Computer Science 4442

Assignment 2

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1.

a)

p(Water = cool|Play = yes) = **1/3**

p(Water = cool|Play = no) = **0**

b)

p(Play = yes|Water = warm) = **2/3**

p(Play = no|Water = warm) = **1/3**

c)

p(Play = yes|Humid = high) = **2/3**

p(Play = yes|Humid = normal) = **1**

d)

With Laplace smoothing:

p(Water = cool|Play = yes) = **2/5**

p(Water = cool|Play = no) = **1/3**

2.

a)

2a)

$$K(x,z) = K_1(x,z)\, K_2(x,z)$$

$K(x,z)$ is symmetric

K_1 is a kernal so $\exists\, \phi^{(1)}$ such that:

$$K_1(x,z) = (\phi^1(x))^T (\phi^1(z))$$

K_2 is a kernal so $\exists\, \phi^{(2)}$ such that:

$$K_2(x,z) = (\phi^2(x))^T (\phi^2(z))$$

$$\therefore K(x,z) = K_1(x,z)\, K_2(x,z)$$

$$= \sum_i \phi_i^{(1)}(x)\, \phi_i^{(1)}(z) \sum_j \phi_j^{(2)}(x)\, \phi_j^{(2)}(z)$$

$$= \sum_i \sum_j \phi_i^{(1)}(x)\, \phi_i^{(1)}(z)\, \phi_j^{(2)}(x)\, \phi_j^{(2)}(z)$$

$$= \sum_i \sum_j \left(\phi_i^{(1)}(x)\, \phi_j^{(2)}(x)\right)\left(\phi_i^{(1)}(z)\, \phi_j^{(2)}(z)\right)$$

$$\text{def } \psi(\cdot) = \phi^{(1)}(\cdot)\, \phi^{(2)}(\cdot)$$

$$\therefore = \sum_i \sum_j \psi_{i \cdot j}(x)\, \psi_{i \cdot j}(z)$$

$$\therefore K(x,z) = \psi(x)^T \psi(z)$$

$$\boxed{\therefore \text{Valid}}$$

b)

Date

2b)

$K(x,z) = f_1(x)\, f_1(z) + f_2(x)\, f_2(z)$ where $f_1, f_2 : R^n \to R$ are real valued functions

$f_1(x)\, f_1(z) = K_1(x,z)$

$f_2(x)\, f_2(z) = K_2(x,z)$

$\therefore K(x,z) = K_1(x,z) + K_2(x,z)$

Let:

$\phi^1(x) = (\phi^1_1(x), \phi^1_2(x), \ldots, \phi^1_{n_1}(x))$

$\phi^2(x) = (\phi^2_1(x), \phi^2_2(x), \ldots, \phi^2_{n_2}(x))$

be the feature map of K_1, K_2

Define $\phi(x)$ by concatinating feature maps

$\phi(x) = (\phi^1_1(x), \phi^1_2(x), \ldots, \phi^1_{n_1}(x), \phi^2_1(x), \ldots, \phi^2_{n_2}(x))$

$\therefore$ satisfies $\phi(x) \cdot \phi(z) = \phi^1(x) \cdot \phi^1(z) + \phi^2(x) \cdot \phi^2(z)$

$\boxed{\therefore \text{Valid}}$

c)

2c) $K(x,z) = \dfrac{K_1(x,z)}{\sqrt{K_1(x,x)\, K_1(z,z)}}$ where $K_1(x,x) > 0$ for any x

$$K_1(x,z) = \frac{K_1(x,z)}{\sqrt{K_1(x,x)\, K_1(z,z)}}$$

$$= \frac{1}{\sqrt{K_1(x,x)}} \cdot K_1(x,z) \cdot \frac{1}{\sqrt{K_2(z,z)}}$$

$$= \frac{1}{\sqrt{\phi_x \phi_x}} \cdot \phi_x\, \phi(z) \cdot \frac{1}{\sqrt{\phi_z \phi_z}}$$

$$= \frac{1}{\|\phi(x)\|} \cdot \phi(x)\, \phi(z) \cdot \frac{1}{\|\phi(z)\|}$$

$$= \frac{\phi(x)\, \phi(z)}{\|\phi_x\|\, \|\phi(z)\|}$$

$= \Psi(x)\, \Psi(z)$ with:

