

Notebook - Maratona de Programação

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1 Algoritmos

1.1 Mochila

1.2 Kadane-DP

1.3 Iterative-BS

```
1 int main()
       int l=1, r=N;
       int res=-1;
4
       while(1 \le r)
           int m = (1 + r)/2;
9
           if(!ver(m))
           {
                1 = m+1;
           }
           else
13
14
                res = m;
                r = m-1;
16
           }
       }
18
       cout << res << endl;</pre>
19
20
       return 0;
21
22 }
```

2 Grafos

2.1 BFS

```
1 //BFS (Breadth First Search) O(V+A)
3 vector < vector < int >> adj; // adjacency list
     representation
 4 int n; // number of nodes
5 int s; // source vertex
7 queue < int > q;
 8 vector < int > d(n, INF);
9 d[s]=0;
10
11 q.push(s);
12 used[s] = true;
13 while (!q.empty()) {
14
   int v = q.front();
15
      q.pop();
16
       for (int u : adj[v]) {
           if (d[u] > d[v] + 1) {
17
18
               q.push(u);
               d[u] = d[v] + 1;
19
20
           }
21
       }
22 }
```

2.2 Find-bridges

```
#define vi vector<int>
3 vector < vector <int> > grafo;
4 vector < bool > visited;
5 vi t, low;
6 int timer=0;
8 void find_bridges(int v, int p=-1)
9 {
       visited[v] = true;
10
       t[v] = low[v] = timer++;
       for(int i=0;i<(int)grafo[v].size();i++)</pre>
12
13
           int vert = grafo[v][i];
14
           if(vert == p)
16
               continue;
           if(visited[vert])
               low[v] = min(low[v], t[vert]);
18
           else
19
20
               find_bridges(vert, v);
               low[v] = min(low[v], low[vert]);
               if(low[to] > t[v])
23
                    IS_BRIDGE(v, vert);
24
25
       }
26
27 }
28
29 int main()
30 {
       timer = 0;
31
       visited.assign(N+1, false);
       t.assign(N+1, 0);
33
       low.assign(N+1, 0);
34
35
36
       for(int i=0;i<N;i++)
           if(!visited[i])
37
               find_bridges(1);
38
39
40
       return 0;
41 }
```

