



## Notebook - Maratona de Programação

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### Sumário

# 1 Algoritmos

## 1.1 Mochila

```
1 int val[MAXN], peso[MAXN], dp[MAXN][MAXS]
2
3 int knapsack(int N, int M) // Objetos | Peso max
4 {
5     for(i=0;i<=N;i++)
6     {
7         for(j=0;j<=M;j++)
8         {
9             if (i==0 || j==0)
10                dp[i][j] = 0;
11             else if (peso[i-1] <= j)
12                dp[i][j] = max(val[i-1]+dp[i-1][j-
13                    peso[i-1]], dp[i-1][j]);
14             else
15                dp[i][j] = dp[i-1][j];
16         }
17     }
18     return dp[N][M];
19 }
```

## 1.2 Kadane-DP

```
1 // Largest Sum Contiguous Subarray
2
3 int maxSubArraySum(vector<int> a)
4 {
5     int size = a.size();
6     int max_so_far = a[0];
7     int curr_max = a[0];
8
9     for (int i=1;i<size;i++)
10     {
11         curr_max = max(a[i], curr_max+a[i]);
12         max_so_far = max(max_so_far, curr_max);
13     }
14     return max_so_far;
15 }
```

## 1.3 Iterative-BS

```
1 int main()
2 {
3     int l=1, r=N;
4     int res=-1;
5
6     while(l <= r)
7     {
8         int m = (l + r)/2;
9         if(!ver(m))
10         {
11             l = m+1;
12         }
13         else
14         {
15             res = m;
16             r = m-1;
17         }
18     }
19     cout << res << endl;
20
21     return 0;
22 }
```

# 2 Grafos

## 2.1 BFS

```
1 //BFS (Breadth First Search) O(V+A)
2
3 vector<vector<int>> adj; // adjacency list
4 // representation
5 int n; // number of nodes
6 int s; // source vertex
7
8 queue<int> q;
9 vector<int> d(n, INF);
10 d[s]=0;
11
12 q.push(s);
13 used[s] = true;
14 while (!q.empty()) {
15     int v = q.front();
16     q.pop();
17     for (int u : adj[v]) {
18         if (d[u] > d[v] + 1) {
19             q.push(u);
20             d[u] = d[v] + 1;
21         }
22     }
23 }
```

## 2.2 Find-bridges

```
1 #define vi vector<int>
2
3 vector< vector<int> > grafo;
4 vector<bool> visited;
5 vi t, low;
6 int timer=0;
7
8 void find_bridges(int v, int p=-1){
9     visited[v] = true;
10     t[v] = low[v] = timer++;
11     for(int i=0;i<(int)grafo[v].size();i++){
12         int vert = grafo[v][i];
13         if(vert == p)
14             continue;
15         if(visited[vert])
16             low[v] = min(low[v], t[vert]);
17         else{
18             find_bridges(vert, v);
19             low[v] = min(low[v], low[vert]);
20             if(low[vert] > t[v])
21                 IS_BRIDGE(v, vert);
22         }
23     }
24 }
25
26 int main()
27 {
28     timer = 0;
29     visited.assign(N+1, false);
30     t.assign(N+1, 0);
31     low.assign(N+1, 0);
32
33     for(int i=0;i<N;i++)
34         if(!visited[i])
35             find_bridges(i);
36
37     return 0;
38 }
```

## 2.3 Dijkstra

```
1 // Dijkstra - Shortest Path
2
3 #define pii pair<int, int>
4 #define vi vector<int>
5 #define vii vector< pair<int,int> >
```

```

6 #define INF 0x3f3f3f3f
7
8 vector<vii> grafo(N+1, vii());
9 vi distancia(N+1, INF);
10 priority_queue< pii, vii, greater<pii> > fila;
11
12 void dijkstra(int k){
13     int dist, vert, aux;
14     distancia[k]=0;
15
16     fila.push(mp(k, 0));
17
18     while(!fila.empty()){
19         aux=fila.top().f;
20         fila.pop();
21
22         for(auto v: grafo[aux]){
23             vert=v.f;
24             dist=v.s;
25             if(distancia[vert]>distancia[aux]+dist){
26                 distancia[vert]=distancia[aux]+dist;
27                 fila.push(mp(vert, distancia[vert]));
28             }
29         }
30     }
31 }
32
33 int main()
34 {
35     for(int i=0; i<M; i++){
36         cin >> a >> b >> p;
37         grafo[a].pb(mp(b, p));
38         grafo[b].pb(mp(a, p));
39     }
40 }