$$\Psi(x) = \frac{\phi(x)}{\|\phi(x)\|}$$

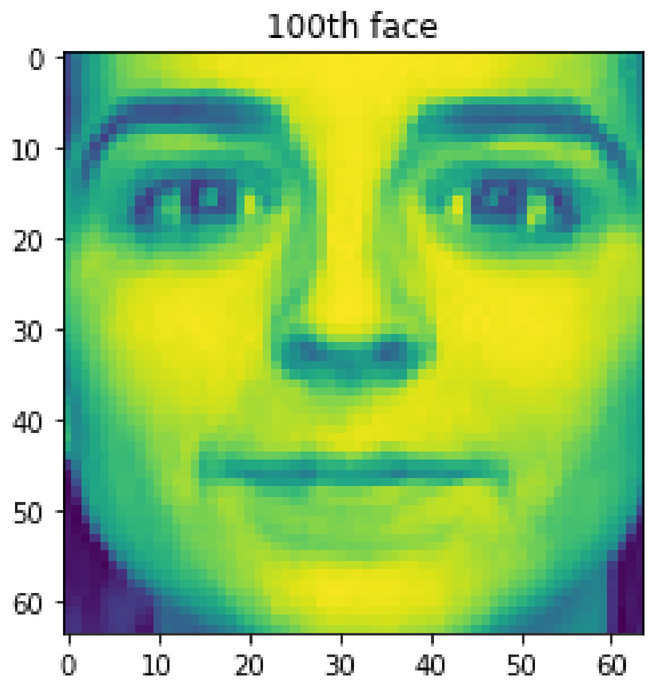
and

$$\Psi(z) = \frac{\phi(z)}{\|\phi(z)\|}$$

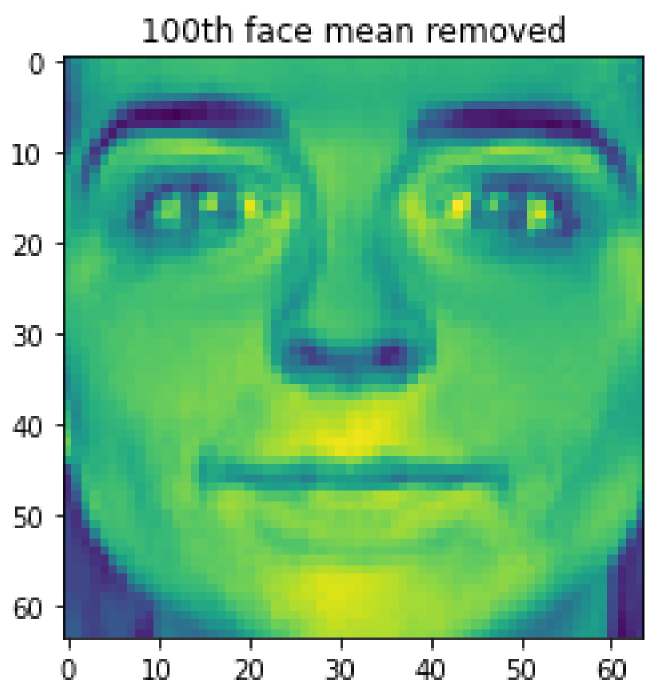
$\boxed{\therefore \text{Valid}}$

3.

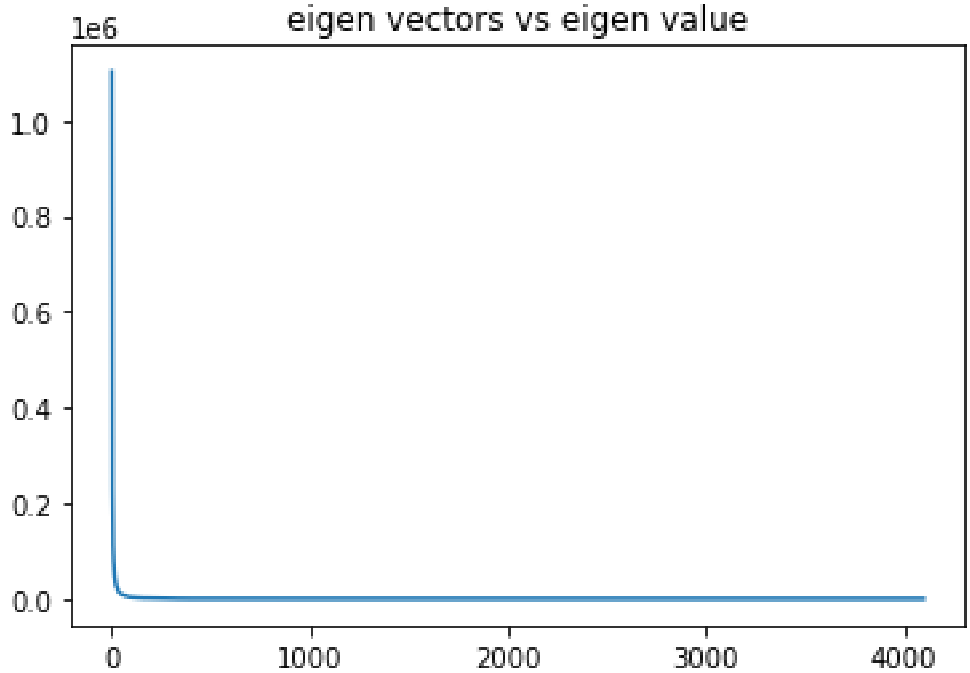
a)


