	$(q\vee p)\wedge\neg(q\wedge p)$	Commutative Law 3
5	$q\oplus p$	Definition 4

b. $p\oplus p$ is a contradiction

1	$p\oplus p$	
2	$(p\vee p)\wedge\neg(p\wedge p)$	Definition 1
3	$p\wedge\neg(p\wedge p)$	Idempotent Law 2
4	$p\wedge\neg p$	Idempotent Law 3
5	False	Complement Law 4

It will always evaluate to false as the expression simplifies to false

c. $r\wedge(p\oplus q) \equiv (r\wedge p)\oplus(r\wedge q)$

1	$(r\wedge p)\oplus(r\wedge q)$	
2	$(r\wedge p)\vee(r\wedge q))\wedge\neg((r\wedge p)\wedge(r\wedge q))$	Definition 1
3	$(r\wedge(p\vee q))\wedge \neg((r\wedge p)\wedge(r\wedge q))$	Distributive Law 2
4	$(r\wedge(p\vee q))\wedge(\neg(r\wedge p)\vee\neg(r\wedge q))$	DeMorgans Law 3
5	$(r\wedge(p\vee q))\wedge((\neg r\vee\neg p)\vee\neg(r\wedge q))$	DeMorgans Law 4
6	$(r\wedge(p\vee q))\wedge((\neg r\vee\neg p)\vee(\neg r\vee\neg q))$	DeMorgans Law 5
7	$(r\wedge(p\vee q))\wedge(\neg r\vee\neg r\vee\neg p\vee\neg q)$	Associative Law 6
8	$(r\wedge(p\vee q))\wedge(\neg(r\wedge r)\vee\neg p\vee\neg q)$	DeMorgans Law 7
9	$(r\wedge(p\vee q))\wedge(\neg r\vee(\neg p\vee\neg q))$	Idempotent Law 8
10	$(r\wedge(p\vee q)\wedge\neg r)\wedge(r\wedge(p\vee q)\wedge\neg p)\vee(r\wedge(p\vee q)\wedge\neg q)$	Distributive Law 9
11	$(r\wedge\neg r\wedge(p\vee q))\wedge(r\wedge(p\vee q)\wedge\neg p)\vee(r\wedge(p\vee q)\wedge\neg q)$	Associative Law 10
12	$(F\wedge(p\vee q))\vee((r\wedge(p\vee q)\wedge\neg p)\vee(r\wedge(p\vee q)\wedge q))$	Negation Law 11
13	$F\vee((r\wedge(p\vee q)\wedge\neg q)\vee(r\wedge(p\vee q)\wedge\neg q))$	Domination Law 12
14	$(r\wedge(p\vee q))\wedge(\neg p\vee\neg q)$	Distributive Law 13
15	$r\wedge(p\vee q)\wedge\neg(p\wedge q)$	DeMorgans Law 14
16	$r\wedge(p\oplus q)$	Definition

6. p: The student got an A on the final.
   q: The student turned in all the homework.
   r: The student is on academic probation
   a. The student is not on academic probation and the student got an A on the final or turned in all the homework.

      $\equiv \neg r \wedge (p \vee q)$
   b. If the student got an A on the final, then the student is not on academic probation

      $\equiv p \rightarrow \neg r$
   c. If the student is on academic probation, then the student did not get an A on the final

      $\equiv r \rightarrow \neg p$