2.3 Dijkstra

```
_{1} // Dijkstra - Shortest Path _{2}
```

```
3 #define pii pair<int, int>
4 #define vi vector <int>
                                                           26
                                                                  return minimum;
5 #define vii vector< pair<int,int> >
                                                           27 }
6 #define INF 0x3f3f3f3f
                                                           28
                                                           29 int start[MAX+1], dfs_time;
8 vector<vii>> grafo(N+1, vii());
                                                           30 int tour[2*MAX+1], id[2*MAX+1];
9 vi distancia(N+1, INF);
                                                           31
10 priority_queue < pii, vii, greater < pii > > fila;
                                                           32 void dfs(int u, int pai=-1){
                                                                  start[u] = dfs_time;
                                                           33
12 void dijkstra(int k)
                                                                  id[dfs_time] = u;
                                                           34
                                                                  tour[dfs_time++] = start[u];
13 €
                                                           35
14
       int dist, vert, aux;
                                                           36
                                                                  for(int v : grafo[u]){
      distancia[k]=0;
                                                           37
                                                                      if(v==pai)
                                                                           continue;
16
                                                           38
      fila.push(mp(k, 0));
                                                           39
                                                                       dfs(v, u);
                                                                       id[dfs_time] = u;
18
                                                           40
19
      while (!fila.empty())
                                                           41
                                                                       tour[dfs_time++] = start[u];
20
                                                           42
           aux=fila.top().f;
21
                                                           43 }
22
          fila.pop();
                                                           44
                                                           45 int LCA(int u, int v)
           for(auto v: grafo[aux])
                                                           46 {
                                                           47
                                                                  if(start[u] > start[v])
               vert=v.f;
                                                                      swap(u, v);
               dist=v.s:
                                                                  return id[query(start[u], start[v])];
                                                           49
               if(distancia[vert]>distancia[aux]+dist)
                                                           50 }
28
29
                    distancia[vert] = distancia[aux] + dist; 52 int main()
30
                   fila.push(mp(vert, distancia[vert])); 53 {
                                                                  int N, k, a, b;
                                                           54
                                                                  cin >> N;
           }
33
                                                           56
      }
                                                                  for(int i=0;i<N-1;i++)
34
35 }
                                                                       cin >> a >> b;
36
                                                           59
37 int main()
                                                                       grafo[a].pb(b);
                                                           60
                                                                       grafo[b].pb(a);
38 €
                                                           61
39
      for(int i=0; i<M; i++)
                                                           62
                                                           63
                                                                  dfs(1);
40
           cin >> a >> b >> p;
                                                           64
41
42
           grafo[a].pb(mp(b, p));
                                                           65
                                                                  make();
                                                                  precompute(2*N, tour);
43
           grafo[b].pb(mp(a, p));
                                                           66
                                                           67
44
45 }
                                                           68
                                                                  cin >> k;
                                                           69
  2.4 LCA
                                                           70
                                                                  for(int i=0;i<k;i++)
                                                                       cin >> a >> b;
1 const int K = 100;
                                                                       cout << LCA(a, b) << endl;</pre>
1 int logv[MAX+1];
                                                           73
                                                           74
3 int st[MAX][K];
                                                           75
4 vector < vi > grafo(200010, vi());
                                                                  return 0;
                                                           76
6 void make(){
      logv[1] = 0; // pre-computar tabela de log
                                                                    Floyd-Warshall
                                                              2.5
      for (int i = 2; i <= MAX; i++)
9
          logv[i] = logv[i/2] + 1;
                                                            1 // Floyd Warshall
10 }
void precompute(int N, int array[]) { //
                                                            3 int dist[MAX][MAX];
13
   for (int i = 0; i < N; i++)
        st[i][0] = array[i];
                                                            5 void Floydwarshall()
14
                                                                  for(int k = 1; k \le n; k++)
    int k = logv[N];
16
                                                                      for(int i = 1; i <= n; i++)
17
    for (int j = 1; j \le k; j++)
                                                                           for(int j = 1; j \le n; j++)
         for (int i = 0; i + (1 << j) <= N; i++)
                                                            9
18
             st[i][j] = min(st[i][j-1], st[i + (1 << (j 10))]
                                                                               dist[i][j] = min(dist[i][j], dist[i][
19
                                                                  k] + dist[k][j]);
      - 1))][j - 1]);
20 }
21
                                                              2.6 Kruskal
22 int query(int L, int R) {
      int j = logv[R - L + 1];
      int\ minimum\ =\ min(st[L][j]\ ,\ st[R\ -\ (1\ <<\ j)\ +\ 1][\ {\tiny 1\ //\ } deve-se\ ter\ dsu\ codada\ com\ as\ funcoes\ make\_set\ ,
24
                                                                  find_set e union_sets
      i]);
```

```
2 struct Edge {
      int u, v, weight;
                                                          5
                                                                vector< pair<int, int> > arestas;
      bool operator < (Edge const& other) {</pre>
                                                                arestas.push_back(make_pair(1, 2));
                                                          6
          return weight < other.weight;
                                                                 arestas.push_back(make_pair(1, 3));
7 }:
                                                          9 // Adjacency Matrix
                                                                 int grafo[10][10];
9 int n:
10 vector < Edge > edges;
                                                          12
                                                          13
                                                                 grafo[1][2] = grafo[2][1] = 1;
12 int cost = 0;
                                                                 grafo[1][3] = grafo[3][1] = 2;
                                                          14
vector < Edge > result;
14 for (int i = 0; i < n; i++)
                                                          16 // Adjacency List
      make_set(i);
                                                          17
16
                                                          18
                                                                 vector < int > vizinhos[10];
17 sort(edges.begin(), edges.end());
                                                          19
                                                          20
                                                                 vizinhos[1].push_back(2);
                                                                 vizinhos[1].push_back(2);
19 for (Edge e : edges) {
      if (find_set(e.u) != find_set(e.v)) {
          cost += e.weight;
                                                            2.10 Centroid
21
          result.push_back(e); // vector com as arestas
       da MST
                                                           vi g[MAX];
          union_sets(e.u, e.v);
                                                           2 int size[MAX];
                                                           3 bool erased[MAX]; // vetor dos vertices apagados na
25 }
                                                                decomp.
  2.7 DFS
                                                           5 int sz(int u, int p) {
                                                              int s = 1;
1 //DFS (Depth First Search) O(V+A)
                                                               for(auto prox : g[u]) {
                                                                 if(prox != p and !erased[prox])
3 void DFS(int x)
                                                                   s += sz(prox, u);
4 {
                                                              }
      for(int i=0; i<(int)vizinhos[x].size(); i++)</pre>
5
                                                              return size[u] = s;
6
                                                          12 }
           int v = vizinhos[x][i];
          if(componente[v] == -1)
                                                          14 int centroid(int u, int p, int n) {
                                                              // chamar funcao sz antes, n = size[u]
               componente[v] = componente[x];
                                                              for(auto prox : g[u]) {
                                                          16
               DFS(v);
                                                          17
                                                                 if(prox != p and !erased[prox]) {
12
          }
                                                                   if(size[prox] > n/2) {
                                                          18
      }
13
                                                                     return centroid(prox, u, n);
14 }
                                                          20
                                                                 }
                                                          21
  2.8 Kosaraju
                                                              }
                                                          22
                                                          23
                                                              return u;
_{\rm 1} // KOSARAJU - O(V+E) - encontra componentes
      fortemente conexos
                                                            2.11 Prim
_2 // g -> grafo, gt -> grafo tempo
3 // vis -> visitado, cor -> componente fortemente
      conexo ordenado topologicamente
                                                          1 // Prim Algorithm
4 vector<int> g[N], gt[N], S; int vis[N], cor[N];
                                                           2 #define MAXN 10100
5 void dfs(int u){
                                                           3 #define INFINITO 999999999
      vis[u] = 1; for(int v : g[u]) if(!vis[v]) dfs(v);
      S.push_back(u);
                                                           5 int n, m;
8 }
                                                           6 int distancia[MAXN];
9 void dfst(int u, int e){
                                                           7 int processado[MAXN];
      cor[u] = e;
                                                           8 vector < pii > vizinhos [MAXN];
      for(int v : gt[u]) if(!cor[v]) dfst(v, e);
12 }
                                                          10 int Prim()
13 void kosaraju(){
                                                          11 {
      for(int i = 1; i <= n; i++) if(!vis[i]) dfs(i);</pre>
14
                                                                 for(int i = 2;i <= n;i++) distancia[i] = INFINITO</pre>
      for(int i = 1; i <= n; i++) for(int j : g[i])
          gt[j].push_back(i);
16
                                                                 distancia[1] = 0;
      int e = 0; reverse(S.begin(), S.end());
18
      for(int u : S) if(!cor[u]) dfst(u, ++e);
                                                                 priority_queue < pii, vector < pii >, greater < pii > >
19 }
                                                                 fila.push( pii(distancia[1], 1) );
                                                          16
  2.9 Represent
                                                                 while(1)
                                                          18
1 // Grafos
                                                          19
                                                                     int davez = -1;
                                                          20
3 // List of edges
                                                          21
```

```
while(!fila.empty())
                                                                 if (q.x<=max(p.x,r.x) && q.x>=min(p.x,r.x) && q.y
                                                          10
                                                                 <=\max(p.y,r.y) \&\& q.y>=\min(p.y,r.y))
               int atual = fila.top().second;
                                                         11
                                                                     return true;
24
              fila.pop();
                                                          12
                                                          13
                                                                 return false;
               if(!processado[atual])
                                                          14 }
27
                   davez = atual:
                                                          16 int orientation(pnt p, pnt q, pnt r)
29
                   break;
                                                          17 {
30
               }
                                                          18
                                                                 int val=(q.y-p.y)*(r.x-q.x)-(q.x-p.x)*(r.y-q.y);
          }
32
                                                          19
                                                           20
                                                                 if(val==0)
          if(davez == -1)
                                                                    return 0;
34
                                                          21
               break;
                                                          22
                                                                  else if(val>0)
35
36
                                                          23
                                                                     return 1;
           processado[davez] = true;
                                                                 else
                                                          24
                                                                     return 2;
          for(int i = 0;i < (int)vizinhos[davez].size() 26 }</pre>
39
      ;i++)
          {
                                                          28 bool intersect(pnt p1, pnt q1, pnt p2, pnt q2)
40
                                                          29 {
41
               int dist = vizinhos[davez][i].first;
                                                                 int o1 = orientation(p1, q1, p2);
42
                                                          30
                                                                 int o2 = orientation(p1, q1, q2);
               int atual = vizinhos[davez][i].second;
43
                                                          31
                                                                 int o3 = orientation(p2, q2, p1);
               if ( distancia[atual] > dist && !
                                                                 int o4 = orientation(p2, q2, q1);
45
                                                          33
      processado[atual])
                                                           34
                                                                 if (o1!=o2 \text{ and } o3!=o4)
46
              {
                                                           35
                   distancia[atual] = dist;
                                                                     return true;
47
                                                          36
                   fila.push( pii(distancia[atual],
48
                                                          37
                                                                 if(o1==0 && collinear(p1, p2, q1))
      atual));
                                                          38
                                                           39
                                                                     return true;
          }
                                                           40
                                                                 if(o2==0 && collinear(p1, q2, q1))
                                                          41
52
                                                          42
                                                                     return true;
      int custo_arvore = 0;
53
                                                          43
      for(int i = 1;i <= n;i++)
                                                                 if(o3==0 && collinear(p2, p1, q2))
                                                          44
          custo_arvore += distancia[i];
                                                                     return true:
                                                          45
56
                                                          46
                                                                 if(o4==0 && collinear(p2, q1, q2))
      return custo_arvore;
                                                          47
57
58 }
                                                                     return true;
                                                          48
                                                          49
60 int main(){
                                                                 return false;
                                                          50
                                                          51
61
62
      cin >> n >> m:
                                                          52 }
63
      for(int i = 1;i <= m;i++){
                                                             3.2 Rotation
64
65
          int x, y, tempo;
                                                           1 // Rotate clockwise 90 degree
          cin >> x >> y >> tempo;
67
                                                           _{2} (x, y) => (y, -x)
68
           vizinhos[x].pb( pii(tempo, y) );
69
           vizinhos[y].pb( pii(tempo, x) );
                                                           4 // Rotate counterclockwise 90 degree
                                                           5 (x, y) => (-y, x)
      cout << Prim() << endl;</pre>
                                                             3.3 Inter-Retangulos
74
      return 0;
                                                           1 typedef struct
76 }
                                                           2 {
                                                                 int x, y;
                                                           3
  3
       Geometria
                                                           4 } Point;
  3.1 Inter-Retas
```

```
1 // Intersection between lines
2
3 typedef struct
4 {
5    int x, y;
6 } pnt;
7
8 bool collinear(pnt p, pnt q, pnt r)
9 {
```


3.4 Analytic-Geometry

return false;

r1.v)

return true;

