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In [ ]: import numpy as np
import sympy as sp
import matplotlib.pyplot as plt
import math
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In [ ]: def Cheb_sum(n: int, x: float):
    k = 0
    sum = 0
    while k <= math.floor(n/2):
        temp = (-1)**k
        temp *= math.factorial(n - k - 1) / (math.factorial(k) * math.factorial(n - 2*k))
        temp *= (2*x)**(n - 2*k)
        sum += temp
        k += 1
    return sum

def Cheb_cos(n: int, x: float):
    return math.cos(n*math.acos(x))
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In [ ]: n_points = int(1e4)
xs = np.linspace(-1, 1, n_points)
ns = [2, 5, 10, 15, 20, 40, 80]
sum_arr = np.zeros((ns.__len__(), xs.__len__()))
cos_arr = np.zeros_like(sum_arr)
for i, n in enumerate(ns):
    for j, x in enumerate(xs):
        sum_arr[i][j] = Cheb_sum(n, x)
        cos_arr[i][j] = Cheb_cos(n, x)
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In [ ]: fig, axs = plt.subplots(len(ns), figsize = (12, 20))
for i in range(len(cos_arr)):
    axs[i].plot(xs, cos_arr[i] - sum_arr[i], label = ns[i])
    axs[i].set_yscale('log')
    axs[i].legend(loc = 'upper right')
    axs[i].grid(True)
    axs[i].set_facecolor((0.9, 0.9, 0.9))
```