Asn 4 X maybe

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1. X finite set of card n and $0 \leq r \leq n$

$A =$ set of all subsets of X of size r
$B =$ Set of all strings of length n with exactly r 1's

Show A and B have a bijection
let us define $f: A \to B$

Suppose $X = \{a_1, a_2, \cdots a_{n-1}, a_n\}$
then if $\{a_1, a_2 \cdots a_{n-1} a_n\} \in A$
then $f(\{a_1, a_2, \cdots a_{n-1}, a_n\}) = \underbrace{11 \cdots 1}_{r\text{ times}} \overbrace{0 \cdots 00}^{n-r\text{ times}}$

to prove that it is a bijection we must find an inverse of f

define $h: B \to A$ such that

if $1101 \cdots 1000$, where ther are n-1's and $n-r$ zeros

then $h(1101 \cdots 10 \cdots 0) = \{a_1\ a_2\ a_3 \cdots a_{n+1}\}$

by this h is the inverse of f
as for any $x \in f$ $h \circ f = f(x)$

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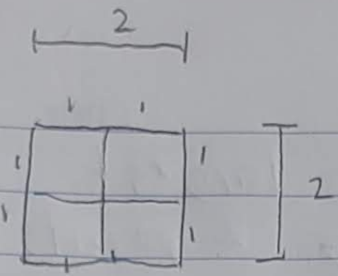
2. 

divide the square with side length of 2 into 4 smaller squares which have length 1

4 squares and 5 points to be placed

by pigeon hole principle when $n+1$ pigeons put in n holes then at least 1 hole contains 2 pigeons

here the pigeons are the points p_1, p_2, p_3, p_4, p_5 Pigeons = points

and holes are the 4 smaller squares

$n+1 = 5$ pigeons put in $n = 4$ holes at least 1 square and square must contain at least 2 points

P_i, P_j for some $1 \leq i,j \leq 5$

then the length of diagonal of the small square is

$= \sqrt{1^2 + 1^2}$

$= \sqrt{2}$

with $\sqrt{2}$ being the largest distance between any 2 points in the square

So since P_i, P_j are in the same small square we say

that they are at most at a distance of $\sqrt{2}$ from each other ∎

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3. Find the num of string with Length=7 with at most 3, 0's

First: Find no zeros

$^7C_0 = 1$

Second: 1 zero

$^7C_1 = 7$

third: 2 zeros

$^7C_2 = (7\times6)/(2\times1) = 42/2 = 21$

Fourth: 3 zeros

$^7C_3 = (7\times6\times5)/(3\times2\times1) = 210/6 = 35$

$\therefore$ # of strings with at most 3 zeros is:

$\boxed{1 + 7 + 21 + 35 = 64}$

4. Find the coeffecients of

x^{99} in $\left(x + \frac{1}{x^2}\right)^{100}$

By binomial thm

$$(a+b)^n = \sum_{j=0}^{n} \binom{n}{j} a^{n-j} \cdot b^j$$

So we can rewrite as

$$= \sum_{j=0}^{100} \binom{100}{j} x^{100-j} \left({}^{1}/_{x^2}\right)^j$$

$$= \sum_{j=0}^{100} \binom{100}{j} x^{100-j} \left({}^{1}/_{x^{2j}}\right)$$

$$= \sum_{j=0}^{100} \binom{100}{j} x^{100-j} \; x^{-2j}$$

$$= \sum_{j=0}^{100} \binom{100}{j} x^{100-j-2j}$$

$$= \sum_{j=0}^{100} \binom{100}{j} x^{100-3j}$$

$\therefore x^{94} = x^{100-3j}$ iff $j = 2$

$= \binom{100}{2} x^{94}$

$\therefore$ coeffecient is $\binom{100}{2}$ but more properly as

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$$\binom{100}{2} = \frac{100!}{(100-2)!\,2!} \quad \text{since} \quad \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

$$= \frac{100!}{98!\,2!}$$

$$= \frac{100 \cdot 99 \cdot \cancel{98!}}{\cancel{98!}\,2!}$$

$$= \frac{100 \cdot 99}{2!}$$

$$= \frac{\cancel{100} \cdot 99}{\cancel{2} \cdot 1}$$

$$= 50 \cdot 99 = 4950$$

$\therefore$ the coeffecient is 4950

$$4950\,(x^{99})$$