9

10

11 }

```
2 {
                                                         70
                                                               if(v>0) return 1;
      double x, y;
                                                               if(v==0) return 0;
      point(double _x=0, double _y=0){
                                                         72
                                                               return -1;
4
          x=_x;y=_y;
                                                        73 }
                                                         75 // Area de um poligono (pontos ordenados por
      void show(){
                                                               adjacencia)
         cout << "x = " << x << endl;
                                                         76 double area(vector <point> p){
          cout << "y = " << y << endl;
                                                        double ret = 0;
                                                             for(int i=2;i<(int)p.size();i++)
                                                         78
                                                         79
                                                              ret += cross(p[i] - p[0], p[i-1] - p[0])/2;
      point operator+(const point &o) const{
13
                                                         80
                                                             return abs(ret);
                                                        81 }
         return \{x + o.x, y + o.y\};
14
                                                         82 // Concavo ou Convexo
      point operator-(const point &o) const{
                                                         83 double ccw(point a, point b, point c){
16
          return {x - o.x, y - o.y};
                                                         double ret = cross(b - a, c - b);
                                                            return ret < 0;
18
                                                         85
                                                         86 }
      bool operator == (const point &o) const{
          return (x == o.x and y == o.y);
20
                                                           4
                                                                ED
21
22
23 };
                                                                Range-query-bigger-than-k-BIT
25 struct line
26 {
                                                         _{1} // C++ program to print the number of elements
27
      point fp, sp;
                                                         2 // greater than k in a subarray of range L-R.
      line(point _fp=0, point _sp=0){
28
                                                         3 #include <bits/stdc++.h>
          fp=_fp;sp=_sp;
                                                         4 using namespace std;
30
31
                                                         6 // Structure which will store both
     //a=y1-y2;
32
                                                         7 // array elements and queries.
     //b = x2 - x1;
33
                                                         8 struct node {
     //c = x2 * y1 - y2 * x1;
                                                             int pos;
35
                                                              int 1;
36 };
                                                         11
                                                               int r;
                                                         12
                                                               int val:
38 // Produto Escalar
                                                        13 };
39 double dot(point a, point b){
                                                        14
     return a.x*b.x + a.y*b.y;
40
                                                        15 // Boolean comparator that will be used
41 }
                                                         16 // for sorting the structural array.
42
                                                         17 bool comp(node a, node b)
43 // Produto Vetorial
                                                        18 {
44 double cross(point a, point b){
                                                               // If 2 values are equal the query will
                                                        19
      return a.x*b.y - a.y*b.x;
45
                                                        20
                                                               // occur first then array element
46 }
                                                               if (a.val == b.val)
                                                        21
                                                         22
                                                                    return a.l > b.l;
48 // Dist entre dois pontos
                                                         23
49 double dist(point a, point b){
                                                        24
                                                               // Otherwise sorted in descending order.
   point c = a - b;
                                                               return a.val > b.val;
                                                        25
51
      return sqrt(c.x*c.x + c.y*c.y);
                                                         26 }
52 }
                                                         28 // Updates the node of BIT array by adding
54 // Colinearidade entre 3 pontos
                                                        _{29} // 1 to it and its ancestors.
55 bool collinear(point a, point b, point c){
                                                        30 void update(int* BIT, int n, int idx)
      return ((c.y-b.y)*(b.x-a.x)==(b.y-a.y)*(c.x-b.x))_{31} {
                                                                while (idx \leq n) {
      // return (a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b<sub>33</sub>
                                                                  BIT[idx]++;
      .y)); // Triangle area
                                                                   idx += idx & (-idx);
      // No caso de pontos tridimensionais, usar
                                                         35
      produto vetorial.
                                                         36 }
                                                         _{\rm 37} // Returns the count of numbers of elements
                                                         38 // present from starting till idx.
61 // Dist entre ponto e reta
                                                         39 int query(int* BIT, int idx)
62 double distr(point a, line b){
                                                         40 {
      double crs = cross(point(a - b.fp), point(b.sp - 41
                                                               int ans = 0;
      b.fp));
                                                               while (idx) {
                                                         42
      return abs(crs/dist(b.fp, b.sp));
                                                                  ans += BIT[idx];
64
                                                         43
65 }
                                                                   idx -= idx & (-idx);
                                                         45
or void esq(point a, point b, point ext)
                                                        46
68 { // Esquerda = 1; Direita = -1; Collinear = 0; 47
                                                               return ans;
      11 v = a.x*b.y+b.x*ext.y+ext.x*a.y - (a.y*b.x+b.y_{48})
```

*ext.x+ext.y*a.x);

