

CS-3305A
Assignment 3
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1. Hash table of size $N=7$, $h(k)=k \bmod 7$, elements $=\{19,27,12,47,15\}$ using separate chaining to handle collisions

0	
1	
2	
3	
4	
5	19
6	

Insertion of 19
 $H(19)=19\%7=5$

0	
1	
2	
3	
4	
5	19
6	27

Insertion of 27
 $H(27)=27\%7=6$

0		
1		
2		
3		
4		
5	19	12
6	27	

Insertion of 12
 $H(12)=12\%7=5$

0			
1			
2			
3			
4			
5	19	12	47
6	27		

Insertion of 47
 $H(47)=47\%7=5$

0			
1	15		
2			
3			
4			
5	19	12	47
6	27		

Insertion of 15
 $H(15) = 15\%7=1$

2. Hash table of size $N=7$, $h(k)=k \bmod 7$, elements $=\{19,27,12,47,15\}$ using linear probing to handle collisions

0	
1	
2	
3	
4	
5	19
6	
Insertion of 19 $H(19)=19\%7=5$	

0	
1	
2	
3	
4	
5	19
6	27
Insertion of 27 $H(27)=27\%7=6$	

0	12
1	
2	
3	
4	
5	19
6	27
Insertion of 12 $H(12)=12\%7=5$ Collision $H(12)+1=6$ Collision $H(12)+2=0$	

0	12
1	47
2	
3	
4	
5	19
6	27
Insertion of 47 $H(47)=47\%7=5$ Collision $H(47)+1=6$ Collision $H(47)+2=0$ Collision $H(47)+3=1$	

0	12
1	47
2	15
3	
4	
5	19
6	27
Insertion of 15 $H(15)=15\%7=1$ Collision $H(15)+1=2$	

3. Hash table of size $N=7$, $h(k)=k \bmod 7$, elements $=\{19,27,12,47,15\}$ using double hashing to handle collisions

0	
1	
2	
3	
4	
5	19
6	
Insertion of 19 $H(19)=19\%7=5$	

0	
1	
2	
3	
4	
5	19
6	27
Insertion of 27 $H(27)=27\%7=6$	

0	
1	12
2	
3	
4	
5	19
6	27
Insertion of 12 $H(12)=12\%7=5$ Collision $H(12)=5-(12\%5)=3$ $H(12)=1$	

0	
1	12
2	
3	
4	47
5	19
6	27
Insertion of 47 $H(47)=47\%7=5$ Collision $H(47)=5-(47\%5)=3$ $H(47)=1$ Collision $H(47)+3=4$	

0	
1	12
2	15
3	
4	47
5	19
6	27
Insertion of 15 $H(15)=15\%7=1$ Collision $H(15)=5-(15\%5)=5$ $H(15)=6$ Collision $H(15)+5=4$ Collision $H(15)+5=2$	

4. $F(1)=3$
 $F(n)=F(n-1)+2n+1$

Build

$$F(n)=f(n-1)+2n+1$$

$$F(n-1)=((f(n-2)+2(n-1)+1)+2n+1 = f(n-2)+2n-1+2n+1 = f(n-2)+4n$$

$$F(n-2)=(f(n-3)+2(n-2)+1)+4n = f(n-3)+2n-3 +4n = f(n-3)+6n-3$$

$$F(n-3) = (f(n-4) + 2(n-3) + 1) + 6n-3 = f(n-4)+2n-5+6n-3 = f(n-4)+8n-8$$

$$F(n-4) = (f(n-5) + 2(n-4)+1) + 8n-8 = f(n-5)+2n-8+1+8n-8 = f(n-5)+10n-15$$

Expand

$$F(n-1) = f((n-1)-1)+2(n-1) +1$$

$$F(n-2) = f((n-3)+2(n-2)+1$$

$$F(n-3) = f(n-4)+2(n-3)+1$$

$$F(n-4) = f(n-5)+2(n-4)+1$$

$$\begin{aligned} F(n) &= f(n-10+2n+1 \\ &= f(n-2)+4n \\ &= f(n-3)+6n-3 \\ &= f(n-4)+8n-8 \\ &= F(n-5)+10n-15 \\ &= F(n-i)+i(2n)+(c) \end{aligned}$$

$$X_n = an^2 + bn + c$$

$$X_1 = a + b + c = 1$$

$$X_2 = 4a + 2b + c = 0$$

$$X_3 = 9a + 3b + c$$

$$X_2 - X_1 = 3a + b \quad 0 - 1 = 3a + b$$

$$X_3 - X_2 = 5a + b \quad 0 - 3 = 5a + b$$

$$-3 + 1 = 5a + b - (3a + b)$$

$$-2 = 2a$$

$$A = -1$$

$$-1 = 3(-1) + b$$

$$B = 2$$

$$A + b + c = 1$$

$$-1 + 2 + c = 1$$

$$C = 0$$

$$T_i = i^2 + 2i$$

$$\text{Therefore } f(n) = f(n-1) + i(2n) + (-1^2 + 2i)$$

$$\text{Since } f(1) = 3$$

$$n - i = 1$$

$$i = n - 1$$

$$\begin{aligned}
& f(1) + (n-1)(2n) + (-(n-1)^2 + 2(n-1)) \\
&= 3 + 2n^2 - 2n + (-(n^2 - 2n + 1) + 2n - 2) \\
&= 3 + 2n^2 - 2n - n^2 + 2n - 1 + 2n - 2 \\
&= 2n^2 - n^2 - 2n + 2n + 2n + 3 - 2 - 1 \\
&= n^2 - 2n
\end{aligned}$$

Therefore, the time complexity is $O(n^2)$

5.

a. **Algorithm** isSymmetric(r)

Input: root r of the tree

Output: true if the tree is symmetric false if it is not

if r = null

 return true

else

 x is child of r if r has child

 for each child c of r do

 if c.value != x.value

 return false

 sym <- true

 for each child c of r do

 sym <- sym and isSymmetric(c)

return sym

b. Worst case is when the parent of the leaf is not symmetric

Let x be the max number of children for a parent in the provided tree:

of iterations = degree(r)

1.

C1 operation in base case; $C1 + C2 + C3 \text{degree}(r)$ in recursive case

2.

One call is performed per node

3.

$\sum(\text{leaves}) C1 + (C2 + C3x\text{Degree}(r))$

$= C1x\#\text{internal} + C2x\#\text{internal} + C3 \sum \text{degree}(r)$

$= \#\text{leaves}(n) + \#\text{internal}$ is $O(n)$

$O(n)$