```

## 2.4 LCA

```

1 const int K = 100;
2 int logv[MAX+1];
3 int st[MAX][K];
4 vector<vi> grafo(200010, vi());
5
6 void make(){
7     logv[1] = 0; // pre-computar tabela de log
8     for (int i = 2; i <= MAX; i++)
9         logv[i] = logv[i/2] + 1;
10 }
11
12 void precompute(int N, int array[]) { //
13     for (int i = 0; i < N; i++)
14         st[i][0] = array[i];
15
16     int k = logv[N];
17     for (int j = 1; j <= k; j++)
18         for (int i = 0; i + (1 << j) <= N; i++)
19             st[i][j] = min(st[i][j-1], st[i + (1 << (j
20 - 1))][j - 1]);
21 }
22
23 int query(int L, int R) {
24     int j = logv[R - L + 1];
25     int minimum = min(st[L][j], st[R - (1 << j) + 1][j]);
26
27     return minimum;
28 }
29
30 int start[MAX+1], dfs_time;
31 int tour[2*MAX+1], id[2*MAX+1];
32 void dfs(int u, int pai=-1){

```

```

33     start[u] = dfs_time;
34     id[dfs_time] = u;
35     tour[dfs_time++] = start[u];
36     for(int v : grafo[u]){
37         if(v==pai)
38             continue;
39         dfs(v, u);
40         id[dfs_time] = u;
41         tour[dfs_time++] = start[u];
42     }
43 }
44
45 int LCA(int u, int v)
46 {
47     if(start[u] > start[v])
48         swap(u, v);
49     return id[query(start[u], start[v])];
50 }
51
52 int main()
53 {
54     int N, k, a, b;
55     cin >> N;
56
57     for(int i=0;i<N-1;i++)
58     {
59         cin >> a >> b;
60         grafo[a].pb(b);
61         grafo[b].pb(a);
62     }
63     dfs(1);
64
65     make();
66     precompute(2*N, tour);
67
68
69     cin >> k;
70     for(int i=0;i<k;i++)
71     {
72         cin >> a >> b;
73         cout << LCA(a, b) << endl;
74     }
75
76     return 0;
77 }

```

## 2.5 Floyd-Warshall

```

1 // Floyd Warshall
2
3 int dist[MAX][MAX];
4
5 void Floydwarshall()
6 {
7     for(int k = 1; k <= n; k++)
8         for(int i = 1; i <= n; i++)
9             for(int j = 1; j <= n; j++)
10                 dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j]);
11 }

```

## 2.6 Kruskal

```

1 // deve-se ter dsu codada com as funcoes make_set,
2 find_set e union_sets
3 struct Edge {
4     int u, v, weight;
5     bool operator<(Edge const& other) {
6         return weight < other.weight;
7     }
8 };

```

```

9 int n;
10 vector<Edge> edges;
11
12 int cost = 0;
13 vector<Edge> result;
14 for (int i = 0; i < n; i++)
15     make_set(i);
16
17 sort(edges.begin(), edges.end());
18
19 for (Edge e : edges) {
20     if (find_set(e.u) != find_set(e.v)) {
21         cost += e.weight;
22         result.push_back(e); // vector com as arestas
23         da MST
24         union_sets(e.u, e.v);
25     }
26 }

```

## 2.7 DFS

```

1 //DFS (Depth First Search) O(V+A)
2
3 void DFS(int x){
4     for(int i=0; i<(int)vizinhos[x].size(); i++){
5         int v = vizinhos[x][i];
6         if(componente[v] == -1){
7             componente[v] = componente[x];
8             DFS(v);
9         }
10    }
11 }

```

## 2.8 Kosaraju

```

1 // KOSARAJU - O(V+E) - encontra componentes
2 // g -> grafo, gt -> grafo tempo
3 // vis -> visitado, cor -> componente fortemente
4 // conexo ordenado topologicamente
5 vector<int> g[N], gt[N], S; int vis[N], cor[N];
6 void dfs(int u){
7     vis[u] = 1; for(int v : g[u]) if(!vis[v]) dfs(v);
8     S.push_back(u);
9 }
10 void dfst(int u, int e){
11     cor[u] = e;
12     for(int v : gt[u]) if(!cor[v]) dfst(v, e);
13 }
14 void kosaraju(){
15     for(int i = 1; i <= n; i++) if(!vis[i]) dfs(i);
16     for(int i = 1; i <= n; i++) for(int j : g[i])
17         gt[j].push_back(i);
18     int e = 0; reverse(S.begin(), S.end());
19     for(int u : S) if(!cor[u]) dfst(u, ++e);
20 }