1 struct point

```
50 // Function to solve the queries offline
                                                          118
                                                                  // 1-based indexing
51 void solveQuery(int arr[], int n, int QueryL[],
                                                                 int QueryL[] = { 1, 2 };
                                                          119
                                                                 int QueryR[] = { 4, 6 };
                   int QueryR[], int QueryK[], int q)
                                                                 // k for each query
       // create node to store the elements
54
       // and the queries
                                                                  int QueryK[] = { 6, 8 };
       node a[n + q + 1];
56
                                                          124
       // 1-based indexing.
                                                                  // number of queries
                                                                 int q = sizeof(QueryL) / sizeof(QueryL[0]);
                                                          126
       // traverse for all array numbers
59
                                                          127
       for (int i = 1; i <= n; ++i) {
60
                                                          128
                                                                  // Function call to get
           a[i].val = arr[i - 1];
                                                                  solveQuery(arr, n, QueryL, QueryR, QueryK, q);
61
                                                          129
           a[i].pos = 0;
                                                          130
62
           a[i].1 = 0;
63
                                                                 return 0;
           a[i].r = i;
                                                          132 }
64
65
66
                                                                  Iterative-SegTree
                                                             4.2
       // iterate for all queries
       for (int i = n + 1; i <= n + q; ++i) {
68
                                                           1 // Segment Tree Iterativa - Range maximum query
           a[i].pos = i - n;
69
           a[i].val = QueryK[i - n - 1];
71
           a[i].1 = QueryL[i - n - 1];
                                                           3 #define N 100010
           a[i].r = QueryR[i - n - 1];
                                                           5 struct Segtree
74
                                                           6 {
       // In-built sort function used to
                                                                 int t[2*N] = \{0\};
       // sort node array using comp function.
76
       sort(a + 1, a + n + q + 1, comp);
                                                                 void build()
                                                           9
78
       // Binary Indexed tree with
                                                                      for(int i=N-1; i>0; i--)
       // initially 0 at all places.
                                                                          t[i]=max(t[i<<1], t[1<<1|1]);
80
                                                           12
       int BIT[n + 1];
81
                                                           13
82
                                                           14
       // initially 0
                                                                 int query(int 1, int r)
                                                           15
83
       memset(BIT, 0, sizeof(BIT));
                                                           16
85
                                                                      int ans=0;
                                                                      for (i+=N, r+=N; 1< r; 1>>=1, r>>=1)
       // For storing answers for each query( 1-based
86
                                                           18
       indexing ).
                                                           19
       int ans[q + 1];
                                                                          if(1&1)
87
                                                           20
                                                                              ans=max(ans, t[1++]);
       // traverse for numbers and query
                                                                          if(r&1)
89
       for (int i = 1; i \le n + q; ++i) {
                                                                              ans=max(ans, t[--r]);
                                                           23
90
91
           if (a[i].pos != 0) {
                                                           24
                                                                      7
92
                                                           25
               // call function to returns answer for
                                                                      return ans;
       each query
               int cnt = query(BIT, a[i].r) - query(BIT, 28
        a[i].1 - 1);
                                                                 void update(int p, int value)
95
                                                           30
               // This will ensure that answer of each
                                                                      for(t[p+=n]=value; p>1; p>>=1)
96
                                                           31
                                                                          t[p>>1] = max(t[p], t[p^1]);
       query
               // are stored in order it was initially
       asked.
                                                           34
               ans[a[i].pos] = cnt;
                                                           35 };
98
           }
99
                                                           36
                                                           37 int main()
           else {
               // a[i].r contains the position of the
                                                           38 {
                // element in the original array.
                                                           39
                                                                  Segtree st:
               update(BIT, n, a[i].r);
                                                           40
           }
104
                                                           41
                                                                 for(int i=0;i<n;i++)
                                                           42
       // Output the answer array
                                                                      cin >> aux;
                                                          43
       for (int i = 1; i <= q; ++i) {
                                                                      st.t[N+i] = aux; //Leaves are stored in
                                                          44
108
           cout << ans[i] << endl;</pre>
                                                                  continuous nodes with indices starting with N
109
                                                          45
110 }
                                                           46
                                                                 st.build();
                                                           47
112 // Driver Code
                                                                 x = st.query(inicio, fim);
                                                           48
113 int main()
                                                                  st.update(ind, value);
                                                           49
114 {
                                                           50
       int arr[] = { 7, 3, 9, 13, 5, 4 };
                                                          51 }
       int n = sizeof(arr) / sizeof(arr[0]);
116
```

```
4.3 Recursive-SegTree
```

```
1 // Segment Tree Recursiva - Range maximum query
                                                          72 int main()
                                                          73 {
3 vector<int> val(MAX, 0);
                                                          74
                                                                 monta(1, N, 1);
                                                                 atualiza(1, 1, \mathbb{N}, pos, valor);
4 vector < int > vet(N);
                                                                 x = consulta(1, 1, N, inicio, fim);
                                                          76
6 void monta(int i, int j, int no)
                                                          78 }
      if(i==j)
                                                            4.4 Delta-Encoding
9
          val[no]=vet[i];
          return:
                                                         1 // Delta encoding
12
                                                          3 for(int i=0;i<q;i++)</pre>
      int esq = 2*no;
                                                          4 {
      int dir = 2*no+1;
                                                                 int l,r,x;
      int meio = (i+j)/2;
                                                                 cin >> 1 >> r >> x;
                                                                 delta[1] += x;
      monta(i, meio, esq);
18
                                                                 delta[r+1] = x;
19
      monta(meio+1, j, dir);
                                                          9 }
20
      val[no]=max(val[esq], val[dir]);
                                                          int atual = 0;
22 }
                                                          12
                                                          13 for(int i=0;i<n;i++)</pre>
24 void atualiza(int no, int i, int j, int pos, int
                                                          14 {
      novo_valor)
                                                                 atual += delta[i]:
                                                          15
25 {
                                                                 v[i] += atual;
                                                          16
      if(i==j)
26
28
          val[no]=novo_valor;
                                                            4.5 Seg-Tree-Farao
      }else
29
30
          int esq = 2*no;
                                                          1 typedef struct
31
          int dir = 2*no+1;
                                                                 pii prefix, sufix, total, maximo;
          int meio = (i+j)/2;
33
                                                          3
                                                           4 } no:
35
          if(pos<=meio)
              atualiza(esq, i, meio, pos, novo_valor); 6 int noleft[MAX], noright[MAX]; //Guarda os valores
36
           else
                                                               dos nos para que nao sejam calculados novamente
              atualiza(dir, meio+1, j, pos, novo_valor)
38
                                                                nas querys
                                                           7 int v[MAX];
39
                                                           8 no arvore[MAX];
          if(val[esq]>val[dir])
40
41
              val[no]=val[esq];
                                                          10 pii somar(pii a, pii b) // une pairs
                                                          11 {
           else
42
43
              val[no]=val[dir];
                                                          12
                                                                 return mp(a.f+b.f, a.s+b.s);
      }
                                                          13 }
44
45 }
                                                          14
                                                          15 no une(no l, no r)
47 int consulta(int no, int i, int j, int A, int B)
                                                          16 {
                                                                 if(1.total.s==0)
48 {
      if(i>B || j<A)
49
                                                          18
                                                                    return r:
50
          return -1;
                                                          19
                                                                 if(r.total.s==0)
      if(i>=A and j<=B)
                                                          20
                                                                    return 1:
          return val[no];
                                                          21
52
                                                          22
      int esq = 2*no;
54
                                                          23
      int dir = 2*no+1;
                                                                 m.prefix = max(l.prefix, somar(l.total, r.prefix)
56
      int meio = (i+j)/2;
                                                                ); //prefixo
                                                                 m.sufix = max(r.sufix, somar(r.total, l.sufix));
      int resp_esq = consulta(esq, i, meio, A, B);
                                                                 //sufixo
      int resp_dir = consulta(dir, meio+1, j, A, B);
                                                                m.total = somar(1.total, r.total); //Soma de
59
60
                                                                todos os elementos da subarvore
      if(resp_dir==-1)
                                                                 m.maximo = max(max(1.maximo, r.maximo), somar(1.
61
62
          return resp_esq;
                                                                 sufix, r.prefix)); //Resultado para cada
63
      if(resp_esq==-1)
                                                                 subarvore
64
         return resp_dir;
                                                          28
                                                                 return m;
                                                          29
      if(resp_esq>resp_dir)
                                                          30 }
66
          return resp_esq;
                                                          32 no makenozero()
68
      else
          return resp_dir;
                                                          33 {
69
```

