Asn 3

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1. $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\quad B = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$

$x'' = ax' + by'$ $\quad x' = px + qy$

$y'' = cx' + dy'$ $\quad y' = rx + sy$

1. $\begin{pmatrix} x'' \\ y'' \end{pmatrix} = A \begin{pmatrix} x' \\ y' \end{pmatrix}$

2. $\begin{pmatrix} x' \\ y' \end{pmatrix} = B \begin{pmatrix} x \\ y \end{pmatrix}$

3. $\begin{pmatrix} x'' \\ y'' \end{pmatrix} = C \begin{pmatrix} x \\ y \end{pmatrix}$ $\quad$ Prove $C = AB$

$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = A \begin{pmatrix} x' \\ y' \end{pmatrix}$ by 1.

$C \begin{pmatrix} x \\ y \end{pmatrix} = A \begin{pmatrix} x' \\ y' \end{pmatrix}$ by 3.

$C \begin{pmatrix} x \\ y \end{pmatrix} = A \left(B \begin{pmatrix} x \\ y \end{pmatrix} \right)$ by 2

$C \begin{pmatrix} x \\ y \end{pmatrix} = AB \begin{pmatrix} x \\ y \end{pmatrix}$

$\therefore C = AB$

1. continued

2. $\begin{pmatrix} px+qy \\ rx+sy \end{pmatrix} = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

1. $\begin{pmatrix} a(px+qy)+b(rx+sy) \\ c(px+qy)+d(rx+sy) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} px+qy \\ rx+sy \end{pmatrix}$

3. $\begin{pmatrix} a(px+qy)+b(rx+sy) \\ c(px+qy)+d(rx+sy) \end{pmatrix} = C \begin{pmatrix} x \\ y \end{pmatrix}$

4. $A \cdot B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} pa+rb & qa+sb \\ pc+rd & qc+sd \end{pmatrix}$

5. $\begin{pmatrix} pa+rb & qa+sb \\ pc+rd & qc+sd \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} xpa+xrb+yqa+ysb \\ xpc+xrd+yqc+ysd \end{pmatrix}$

6. $\begin{pmatrix} a(px+qy)+b(rx+sy) \\ c(px+qy)+d(rx+sy) \end{pmatrix}$

$(A \cdot B)\begin{pmatrix} x \\ y \end{pmatrix} = C\begin{pmatrix} x \\ y \end{pmatrix}$ proven by $3=6$
with 3 being $A\begin{pmatrix} x' \\ y' \end{pmatrix}$ (with $\begin{pmatrix} x' \\ y' \end{pmatrix}$ being derived from $B\begin{pmatrix} x \\ y \end{pmatrix}) = \begin{pmatrix} x'' \\ y'' \end{pmatrix} = C\begin{pmatrix} x \\ y \end{pmatrix}$
and 6 coming from $(A \cdot B)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x'' \\ y'' \end{pmatrix} = C\begin{pmatrix} x \\ y \end{pmatrix}$

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2. $\sum_{k=0}^{n} (3k+1)^2 = 1^2 + 4^2 + \cdots + (3n+1)^2$

$= \sum_{k=0}^{n} (3k+1)^2 = \sum_{k=0}^{n} (3k+1)(3k+1)$

$= \sum_{k=0}^{n} (9k^2 + 6k + 1)$

$= \sum_{k=0}^{n} 9k^2 + \sum_{k=0}^{n} 6k + \sum_{k=0}^{n} 1$

$= 9\sum_{k=0}^{n} k^2 + 6\sum_{k=0}^{n} k + \sum_{k=0}^{n} 1$

$9\sum_{k=0}^{n} k^2 = 9(0^2 + 1^2 + \cdots + n^2)$

$= 9\,\dfrac{n(n+1)(2n+1)}{6}$

$= \dfrac{3n(n+1)(2n+1)}{6}$

$6\sum_{k=0}^{n} k = 6\,\dfrac{n(n+1)}{2} = 3n(n+1)$

$\sum_{k=0}^{n} 1 = 1 + 1 + 1 \cdots + 1 = n+1$

$\therefore \dfrac{3n(n+1)(2n+1)}{2} + 3n(n+1) + (n+1)$

$= (n+1)\left(\dfrac{6n^2 + 3n + 6n + 2}{2}\right)$

$\therefore = \dfrac{(n+1)(6n^2 + 9n + 2)}{2}$ = closed form

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3. prove $3^{n+1} + 4^{2n-1}$ = P is divisable by 13 for $n \geq 1$, we will prove this with induction

Suppose $n = 1$, then

$$3^{1+1} + 4^{2(1)-1} = 3^2 + 4^1 = 9 + 4 = 13.$$

and $13 \mid 13$ ∴ when $n = 1$ holds true so the base case is true

Let if $p(n)$ is true when $n = k$ then

$$3^{k+1} + 4^{2(k)-1} \text{ is divsable by } 13$$

So, $3^{(k+1)+1} + 4^{2(k+1)-1} = 13p$ for some p

Now we will prove it is true when $n = k+1$

So, $3^{(k+1)+1} + 4^{2(k+1)-1}$

$$= 3^{k+2} + 4^{2k+1}$$
$$= 3^{k+1} \cdot 3 + 4^{2k} \cdot 4$$
$$= 3(13p - 4^{2k-1}) + 4^{2k} \cdot 4$$
$$= 39p - 3 \cdot 4^{2k-1} + 4^{2k} \cdot 4$$
$$= 39p - 3 \cdot \frac{4^{2k}}{4} + 4^{2k} \cdot 4$$
$$= 39p + 4^{2k}\left(4 - \frac{3}{4}\right)$$

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4. $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

$$A_n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix}$$

Base Case $F(1)$:

$$A = \begin{pmatrix} F_2 & F_1 \\ F_1 & F_0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \therefore \text{True}$$

now let $F(K)$ be true

$$A_K = \begin{pmatrix} F_{K+1} & F_K \\ F_K & F_{K-1} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} F_{K+1} + F_K & F_{K+1} \\ F_K + F_{K-1} & F_K \end{pmatrix}$$

$$= \begin{pmatrix} F_{K+2} & F_{K+1} \\ F_{K+1} & F_K \end{pmatrix} \quad \therefore F_n = F_{n-1} + F_{n-2}$$

Now, $F(1)$ is true as is $F(k+1)$, whenever $F(K)$ is true

$\therefore F(n)$ is true for all $n \geq 1$

$$\therefore A_n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix} \text{ for } n \geq 1 \ \blacksquare$$