```

## 2.9 Centroid

```

1 vi g[MAX];
2 int size[MAX];
3 bool erased[MAX]; // vetor dos vertices apagados na
4 // decomp.
5
6 int sz(int u, int p) {
7     int s = 1;
8     for(auto prox : g[u]) {
9         if(prox != p and !erased[prox])
10             s += sz(prox, u);
11     }
12     return size[u] = s;
13 }

```

```

13
14 int centroid(int u, int p, int n) {
15     // chamar funcao sz antes, n = size[u]
16     for(auto prox : g[u]) {
17         if(prox != p and !erased[prox]) {
18             if(size[prox] > n/2) {
19                 return centroid(prox, u, n);
20             }
21         }
22     }
23     return u;
24 }

```

## 2.10 Prim

```

1 // Prim Algorithm
2 #define MAXN 10100
3 #define INFINITO 999999999
4
5 int n, m;
6 int distancia[MAXN];
7 int processado[MAXN];
8 vector<pii> vizinhos[MAXN];
9
10 int Prim()
11 {
12     for(int i = 2; i <= n; i++) distancia[i] = INFINITO;
13     distancia[1] = 0;
14
15     priority_queue< pii, vector<pii>, greater<pii> >
16     fila;
17     fila.push( pii(distancia[1], 1) );
18
19     while(1){
20         int davez = -1;
21
22         while(!fila.empty()){
23             int atual = fila.top().second;
24             fila.pop();
25
26             if(!processado[atual]){
27                 davez = atual;
28                 break;
29             }
30         }
31
32         if(davez == -1)
33             break;
34
35         processado[davez] = true;
36
37         for(int i = 0; i < (int)vizinhos[davez].size();
38             i++){
39             int dist = vizinhos[davez][i].first;
40             int atual = vizinhos[davez][i].second;
41
42             if( distancia[atual] > dist && !
43                 processado[atual])
44             {
45                 distancia[atual] = dist;
46                 fila.push( pii(distancia[atual],
47                     atual) );
48             }
49         }
50     }
51
52     int custo_arvore = 0;
53     for(int i = 1; i <= n; i++)
54         custo_arvore += distancia[i];
55
56     return custo_arvore;
57 }

```

```

54
55 int main(){
56
57     cin >> n >> m;
58
59     for(int i = 1; i <= m; i++){
60
61         int x, y, tempo;
62         cin >> x >> y >> tempo;
63
64         vizinhos[x].pb( pii(tempo, y) );
65         vizinhos[y].pb( pii(tempo, x) );
66     }
67
68     cout << Prim() << endl;
69
70     return 0;
71 }

```

## 3 Geometria

### 3.1 Convex-polygon-intersection

```

1  cod cross(point a, point b){
2      return a.x*b.y - a.y*b.x;
3  }
4
5  int ccw(point a, point b, point e) //-1=esq; 0=
6      collinear; 1=dir;
7
8  {
9      cod tmp = cross(b-a, e-a); // from a to b
10     return (tmp > EPS) - (tmp < -EPS);
11 }
12
13 int n=4;
14 vector<point> A, B;
15
16 bool intersect()
17 {
18     A.pb(A[0]);
19     B.pb(B[0]);
20     point centerA=point();
21     point centerB=point();
22
23     for(int i=0; i<n; i++){
24         centerA=centerA+A[i];
25         centerB=centerB+B[i];
26     }
27     centerA=centerA/n;
28     centerB=centerB/n;
29     A.pb(centerA);
30     B.pb(centerB);
31
32     bool d, e;
33
34     for(int j=1; j<n+2; j++){
35     {
36         d=false, e=false;
37         for(int i=0; i<n; i++){
38             {
39                 int t = esq(A[i], A[i+1], B[j]);
40                 if(t==1) e=true;
41                 else if(t==-1) d=true;
42             }
43
44             if(!(e and d))
45                 return 1;
46         }
47     }
48