70 }

```
4.6
                                                                    BIT-2D
34
       no m:
35
       m.prefix=m.sufix=m.total=m.maximo=mp(0,0);
36
       return m;
                                                             1 // BIT 2D
37 }
38
                                                             3 int bit[MAX][MAX];
39 no makeno(int k)
40 {
                                                             5 int sum(int x, int y)
41
                                                             6 {
       m.prefix=m.sufix=m.total=m.maximo=mp(k,1);
42
                                                                   int resp=0;
       return m;
43
44 }
                                                                   for (int i=x; i>0; i-=i&-i)
                                                             9
                                                                        for (int j=y; j>0; j-=j&-j)
46 void monta(int n)
                                                                            resp+=bit[i][j];
47 {
                                                             12
       if(noleft[n] == noright[n])
48
                                                             13
                                                                   return resp;
49
                                                            14 }
            arvore[n]=makeno(v[noleft[n]]);
           return;
                                                            16 void update(int x, int y, int delta)
52
                                                             17 {
                                                                    for (int i=x; i < MAX; i+=i&-i)
                                                             18
       int mid = (noleft[n]+noright[n])/2;
54
                                                                        for (int j=y; j < MAX; j+=j&-j)
                                                             19
       noleft[2*n]=noleft[n]; noright[2*n]=mid;
                                                                            bit[i][j]+=delta;
       noleft[2*n+1]=mid+1; noright[2*n+1]=noright[n];
56
                                                            21 }
       monta(2*n):
58
                                                             23 int query(int x1, y1, x2, y2)
       monta(2*n+1);
59
                                                            24 {
60
                                                                    return sum(x2,y2) - sum(x2,y1) - sum(x1,y2) + sum
       arvore[n] = une(arvore[2*n], arvore[2*n+1]);
61
                                                                   (x1,y1);
62 }
63
64 no busca(int n, int esq, int dir)
                                                               4.7
                                                                    \operatorname{BIT}
65 {
       if(noleft[n]>=esq and noright[n]<=dir)</pre>
66
                                                             1 struct FT {
67
           return arvore[n];
                                                                   vector<int> bit; // indexado em 1
       \verb|if(noright[n]<| esq or noleft[n]>| dir)|\\
                                                             2
68
           return makenozero();
                                                                   FT(int n) {
71
       return une(busca(2*n, esq, dir),busca(2*n+1, esq,
                                                                        this->n = n + 1;
        dir));
                                                                        bit.assign(n + 1, 0);
72 }
                                                             9
74 int main()
                                                             10
                                                                   int sum(int idx) {
75 {
                                                                        int ret = 0;
76
       int T, N, Q, A, B;
                                                                        for (++idx; idx > 0; idx -= idx & -idx)
                                                             12
77
       no aux;
                                                                            ret += bit[idx];
                                                                        return ret;
       scanf("%d", &T);
                                                             14
79
                                                             15
80
                                                             16
       while (T--)
81
                                                                    int sum(int 1, int r) {
82
                                                                        return sum(r) - sum(l - 1);
                                                             18
            scanf("%d", &N);
83
           for(int i=1;i<=N;i++)
                                                             19
84
                scanf("%d", &v[i]); //Elementos da arvore 20
                                                                   void add(int idx, int delta) {
                                                            21
86
                                                                        for (++idx; idx <= n; idx += idx & -idx)
                                                             22
87
            noleft[1]=1; noright[1]=N;
                                                                            bit[idx] += delta;
                                                            23
88
           monta(1);
                                                            24
89
                                                             25 };
           cin >> 0:
90
           while(0--)
91
                                                                     Sparse-Table
92
                scanf("%d%d", &A, &B); //Intervalo da
93
       query
                                                             1 logv[1] = 0; // pre-computar tabela de log
                aux = busca(1, A, B);
                                                             2 for (int i = 2; i <= MAXN; i++)</pre>
94
                printf("%d %d\n", aux.maximo.f, aux.
95
                                                                  logv[i] = logv[i/2] + 1;
       maximo.s);
96
           }
                                                             5 int logv[MAXN+1];
97
                                                             6 int st[MAXN][K];
98
99
                                                             8 // operacao da sparse table deve ser idempotente ->
100
       return 0;
                                                                   op(x, x) = x
101 }
                                                             9 void precompute(int N) { //
                                                               for (int i = 0; i < N; i++)
                                                             10
                                                                      st[i][0] = array[i];
```

```
sort(queries.begin(), queries.end());
                                                          16
13
    int k = logv[N];
                                                          17
    for (int j = 1; j <= k; j++)
                                                               int 1 = 0, r = -1;
14
                                                          18
        for (int i = 0; i + (1 << j) <= N; i++)
                                                               for(Query q : queries) {
                                                          19
            st[i][j] = max(st[i][j-1], st[i + (1 << (j 20))]
                                                                    while(1 > q.1) \{
      - 1))][j - 1]);
                                                                        1--:
                                                                        add(1);
18
19 int query(int L, int R) {
                                                                    while(r < q.r) {
                                                          24
                                                                        r++;
      int j = logv[R - L + 1];
      int maximum = \max(st[L][j], st[R - (1 << j) + 1][26]
                                                                        add(r);
21
                                                                    }
                                                                    while(1 < q.1) {
      return maximum;
                                                                        remove(1);
23
                                                          29
24 }
                                                          30
                                                                        1++;
                                                          31
                                                                    }
      Union-Find
                                                          32
                                                                    while(r > q.r) {
                                                                        remove(r);
                                                          33
                                                          34
1 // Union-Find Functions
                                                          35
                                                          36
                                                                    res.pb(mp(q.idx, RESPOSTA)); // adicionar
3 int pai[MAX], peso[MAX];
                                                                 resposta de acordo com o problema
                                                          37
5 int find(int aux)
                                                                return res; // ordernar o vetor pelo indice e
6 {
                                                                 responder queries na ordem
      if(pai[aux] == aux)
7
8
         return aux;
9
      else
                                                                  Math
          return pai[aux]=find(pai[aux], pai);
11 }
12
                                                                  Totient
                                                            5.1
13 void join(int x, int y)
14 {
                                                          _{1} // phi(p^k) = (p^(k-1))*(p-1) com p primo
      x = find(x);
                                                          2 // O(sqrt(m))
      y = find(y);
16
                                                          3 ll phi(ll m) {
                                                                ll res = m;
                                                          4
      if(pesos[x]<pesos[y])</pre>
18
                                                                 for(11 d = 2; d*d <= m; d++) {
                                                           5
         pai[x] = y;
19
                                                                   if(m \% d == 0) {
                                                           6
      else if(pesos[x]>pesos[y])
20
                                                                      res = (res/d) * (d-1);
         pai[y] = x;
21
                                                                       while (m \% d == 0) {
      else if(pesos[x]==pesos[y])
                                                                        m /= d:
                                                          9
          pai[x] = y;
24
                                                                   }
          pesos[y]++;
25
                                                                 }
                                                          12
26
                                                          13
                                                                 if(m > 1) {
27 }
                                                                 res /= m;
                                                          14
                                                          15
                                                                  res *= (m-1);
29 int main()
                                                          16
                                                          17
      for(int i=1;i<=N;i++)
31
                                                          18
                                                                 return res;
          pai[i]=i;
32
                                                          19 }
33 }
                                                          21 // modificacao do crivo, O(n*log(log(n)))
  4.10 Mo
                                                          22 vector<ll> phi_to_n(ll n) {
                                                                vector < bool > isprime(n+1, true);
                                                          23
1 const int BLK = 500; // tamanho do bloco, algo entre 24
                                                                 vector < ll> tot(n+1);
      300 e 500 e nice
                                                                 tot[0] = 0; tot[1] = 1;
                                                                 for(ll i = 1; i <= n; i++) {
                                                          26
3 struct Query {
                                                                   tot[i] = i;
                                                          27
  int l, r, idx;
                                                          28
    bool operator < (Query other) const</pre>
                                                          29
                                                          30 for(11 p = 2; p <= n; p++) {
      return make_pair(1 / BLK, r) <
                                                               if(isprime[p]) {
                                                         31
      make_pair(other.1 / BLK, other.r);
                                                         32
                                                                  tot[p] = p-1;
                                                                   for(ll i = p+p; i <= n; i += p) {
    }
9
                                                          33
                                                                       isprime[i] = false;
10 }:
                                                                       tot[i] = (tot[i]/p)*(p-1);
void add(); void remove() // implementar operacoes de 36
                                                                 }
      acordo com o problema, cuidado com TLE ao 37
      utilizar MAP
                                                          38 }
14 vector < pair < int, ll >> mo() {
                                                          40
                                                                 return tot;
   vector < pair < int, ll >> res;
                                                          41 }
```

${\bf 5.2}\quad {\bf Linear\text{-}Diophantine\text{-}Equation}$

1 // Linear Diophantine Equation

```
2 int gcd(int a, int b, int &x, int &y)
       if (a == 0)
           x = 0; y = 1;
           return b;
7
       }
      int x1, y1;
9
      int d = gcd(b%a, a, x1, y1);
      x = y1 - (b / a) * x1;
12
      y = x1;
13
       return d;
14 }
16 bool find_any_solution(int a, int b, int c, int &x0, 8 #define ll long long
       int &y0, int &g)
       g = gcd(abs(a), abs(b), x0, y0);
18
       if (c % g)
19
          return false;
20
21
      x0 *= c / g;
22
      y0 *= c / g;
23
      if (a < 0) x0 = -x0;
      if (b < 0) y0 = -y0;
25
       return true;
27 }
_{29} // All solutions
_{30} // x = x0 + k*b/g
_{31} // y = y0 - k*a/g
```

