Advanced Problem

We can use proof by contradiction to show that the algorithm will also find the $i-1$ smaller and $n-i$ larger elements.

We can let variable y represent the ith smallest element returned by the algorithm. For y to be the ith smallest, there has to be a set of $i-1$ elements such that for all elements in the subset a, the element is less than y. There also has to be a set of exactly $n-i$ elements such that for all elements in the subset b, all elements are greater than y.

Suppose the algorithm ends early and there is an element w that is not equal to y, and the comparison between the two elements was not determined. Without the comparison, we could change the value of w to be smaller or larger than y without contradicting any comparisons. Changing the value of w to be greater or less than y will change the rank of y.

This means that for the algorithm to be correct, it must have done enough comparisons to fix the rank of y to be exactly i. This means that every element in the list has a relationship through direct comparison or the transitive property of y. This therefore proves that to know y is the ith element in the list, we implicitly partition the elements into two sets, one that is less than y and another greater than y.