```

```

49     for(int j=1; j<n+2; j++){
50     {
51         d=false, e=false;
52         for(int i=0; i<n; i++){
53             {
54                 int t = esq(B[i], B[i+1], A[j]);
55                 if(t==1) e=true;
56                 else if(t==-1) d=true;
57             }
58
59             if(!(e and d))
60                 return 1;
61         }
62     }
63     return 0;
64 }

```

### 3.2 Angle-adjacent-vertices-regular-polygon

$$a = 180/N$$

### 3.3 Inter-Retas

```

1  // Intersection between lines
2
3  typedef struct
4  {
5      int x, y;
6  } pnt;
7
8  bool collinear(pnt p, pnt q, pnt r)
9  {
10     if(q.x<=max(p.x,r.x) && q.x>=min(p.x,r.x) && q.y
11     <=max(p.y,r.y) && q.y>=min(p.y,r.y))
12         return true;
13
14     return false;
15 }
16
17 int orientation(pnt p, pnt q, pnt r)
18 {
19     int val=(q.y-p.y)*(r.x-q.x)-(q.x-p.x)*(r.y-q.y);
20
21     if(val==0)
22         return 0;
23     else if(val>0)
24         return 1;
25     else
26         return 2;
27 }
28
29 bool intersect(pnt p1, pnt q1, pnt p2, pnt q2)
30 {
31     int o1 = orientation(p1, q1, p2);
32     int o2 = orientation(p1, q1, q2);
33     int o3 = orientation(p2, q2, p1);
34     int o4 = orientation(p2, q2, q1);
35
36     if(o1!=o2 and o3!=o4)
37         return true;
38
39     if(o1==0 && collinear(p1, p2, q1))
40         return true;
41
42     if(o2==0 && collinear(p1, q2, q1))
43         return true;
44
45     if(o3==0 && collinear(p2, p1, q2))
46         return true;
47
48     if(o4==0 && collinear(p2, q1, q2))
49         return true;
50 }

```

```

49     return false;
50 }
51
52 }

```

### 3.4 Pick's-theorem

- The area of a polygon with integer coordinates:  $A = i + \frac{b}{2} - 1$
- $i$  is the number of points inside the polygon;
- $b$  is the number of points on the boundary;
- $2A$  is necessarily an integer value.

### 3.5 linesweep

```

1 typedef pair<double, double> dd;
2
3 bool compare(dd a, dd b){
4     return a.st < b.st;
5 }
6
7 double closest(dd v[], int n){
8     sort(v, v+n, compare);
9     double best = FLT_MAX;
10    set<dd> box;
11    box.insert(v[0]);
12    int left = 0;
13    rep2(i, 1, n){
14        while(left < i && v[i].st-v[left].st > best){
15            box.erase(v[left++]);
16        }
17        for(set<dd>::iterator it = box.lower_bound(mp
(v[i].nd-best, v[i].st-best)); it!=box.end() && v[
i].nd + best >= it->nd; it++){
18            best = min(best, sqrt(pow(v[i].nd - it->
nd, 2.0) + pow(v[i].st - it->st, 2.0)));
19        }
20        box.insert(v[i]);
21    }
22    return best;
23 }

```

### 3.6 Sort-by-Angle

```

1 int quarter(point a)
2 {
3     if(a.x>=0 and a.y>0) return 0;
4     if(a.x<0 and a.y>=0) return 1;
5     if(a.x<=0 and a.y<0) return 2;
6     return 3;
7 }
8
9 bool comp(point a, point b)
10 {
11     int qa = quarter(a);
12     int qb = quarter(b);
13     if(qa==qb)
14         return cross(a,b)>0;
15     else
16         return quarter(a)<quarter(b);
17 }

```

### 3.7 Cross-properties

- It equals zero if the vectors **a** and **b** are collinear (coplanar in triple product).
- It is negative if the rotation from the first to the second vector is clockwise and positive otherwise.