100th face
0
10
20
30
40
50
60
0
10
20
30
40
50
60

b)


100th face mean removed
0
10
20
30
40
50
60
0
10
20
30
40
50
60

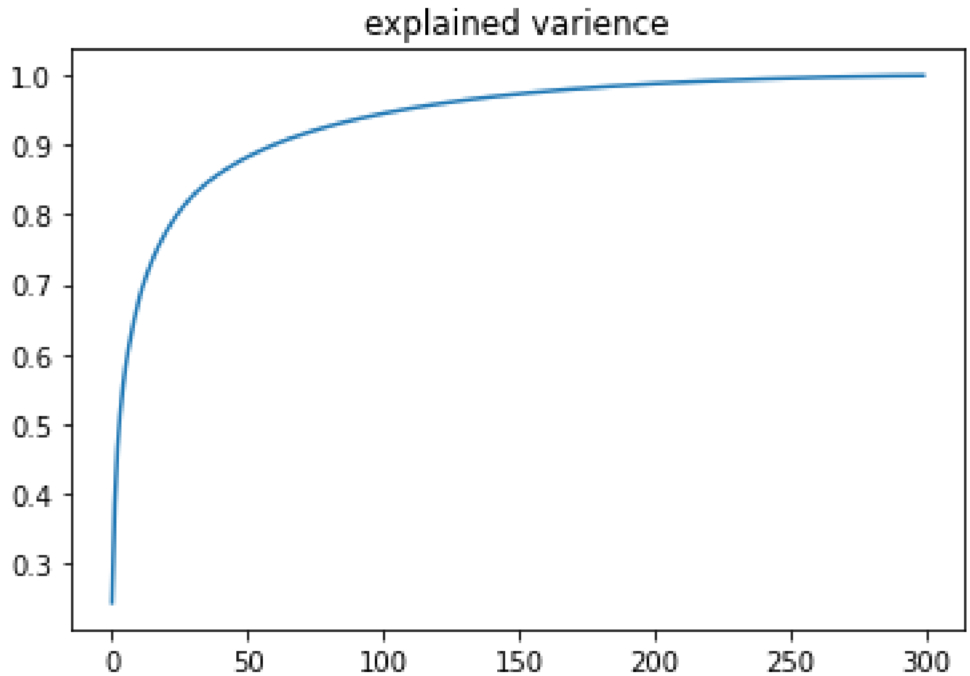
c)



d)

This means that the null space is nontrivial in other terms the image is slightly compressed as it is of a lower dimension as well this means the matrix is not invertible.