5.3 Factorization-sqrt

```
1 // Factorization of a number in sqrt(n)
3 int main()
4 {
       11 N;
      vector < int > div;
6
      cin >> N;
9
       for(11 i=2;i*i<=N;i++)
           if(N\%i==0)
12
           {
13
               vet.pb(i);
14
                while (N\%i==0)
                   N/=i;
16
           }
      7
18
       if(N!=1)
19
           vet.pb(N);
20
       return 0;
23 }
```

5.4 Modular-Exponentiation

```
1 // Modular exponentiaion - (x^y)%mod in O(log y)
2 ll power(ll x, ll y, ll mod)
3 {
4          ll res = 1;
5          x%=mod;
6          while(y)
8          {
9                if(y&1)
```

```
res=(res*x)%mod;
            y = y >> 1;
12
13
            x=(x*x)\%mod;
       }
14
       return res:
16 }
   5.5 Miller-Habin
1 #include <bits/stdc++.h>
2 #define mod 100000007
 3 #define Pi 3.14159265358979311599796346854
 4 #define INF 0x3f3f3f3f
 5 #define MAX 1000010
 6 #define f first
 7 #define s second
 9 #define pb push_back
10 #define mp make_pair
#define pii pair<int, int>
12 #define vi vector<int>
13 #define vii vector < pii >
{\tt 14} \ \ \texttt{\#define} \ \ \texttt{sws} \ \ \texttt{ios\_base} :: \texttt{sync\_with\_stdio(false)}; \texttt{cin.tie(}
       NULL)
15 #define forn(i, n) for(int i=0; i<(int)(n); i++)</pre>
16 #define mdc(a, b) (__gcd((a), (b)))
17 #define mmc(a, b) (((a)/__gcd(a, b)) * b)
18 #define endl '\n'
19 #define teto(a, b) (a+b-1)/b
20
21 using namespace std;
23 ll llrand()
24 {
       11 tmp = rand();
       return (tmp << 31) | rand();
26
27 }
28
29 ll add(ll a, ll b, ll c)
30 €
        return (a + b)%c;
31
32 }
33
34 ll mul(ll a, ll b, ll c)
35 {
       11 \text{ ans} = 0;
36
       while(b)
37
38
            if(b & 1)
39
40
               ans = add(ans, a, c);
            a = add(a, a, c);
41
            b /= 2;
42
       }
43
       return ans;
44
45 }
47 ll fexp(ll a, ll b, ll c)
48 {
       ll ans = 1;
49
        while(b)
50
            if(b & 1)
52
               ans = mul(ans, a, c);
            a = mul(a, a, c);
54
            b /= 2;
56
57
       return ans;
```

58 **}**

61 {

62

60 bool rabin(ll n)

if(n <= 1)

```
6 using namespace std;
           return 1;
64
       if(n \le 3)
                                                               8 ll llrand()
           return 1;
65
66
                                                               9 {
67
       11 s=0, d=n-1;
                                                              10
                                                                      11 tmp = rand();
       while (d\%2==0)
                                                                      return (tmp << 31) | rand();
68
                                                               12 }
69
            d/=2:
                                                              13
                                                              14 ll add(ll a, ll b, ll c)
            s++;
       }
                                                              15 {
                                                                      return (a + b)%c;
                                                              16
       for(int k = 0; k < 64*4; k++)
74
                                                              17 }
                                                              18
            11 a = (11rand()\%(n - 3)) + 2;
                                                              19 ll mul(ll a, ll b, ll c)
76
            11 x = fexp(a, d, n);
                                                              20 {
            if(x != 1 and x != n-1)
                                                                      11 \text{ ans} = 0;
                                                              21
78
                                                              22
                                                                      while(b)
                for(int r = 1; r < s; r++)
                                                                      {
80
                                                                          if(b & 1)
81
                                                              24
                     x = mul(x, x, n);
                                                                             ans = add(ans, a, c);
82
                                                              25
                                                                          a = add(a, a, c);
                     if(x == 1)
                                                              26
83
                                                                          b /= 2;
                         return 0;
                                                              27
                     if(x == n-1)
                                                              28
85
                         break;
                                                              29
                                                                      return ans;
                }
                                                              30 }
87
                if(x != n-1)
                                                              31
88
                                                              32 ll rho(ll n)
89
                     return 0;
            }
                                                              33 {
90
91
       }
                                                              34
                                                                      ll x, c, y, d, k;
                                                                      int i:
92
                                                              35
93
       return 1;
                                                              36
                                                                      do{
                                                                          i = 1;
94 }
                                                              37
95
                                                                          x = llrand()%n;
                                                              38
                                                              39
                                                                          c = llrand()%n;
97 int main()
                                                                          y = x, k = 4;
                                                              40
                                                                          do{
98 {
                                                               41
                                                                              if(++i == k)
       //sws:
99
                                                              42
       //freopen("input.txt", "r", stdin);
//freopen("output.txt", "w", stdout);
100
                                                              43
                                                                               {
101
                                                              44
                                                                                   y = x;
                                                                                   k *= 2;
                                                              45
103
       11 N;
                                                               46
                                                                               }
       cin >> N;
                                                                               x = add(mul(x, x, n), c, n);
104
                                                              47
                                                                               d = \_gcd(abs(x - y), n);
                                                              48
106
       cout << rabin(N) << endl;</pre>
                                                              49
                                                                          7
                                                                          while(d == 1);
                                                               50
108
       return 0;
                                                              51
                                                                      while(d == n);
109
110 }
                                                                      return d;
                                                              54
   5.6 Inverso-Mult
                                                              55 }
                                                              56
                                                              57 int main()
 1 // gcd(a, m) = 1 para existir solucao
                                                              58 {
 _{2} // ax + my = 1, ou a*x = 1 (mod m)
                                                                      srand(time(0));
                                                              59
 3 ll inv(ll a, ll m) { // com gcd
                                                              60
       11 x, y;
                                                                      11 N:
                                                              61
       gcd(a, m, x, y);
 5
                                                                      cin >> N;
                                                              62
       return (((x % m) +m) %m);
                                                              63
 7 }
                                                                      11 \text{ div} = \text{rho}(N);
                                                              64
                                                                      cout << div << " " << N/div << endl;
 9 ll inv(ll a, ll phim) { // com phi(m), se m for primo _{66}^{\circ}
        entao phi(m) = p-1
                                                              67
       11 e = phim - 1;
                                                                      // Finding all divisors
                                                               68
       return fexp(a, e);
                                                              69
                                                               70
                                                                      vector<ll> div;
   5.7 Pollard-Rho
                                                               72
                                                                      while(N>1 and !rabin(N))
                                                              73
                                                                      {
                                                              74
                                                                          11 d = rho(N);
 1 // Pollard Rho Algorithm
                                                                          div.pb(d);
                                                                          while (N\%d==0)
                                                              76
 3 #include <bits/stdc++.h>
                                                                               N/=d;
                                                               77
 4 #define ll long long
                                                                      }
                                                               78
```