## 3.8 Inter-Retangulos

```

1 typedef struct
2 {
3     int x, y;
4 } Point;
5
6 bool doOverlap(Point l1, Point r1, Point l2, Point r2)
7 {
8     if (l1.x>r2.x or l2.x>r1.x or l1.y<r2.y or l2.y<
r1.y)
9         return false;
10    return true;
11 }

```

### 3.9 Heron

$$A_{\text{triangulo}} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A_{\text{quadrilatero}} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

### 3.10 3D

```

1 typedef ld cod;
2
3 bool eq(cod a, cod b){ return abs(a - b) <= EPS; }
4
5 struct point
6 {
7     cod x, y, z;
8     point(cod x=0, cod y=0, cod z=0): x(x), y(y), z(z)
9     {}
10
11     point operator+(const point &o) const{
12         return {x+o.x, y+o.y, z+o.z};
13     }
14     point operator-(const point &o) const{
15         return {x-o.x, y-o.y, z-o.z};
16     }
17     point operator*(cod t) const{
18         return {x*t, y*t, z*t};
19     }
20     point operator/(cod t) const{
21         return {x/t, y/t, z/t};
22     }
23     bool operator==(const point &o) const{
24         return eq(x, o.x) and eq(y, o.y) and eq(z, o.
z);
25     }
26 };
27
28 // Produto Escalar
29 cod dot(point a, point b){
30     return a.x*b.x + a.y*b.y + a.z*b.z;
31 }
32
33 // Produto Vetorial
34 point cross(point a, point b){
35     return point(a.y*b.z - a.z*b.y,
36                 a.z*b.x - a.x*b.z,
37                 a.x*b.y - a.y*b.x);
38 }
39
40 ld abs(point a){ // Modulo
41     return sqrt(dot(a, a));
42 }
43
44 ld proj(point a, point b){ // a sobre b
45     return dot(a, b)/abs(b);
46 }

```

```

45 ld angle(point a, point b){ // em radianos
46     return acos(dot(a, b) / abs(a) / abs(b));
47 }
48
49 cod triple(point a, point b, point c){
50     return dot(a, cross(b, c)); // Area do
    paralelepipedo
51 }

```

### 3.11 Dot-properties

- Length of **a**:  $|\mathbf{a}| = \sqrt{\mathbf{a} \cdot \mathbf{a}}$ .
- Projection of **a** onto **b**:  $\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$ .
- Angle between vectors:  $\arccos\left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| \cdot |\mathbf{b}|}\right)$ .
- The dot product is positive if the angle between them is acute, negative if it is obtuse and it equals zero if they are orthogonal, i.e. they form a right angle.

### 3.12 Minkowski-Sum

```

1 vector<point> mk(const vector<point> &a, const vector<
    point> &b){
2     int i = 0, j = 0;
3     for(int k = 0; k < (int)a.size(); k++){ if(a[k]<a[i
    ]])
4         i = k;
5     for(int k = 0; k < (int)b.size(); k++){ if(b[k]<b[j
    ]])
6         j = k;
7
8     vector<point> c;
9     c.reserve(a.size() + b.size());
10    for(int k = 0; k < int(a.size()+b.size()); k++){
11        point pt{a[i] + b[j]};
12        if((int)c.size() >= 2 and !ccw(c[c.size()-2],
            c.back(), pt))
13            c.pop_back();
14        c.pb(pt);
15        int q = i+1, w = j+1;
16        if(q == int(a.size())) q = 0;
17        if(w == int(b.size())) w = 0;
18        if(ccw(c.back(), a[i]+b[w], a[q]+b[j]) < 0) i
            = q;
19        else j = w;
20    }
21
22    if(!ccw(c[0], c[(int)c.size()-1], c[(int)c.size()
        -2]))
23        c.pop_back();
24    if(!ccw(c.back(), c[0], c[1])){
25        c[0]=c.back();
26        c.pop_back();
27    }
28    c.shrink_to_fit();
29
30    return c;
31 }

```

### 3.13 2D

```

1 typedef ld cod;
2 bool eq(cod a, cod b){ return fabs1(a - b) <= EPS; }
3
4 // typedef int cod;
5 // bool eq(cod a, cod b){ return (a==b); }
6
7 struct point

