CSCE 423: Algorithms and Complexity Homework 1

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Due: February 5, 2026

Problem 1

Algorithm

For this problem we will divide our algorithm into 3 steps which involve the use of median of medians or the `SELECT` algorithm. The steps will be:

1. `SELECT`$(A, j) \to a$

2. `SELECT`$(A, k) \to b$

3. Simple comparison loop through list to find all $x \in A$ where

$$a \leq x \leq b.$$

   This would be equivalent to $j \leq \text{RANK}(x) \leq k$.

Analyzing Time Complexity

We have proven in class `SELECT` or median of medians is $O(n)$. Step three is a simple loop through all elements of A so has a time complexity of $O(n)$. So,

$$O(n) + O(n) + O(n)$$

is our time complexity. Or,
$$O(n).$$

Proof of Correctness

Given how we know `SELECT` is correct:

- `SELECT`$(A, j) \to a$ is the element of rank j

- `SELECT`$(A, k) \to b$ is the element of rank k

If our algorithm outputs an element x, then $a \leq x \leq b$. So, there are at least j elements $\leq x$ and at most k elements $\leq x$. So,

$$\mathrm{RANK}(x) \in [j, k]$$

and all output elements are correct.

If element x has $\mathrm{rank}(x) \in [j, k]$, then x is between the jth and k^{th} smallest elements. So, $a \leq x \leq b$ and x is an output. Therefore all correct elements are output.

Problem 2

[Your solution here]

Problem 3

[Your solution here]

Problem 4

Algorithm

This problem is always asking for the 19th element we will simply loop through A, of size n, 19 times. Steps are as follows:

1. Check if array length ≥ 19

2. Loop through array keeping track of smallest element

3. Put smallest element at index (iteration) $\rightarrow$ first case 0

4. Now loop from position 1 to $n - 1$, finding smallest and swapping w/ position 1

5. Keep doing this for all 19 iterations

6. Return element $A[18]$

We are partially sorting the array.

Time Complexity

Total iterations is

$$\sum_{i=0}^{18}(n - i - 1) \quad \text{or} \quad \sum_{i=1}^{19}(n - i) = 19n - 190$$

So,

$$T(n) = 19n - 190 \leq 19n \leq cn = O(n)$$

Proof of Correctness

This algorithm is essentially the first 19 steps of selection sort. We have proved selection sort's correctness within class or prerequisites. Given this it has elements located at $A[i-1]$ where i is the RANK of these elements.

Therefore,

Position $A[18]$ after our algorithm is the 19th element of A.

Problem 5

[Your solution here]

Problem 6 (Bonus)

[Your solution here]