63

```
if(N!=1)
80
          div.pb(N);
                                                           25 cpx b[N+100];
                                                           26 cpx c[N+100];
81
      return 0;
                                                           27 cpx B[N+100];
82
                                                           28 cpx C[N+100];
84 }
                                                           29 int a[N+100];
                                                           30 int x[N+100];
       Verif-primo
                                                           31 double coss[N+100], sins[N+100];
                                                           32 int n.m.p:
                                                           33
1 // prime verification sqrt(N)
                                                           34 cpx operator +(cpx a,cpx b)
                                                           35 {
3 bool eh_primo(long long N)
                                                                  return cpx(a.a+b.a,a.b+b.b);
                                                           36
4 {
                                                           37 }
      if(N==2)
                                                           38
          return true;
                                                           39 cpx operator *(cpx a,cpx b)
      else if (N==1 \text{ or } N\%2==0)
                                                           40 {
          return false;
                                                                  return cpx(a.a*b.a-a.b*b.b,a.a*b.b+a.b*b.a);
                                                           41
      for(long long i=3;i*i<=N;i+=2)</pre>
                                                           42 }
          if(N\%i==0)
                                                           43
               return false;
                                                           44 cpx operator /(cpx a,cpx b)
12
      return true;
                                                           45 {
13 }
                                                                   cpx r = a*b.bar();
                                                           46
                                                                  return cpx(r.a/b.modsq(),r.b/b.modsq());
                                                           47
  5.9 Crivo
                                                           48 }
                                                           49
                                                           50 cpx EXP(int i,int dir)
1 // Sieve of Eratosthenes
                                                           51 {
                                                           52
                                                                  return cpx(coss[i],sins[i]*dir);
4 vector < bool > primos(100010, true);
                                                           53 }
5 cin >> N;
                                                           55 void FFT(cpx *in,cpx *out,int step,int size,int dir)
7 primos[0]=false;
                                                           56 {
8 primos[1]=false;
                                                           57
                                                                  if(size<1) return;
                                                                  if(size==1)
                                                           58
10 for(int i=2;i<=N;i++)
                                                                       out [0] = in [0]:
      if(primos[i])
                                                           60
          for(int j=i+i; j<=N; j+=i)</pre>
                                                           61
                                                                      return;
               primos[j]=false;
                                                           62
                                                                  FFT(in,out,step*2,size/2,dir);
                                                           63
  5.10 Formulas
                                                           64
                                                                  FFT(in+step,out+size/2,step*2,size/2,dir);
                                                                  for(int i=0;i<size/2;++i)
                                                           65
                                                           66
1 int sum_x2(11 N)
                                                           67
                                                                       cpx even=out[i];
2 {
                                                                       cpx odd=out[i+size/2];
                                                           68
      return (2*N*N*N + 3*N*N + N)/6:
                                                           69
                                                                       out[i] = even+EXP(i*step,dir)*odd;
                                                                       out[i+size/2] = even+EXP((i+size/2)*step,dir)*
                                                                  odd;
  5.11 FFT-golfbot
                                                                  }
                                                           72 }
# #include <bits/stdc++.h>
                                                           73
                                                           74 int main()
                                                           75 {
3 using namespace std;
                                                                  for(int i=0;i<=N;++i)
                                                           76
5 const int N = (1<<19);</pre>
                                                           78
                                                                       coss[i]=cos(two_pi*i/N);
6 const double two_pi = 4 * acos(0);
                                                                      sins[i]=sin(two_pi*i/N);
                                                           79
                                                           80
8 struct cpx
                                                           81
                                                                  while(cin >> n) // Numero de tacadas possiveis
9 {
                                                           82
      cpx(){}
                                                                       fill(x,x+N+100,0);
                                                           83
      cpx(double aa): a(aa){}
                                                                       fill(a,a+N+100,0);
                                                           84
       cpx(double aa, double bb):a(aa),b(bb){}
                                                                       for(int i=0;i<n;++i)
      double a;
                                                           85
      double b;
                                                           86
      double modsq(void) const
                                                           87
                                                                           cin >> p; // Distancia das tacadas
                                                           88
                                                                           x[p]=1:
16
                                                           89
           return a*a+b*b;
                                                                       for (int i=0; i<N+100; ++i)
      }
                                                           90
18
                                                           91
      cpx bar(void) const
19
                                                           92
                                                                           b[i]=cpx(x[i],0);
20
                                                           93
           return cpx(a,-b);
21
                                                                       cin >> m; // Querys
                                                           94
      }
22
                                                           95
                                                                       for(int i=0;i<m;++i)
23 };
```

```
tmp.pb(0);
           {
96
                                                                   }
97
               cin >> a[i]; // Distancia da query
           }
98
                                                                   res.pb(tmp);
           FFT(b,B,1,N,1);
                                                             9
99
           for(int i=0;i<N;++i)</pre>
               C[i]=B[i]*B[i];
                                                                 for(int row = 0; row < n; row++) {
           FFT(C,c,1,N,-1);
                                                                   for(int col = 0; col < n; col++) \{
           for(int i=0:i<N:++i)
                                                                     11 \text{ val} = 0:
                                                            13
               c[i]=c[i]/N;
                                                                     for(int k = 0; k < n; k++) {
104
                                                            14
           int cnt=0;
                                                                       val += (a[row][k]*b[k][col]);
           for(int i=0;i<m;++i)
106
                                                            16
               if(c[a[i]].a>0.5 || x[a[i]])
                                                                     res[row][col] = val;
108
                   cnt++;
                                                            18
           cout << cnt << endl;</pre>
                                                                 }
                                                            19
       }
                                                            20
       return 0;
                                                            21
                                                                 return res:
112 }
                                                            22 }
   5.12 Modular-Factorial
                                                            24 vector < vl > fexp(vector < vl > b, ll e, int n) {
                                                                 if(e == 0) {
                                                            25
                                                                   vector<vl> id;
                                                            26
 1 // C++ program to comput n! % p using Wilson's
                                                                   for(int i = 0; i < n; i++) {
                                                            27
                                                                     vl tmp:
                                                            28
 # #include <bits/stdc++.h>
                                                                     for(int j = 0; j < n; j++) {
 3 using namespace std;
                                                                       if(i == j)
                                                            30
                                                                          tmp.pb(1);
                                                            31
 5 int power(int x, unsigned int y, int p)
                                                                        else
 6 {
                                                                         tmp.pb(0);
       int res = 1;
                                                                     7
                                                            34
       x = x \% p;
                                                                     id.pb(tmp);
                                                            35
 9
                                                            36
       while(y > 0)
                                                            37
                                                            38
                                                                   return id;
           if(y & 1)
                                                            39
               res = (res * x) % p;
                                                            40
14
                                                                 vector < vl> res = fexp(b, e/2, n);
                                                            41
           y = y >> 1;
                                                                 res = mult(res, res, n);
                                                            42
           x = (x * x) % p;
16
                                                            43
                                                                 if(e%2)
                                                            44
18
       return res;
                                                                   res = mult(res, b, n);
                                                            45
19 }
                                                            46
                                                            47
                                                                 return res;
21 int modInverse(int a, int p)
                                                            48 }
22 {
                                                            49
23
       return power(a, p-2, p);
                                                            50 // k = tamanho da recorrencia/matriz, n = n-esimo
24 }
                                                            51 // f(n) = c1*f(n-1) + c2*f(n-2) + ... + ck*f(n-k)
26 int modFact(int n, int p)
                                                            52 // base -> [f(k-1), f(k-2), ..., f(0)]
                                                            _{53} // coeficientes -> [c1, c2, ..., ck]
       if (p \le n)
28
                                                            54 vl solve(int k, int n, vl base, vl coef) {
           return 0;
29
                                                                 vector < vl> inicial;
30
                                                                 inicial.pb(coef);
                                                            56
       int res = (p - 1);
31
                                                                 for(int row = 0; row < k-1; row++) {
                                                                   vl tmp;
                                                            58
       for(int i = n + 1; i < p; i++)
                                                            59
                                                                   for(int col = 0; col < k; col++) {</pre>
34
          res = (res * modInverse(i, p)) % p;
                                                            60
                                                                     if(col == row)
35
       return res;
                                                                       tmp.pb(1);
                                                            61
36 }
                                                                     else
                                                            62
                                                            63
                                                                       tmp.pb(0);
38 int main()
                                                            64
39 {
                                                            65
                                                                   inicial.pb(tmp);
40
       int n = 25, p = 29;
                                                            66
       cout << modFact(n, p);</pre>
41
                                                            67
       return 0;
42
                                                                 vector < vl > matexp = fexp(inicial, max(0, n-k+1), k)
                                                            68
43 }
                                                                 vl res(k):
  5.13 Recursao-linear
                                                                 for(int row = 0; row < k; row++) {
                                                                   11 \ val = 0;
 _{\rm 1} vector<vl> mult(vector<vl> a, vector<vl> b, int n) { ^{72}
                                                                   for(int aux = 0; aux < k; aux++) {
    vector < vl> res;
                                                                     val += matexp[row][aux]*base[aux];
     for(int i = 0; i < n; i++) {
                                                            74
                                                            75
      vl tmp;
                                                            76
                                                                   res[row] = val; // res = (f(n), f(n-1), ..., f(n-1))
       for(int j = 0; j < n; j++) {
```

```
k+1))
77
    }
                                                           22
                                                                  // Turn on the j-th bit
78
                                                           23
                                                                      int S = 34; //(100010)
79
    return res;
80 }
                                                                      int j = 3;
                                                           26
  5.14 Kamenetsky
                                                                      S = S | (1 << j);
                                                           27
                                                           28
                                                                  // Turn off the j-th bit
                                                           29
1 // Number of digits in n! O(1)
                                                           30
                                                                      int S = 42; //(101010)
                                                           31
3 #define Pi 3.14159265358979311599796346854
                                                                      int j = 1;
4 #define Eul 2.71828182845904509079559829842
                                                                      S &= ~(1<<j)
                                                           34
6 long long findDigits(int n)
                                                                      S == 40 //(101000)
                                                           36
      double x;
9
                                                                  // Check the j-th element
                                                           38
      if (n < 0)
          return 0:
                                                                      int S = 42; //(101010)
                                                           40
      if (n == 1)
                                                                      int j = 3;
                                                           41
          return 1;
14
                                                                      T = S & (1 << i); // T = 0
                                                           43
      x = ((n * log10(n / euler) + log10(2 * Pi * n))
      /2.0));
                                                                  // Least significant bit (lsb)
                                                           45
                                                           46
      return floor(x) + 1;
                                                                      int lsb(int x){ return x&-x; }
                                                           47
18 }
                                                           48
                                                                  // Exchange o j-th element
                                                           49
       Misc
  6
                                                                      S = (1 << j)
  6.1 LIS
                                                           53
                                                                  // Position of the first bit on
nultiset < int > S:
                                                                      T = (S & (-S))
2 for(int i = 0; i < n; i++){</pre>
                                                                      T \rightarrow 4 \text{ bit ligado } //(1000)
      auto it = S.upper_bound(vet[i]); // low for inc
      if(it != S.end())
                                                                  // Most significant digit of N
          S.erase(it):
5
      S.insert(vet[i]);
6
                                                                      double K = log10(N);
                                                           60
7 }
                                                           61
                                                                      K = K - floor(K);
8 // size of the lis
                                                                      int X = pow(10, K);
9 int ans = S.size();
                                                           63
                                                                  // Number of digits in N
                                                           64
  6.2 Bitwise
                                                           66
                                                                      X =floor(log10(N)) + 1;
1 // Bitwise
                                                           67
                                                                  // Power of two
      unsigned char a = 5, b = 9; // a = (00000101), b
                                                                      bool isPowerOfTwo(int x){ return x && (!(x&(x
      = (00001001)
                                                                  -1))); }
      AND -
                      a&b
                           // The result is 00000001
                                                                  // Turn off the first bit 1
      (1)
                                                                      m = m & (m-1):
                      a|b
                            // The result is 00001101
      OR -
      (13)
                                                                  // Built-in functions
                            // The result is 00001100
                                                           75
      XOR. -
                      a^b
      (12)
                                                                      // Number of bits 1
      NOT -
                      ~a
                             // The result is 11111010
                                                           77
                                                                      __builtin_popcount()
      (250)
                                                           78
                                                                      __builtin_popcountl1()
                                                           79
      Left shift -
                      b<<1 // The result is 00010010
9
                                                           80
                                                                      // Number of leading zeros
      Right shift - b >> 1 // The result is 00000100
                                                           81
                                                                      __builtin_clz()
                                                           82
                                                                      __builtin_clzl1()
                                                           83
      // Exchange two int variables
                                                                      // Number of trailing zeros
                                                           85
                                                                      __builtin_ctz()
                                                           86
14
           a^=b;
          b^=a;
                                                           87
                                                                      __builtin_ctzl1()
          a^=b;
                                                           88
                                                                  // floor(log2(x))
                                                           89
      // Even or Odd
                                                           90
                                                                      int flog2(int x){ return 32-1-__builtin_clz(x
                                                           91
           (x & 1)? printf("Odd"): printf("Even");
                                                                  ); }
```