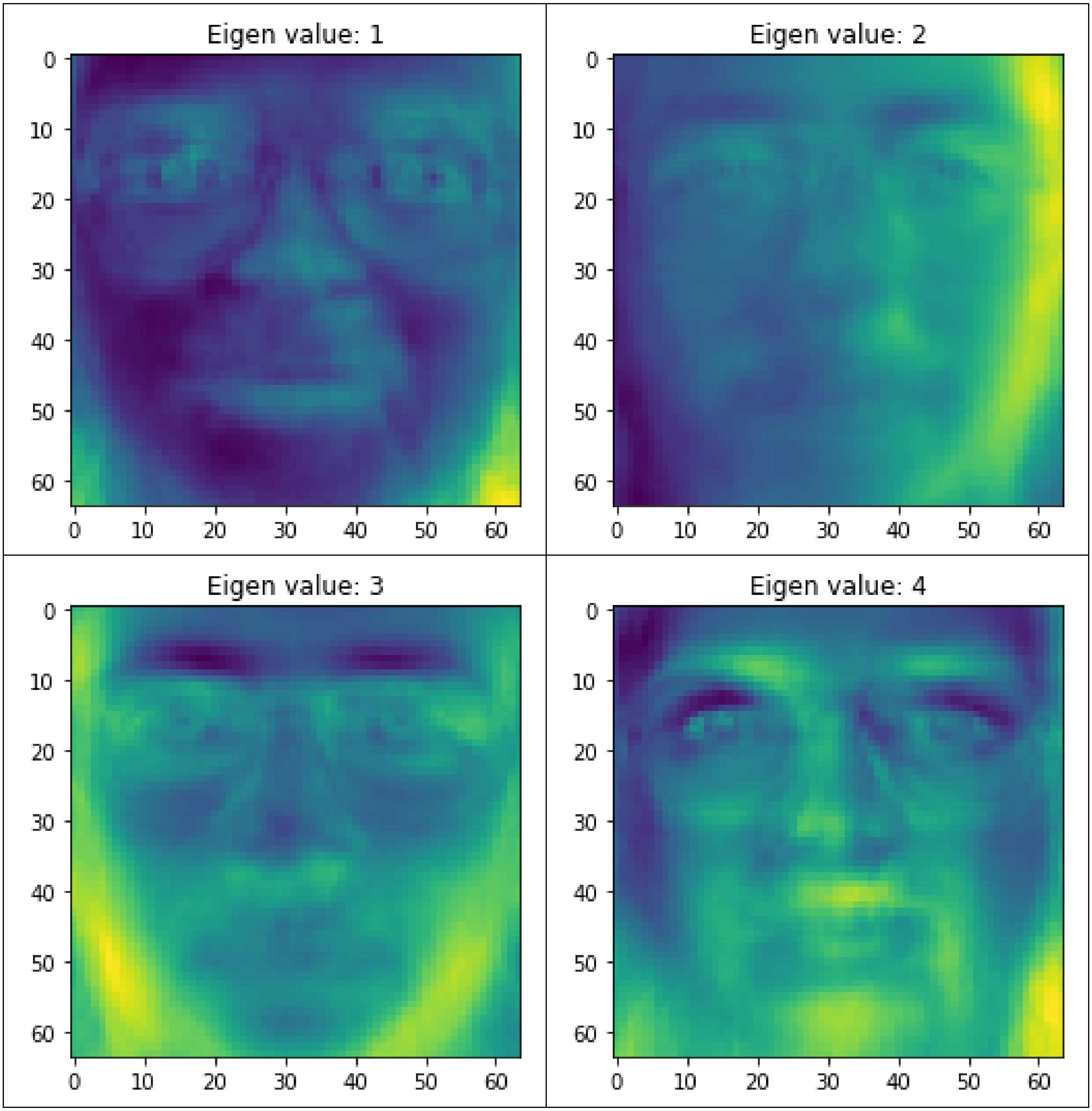
e)

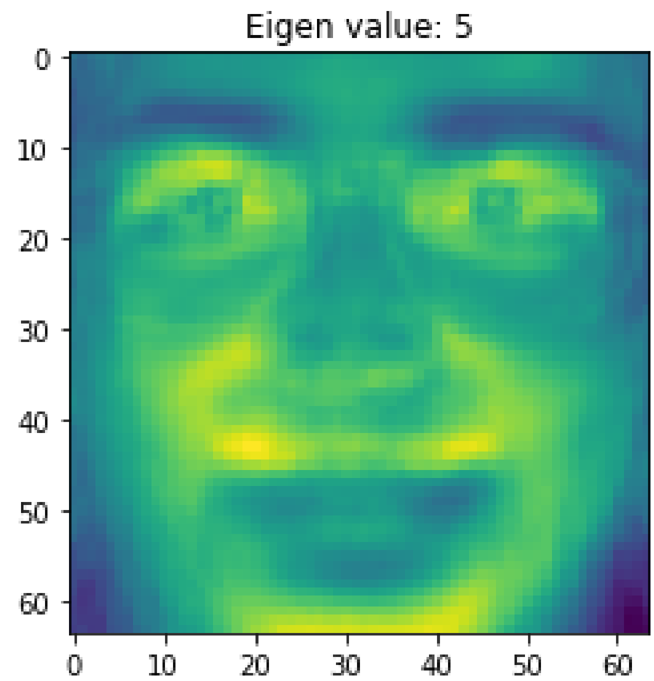


In order to keep 95% variance, we are required to keep a dimensionality of ~140 meaning we only need to keep ~100 components.

