CS-2214 Assignment 1
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1. Construct a compound propositions using the propositions p, q, r and the connectives ∨, ∧, ¬ such that (Justify with truth table)
   a. s = T if and only if (p = q = T, r = F) and (p = F, q = r = T).
   s=(p∧q∧¬r)∨(¬p∧q∧r)

P	Q	R	¬p	¬R	p∧q	(p∧q)∧¬r	¬p∧q	(¬p∧q)∧r)	((p∧q)∧¬r)∨((¬p∧q)∧r)
T	T	T	F	F	T	F	F	F	F
T	**T**	**F**	**F**	**T**	**T**	**T**	**F**	**F**	**T**
T	F	T	F	F	F	F	F	F	F
T	F	F	F	T	F	F	F	F	F
F	**T**	**T**	**T**	**F**	**F**	**F**	**T**	**T**	**T**
F	T	F	T	T	F	F	T	F	F
F	F	T	T	F	F	F	F	F	F
F	F	F	T	T	F	F	F	F	F

   b. s = T if and only if exactly one of p, q or r are true.
   s=(p∧¬q∧¬r)∨(¬p∧q∧¬r)∨(¬p∧¬q∧r)

P	Q	R	(p∧¬q∧¬r)	(¬p∧q∧¬r)	(¬p∧¬q∧r)	(p∧¬q∧¬r)∨(¬p∧q∧¬r)∨(¬p∧¬q∧r)
T	T	T	F	F	F	F
T	T	F	F	F	F	F
T	F	T	F	F	F	F
T	**F**	**F**	**T**	**F**	**F**	**T**
F	T	T	F	F	F	F
F	**T**	**F**	**F**	**T**	**F**	**T**
F	**F**	**T**	**F**	**F**	**T**	**T**
F	F	F	F	F	F	F

2. Decide which of the following propositions are tautology, contradiction or satisfiable with justification. The justification should use truth tables and/or laws of propositional logic:
   a. (p ∧ q ∧ r) → (p ∧ q) ∨ r

P	Q	R	P∧Q	(P∧Q∧R)	(P∧Q)∨R	(P∧Q∧R)→ (P∧Q)∨R
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	F	T	T
T	F	F	F	F	F	T
F	T	T	F	F	T	T
F	T	F	F	F	F	T
F	F	T	F	F	T	T
F	F	F	F	F	F	T

Therefore, by the truth table it is a tautology as it is always true

b. $p \rightarrow \neg p$

P	$\neg$P	P$\rightarrow\neg$P
T	F	F
F	T	T

	Equation	Steps
1	$p\rightarrow\neg p$	
2	$\neg p\vee\neg p$	Conditional identity 1
3	$\neg(p\wedge p)$	DeMorgans Law 2
4	$\neg p$	Idempotent Law 3

Therefore, by truth table and rules of propositional logic $p\rightarrow\neg p$ is satisfiable when p=F

c. $((p \rightarrow q) \wedge q) \rightarrow p$

P	Q	P$\rightarrow$Q	((P$\rightarrow$Q)$\wedge$Q)	((P$\rightarrow$Q)$\wedge$Q)$\rightarrow$P
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Therefore, by truth table the proposition is satisfiable when (p,q) = (T,T), (T,F), (F,F)

3. Decide if the following sentences are equivalent. Use appropriate labeling of propositions and use logical reasoning to justify your conclusion.

i - If Gail scored a 100 in the final and got more than 80% in the assignments then she will get an A in the course.

ii - If Gail scored a 100 in the final then she got less than 80% on the assignments or she will get an A in the course. Note: Here 'or' is used in the sense of disjunction, not exclusive or.

A= Got an A in the course

F = got 100% on the final

G = got more than 80% on the assignments

i = $(f\wedge g) \rightarrow a$

ii = $f\rightarrow(\neg g\vee a)$

Prove: $(f\wedge g) \rightarrow a \equiv f\rightarrow(\neg g\vee a)$

	steps	LHS	RHS	Steps
1		$(f\wedge g) \rightarrow a$		
2	Cond Identity 1	$\neg(f\wedge g)\vee a$		
3	Demorgans 2	$\neg f\vee\neg g\vee a$		
4	Cond Identity 3	$f\rightarrow(\neg g\vee a)$	$f\rightarrow(\neg g\vee a)$	
5			$\neg f\vee(\neg g\vee a)$	Cond Identity 4
6			$\neg(f\wedge g)\vee a$	DeMorgans 5
7			$(f\wedge g)\rightarrow a$	Cond Identity 6

Therefore, proved through the laws of propositional logic the 2 statements are equivalent

4. Prove the validity or invalidity of the following argument. If it is valid then prove it using rules of inference. If it is invalid give appropriate justification. Assume the domain for all statements is the set of all family members in a large family.

   If someone in the family eats tuna then they eat salmon.
   There is someone in the family who eats shrimp.
   No one in the family eats both salmon and shrimp.
   Therefore, there is someone in the family who does not eat tuna

   T(x)= x eats tuna
   S(x) = x eats salmon
   H(x)= x eats shrimp
   $\forall$x = everyone in the family
   $\exists$x = someone in the family

   $\exists x\,(T(x) \to S(x))$
   $\exists x(H(x))$
   $\neg\forall x(S(x)\wedge H(x))$

   $\exists x(\neg T(x))$

1	$\exists x\,(T(x) \to S(x))$	Hypothesis
2	$\exists x(H(x))$	Hypothesis
3	$\neg\forall x(S(x)\wedge H(x))$	Hypothesis
4	$\exists x\neg(\,S(x)\wedge H(x))$	Demorgans Law 3
5	$\exists x(\neg S(x)\vee\neg H(x))$	Demorgans Law 4
6	H(c) for some c	Existential Instantiation 2
7	$T(c)\to S(c)$ for some c	Existential Instantiation 1
8	$(\neg S(c)\vee\neg H(c))$ for some c	Existential Instantiation 5
9	$S(c)\to\neg H(c)$ for some c	Conditional Identity 8
10	$T(c)\to\neg H(c)$ for some c	Hypothetical Syllogism 7,9
11	$\neg T(c)\vee\neg H(c)$ for some c	Conditional Identity 10
12	$\neg T(c)\vee\neg H(c)\wedge H(c)$ for some c	Conjunction 6, 11
13	$\neg T(c)\vee F$ for some c	Complement Law 12
14	$\neg T(c)$ for some c	Identity Law 13
15	$\exists x\neg T(x)$	Existential Generalization 14

Therefore, through the rules of inference and laws of propositional logic the above statements and conclusion is valid.