7.3 Pal-int 92 93 int flog211(11 x){ return 64-1-__builtin_clzll(x); } bool ehpalindromo(ll n) 2 { Strings if(n<0)3 4 return false; 7.1 KMP int divisor = 1; while(n/divisor >= 10) vector<int> preffix_function(const string &s){ divisor *= 10; int n = s.size(); vector<int> b(n+1); b[0] = -1; int i = 0, j = -1; while(n != 0) while(i < n){ while(j >= 0 && s[i] != s[j]) j = b[j];int leading = n / divisor; 12 b[++i] = ++j;int trailing = n % 10; } 14 return b; if(leading != trailing) 9 } return false; 16 void kmp(const string &t, const string &p){ 17 vector<int> b = preffix_function(p); n = (n % divisor)/10;18 int n = t.size(), m = p.size(); 19 int j = 0; divisor = divisor/100; 20 for(int i = 0; i < n; i++){ 14 } 21 while(j >= 0 && t[i] != p[j]) j = b[j]; 22 16 j++; 23 return true; $if(j == m){$ 24 } 18 j = b[j];7.4 **Z-Func** 19 20 } 21 } vector < int > z_algo(const string &s) 22 } int n = s.size(); 7.2 LCS 3 int L = 0, R = 0; 4 vector < int > z(n, 0); string LCSubStr(string X, string Y) for(int i = 1; i < n; i++) 6 2 { int m = X.size(); 3 if(i <= R) 8 int n = Y.size(); z[i] = min(z[i-L], R - i + 1);9 while(z[i]+i < n && s[z[i]+i] == s[z[i]]) int result = 0, end; z[i]++; int len[2][n]; if(i+z[i]-1 > R)int currRow = 0; { L = i;14 for(int i=0;i<=m;i++){ R = i + z[i] - 1;for(int j=0;j<=n;j++){ 16 if(i==0 || j==0) 17 len[currRow][j] = 0; return z: 18 else if(X[i-1] == Y[j-1]){ len[currRow][j] = len[1-currRow][j-1] + 1; 7.5 \mathbf{Hash} if(len[currRow][j] > result){ result = len[currRow][j]; end = i - 1; 1 ll compute_hash(string const& s) { } const 11 p = 31; // primo, melhor = perto da } quantidade de caracteres else const 11 m = 1e9 + 9; // maior mod = menor len[currRow][j] = 0; probabilidade de colisao } 11 hash_value = 0; 23 $ll p_pow = 1;$ 5 currRow = 1 - currRow; 25 for (char c : s) { } $hash_value = (hash_value + (c - 'a' + 1) *$ 26 p_pow) % m; if(result==0) $p_pow = (p_pow * p) % m;$ 28 29 return string(); 9 30 10 return hash_value; return X.substr(end - result + 1, result); 11 } 31

32 }