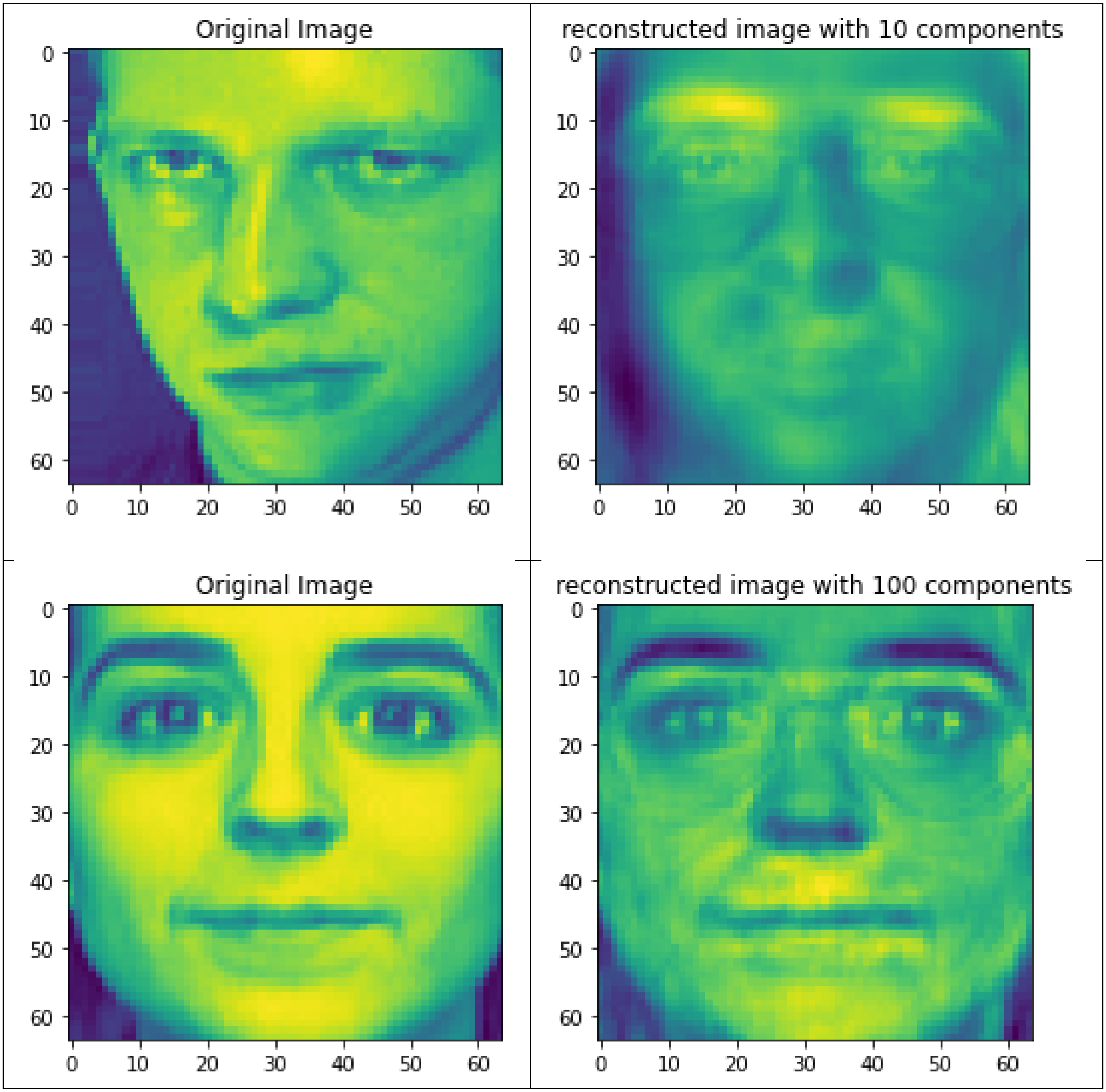
Required Dimensionality: 123

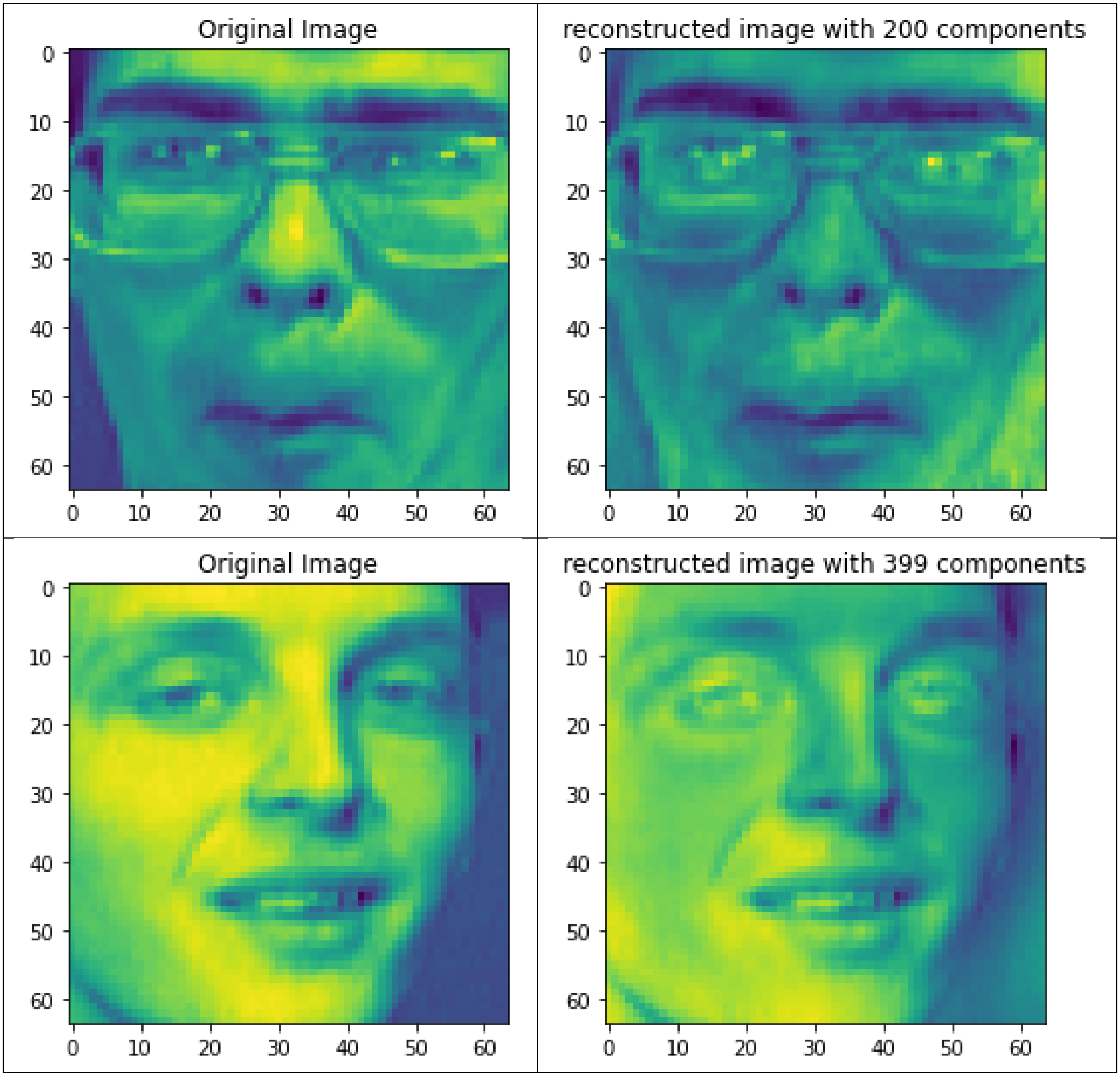
f)


Eigen value: 1
Eigen value: 2
Eigen value: 3
Eigen value: 4
0
10
20
30
40
50
60

Eigen value: 5 0 10 20 30 40 50 60	

g)


Original Image
reconstructed image with 10 components
Original Image
reconstructed image with 100 components
0 10 20 30 40 50 60


Original Image
reconstructed image with 200 components
Original Image
reconstructed image with 399 components