```

```

8 {
9     cod x, y;
10    int id;
11    point(cod x=0, cod y=0): x(x), y(y){}
12
13
14    point operator+(const point &o) const{
15        return {x+o.x, y+o.y};
16    }
17    point operator-(const point &o) const{
18        return {x-o.x, y-o.y};
19    }
20    point operator*(cod t) const{
21        return {x*t, y*t};
22    }
23    point operator/(cod t) const{
24        return {x/t, y/t};
25    }
26    bool operator<(const point &o) const{
27        if(!eq(x, o.x)) return x < o.x;
28        return y < o.y;
29    }
30    bool operator==(const point &o) const{
31        return eq(x, o.x) and eq(y, o.y);
32    }
33
34 };
35
36 struct line
37 {
38     point fp, sp;
39     point(point fp=0, point sp=0): fp(fp), sp(sp){}
40
41     //a=y1-y2;
42     //b=x2-x1;
43     //c=x2*y1-y2*x1;
44
45 };
46
47 // Produto Escalar
48 cod dot(point a, point b){
49     return a.x*b.x + a.y*b.y;
50 }
51
52 // Produto Vetorial
53 cod cross(point a, point b){
54     return a.x*b.y - a.y*b.x;
55 }
56
57 ld norm(point a){ // Modulo
58     return sqrt(dot(a, a));
59 }
60
61 ld proj(point a, point b){ // a sobre b
62     return dot(a, b)/norm(b);
63 }
64
65 ld angle(point a, point b){ // em radianos
66     return acos(dot(a, b) / norm(a) / norm(b));
67 }
68
69 int ccw(point a, point b, point e) //-1=esq; 0=
    collinear; 1=dir;
70
71 {
72     cod tmp = cross(b-a, e-a); // from a to b
73     return (tmp > EPS) - (tmp < -EPS);
74 }
75
76 bool collinear(point a, point b, point c){
77     return eq(cross(a-c, b-c), 0);
78 }
79
80 point rotccw(point p, ld a) // em radianos
81 {
82     //a = a*acos(0.0)/90; // graus
83     return point((p.x*cos(a)-p.y*sin(a)), (p.y*cos(a)

```

```

    +p.x*sin(a));
80 }
81
82 point rot90cw(point a) { return {a.y, -a.x} };
83 point rot90ccw(point a) { return {-a.y, a.x} };
84
85 // Area de um poligono (pontos ordenados por
86 // adjacencia)
87 ld area(vector <point> p){
88     ld ret = 0;
89     for(int i=2;i<(int)p.size();i++)
90         ret += cross(p[i] - p[0], p[i-1] - p[0]);
91     return fabsl(ret/2);
92 }
93
94 // Dist entre ponto e reta
95 cod distr(point a, line b){
96     cod crs = cross(point(a - b.fp), point(b.sp - b.
97     fp));
98     return norm(crs/dist(b.fp, b.sp));
99 }

```

## 4 ED

### 4.1 Range-query-bigger-than-k-BIT

```

1 // C++ program to print the number of elements
2 // greater than k in a subarray of range L-R.
3 #include <bits/stdc++.h>
4 using namespace std;
5
6 // Structure which will store both
7 // array elements and queries.
8 struct node{
9     int pos;
10    int l;
11    int r;
12    int val;
13 };
14
15 // Boolean comparator that will be used
16 // for sorting the structural array.
17 bool comp(node a, node b){
18     // If 2 values are equal the query will
19     // occur first then array element
20     if (a.val == b.val)
21         return a.l > b.l;
22
23     // Otherwise sorted in descending order.
24     return a.val > b.val;
25 }
26
27 // Updates the node of BIT array by adding
28 // 1 to it and its ancestors.
29 void update(int* BIT, int n, int idx){
30     while (idx <= n){
31         BIT[idx]++;
32         idx += idx & (-idx);
33     }
34 }
35
36 // Returns the count of numbers of elements
37 // present from starting till idx.
38 int query(int* BIT, int idx){
39     int ans = 0;
40     while (idx){
41         ans += BIT[idx];
42         idx -= idx & (-idx);
43     }
44     return ans;
45 }

```

```

46
47 // Function to solve the queries offline
48 void solveQuery(int arr[], int n, int QueryL[],
49                 int QueryR[], int QueryK[], int q){
50     // create node to store the elements
51     // and the queries
52     node a[n + q + 1];
53     // 1-based indexing.
54
55     // traverse for all array numbers
56     for(int i = 1; i <= n; ++i){
57         a[i].val = arr[i - 1];
58         a[i].pos = 0;
59         a[i].l = 0;
60         a[i].r = i;
61     }
62
63     // iterate for all queries
64     for(int i = n + 1; i <= n + q; ++i){
65         a[i].pos = i - n;
66         a[i].val = QueryK[i - n - 1];
67         a[i].l = QueryL[i - n - 1];
68         a[i].r = QueryR[i - n - 1];
69     }
70
71     // In-built sort function used to
72     // sort node array using comp function.
73     sort(a + 1, a + n + q + 1, comp);
74
75     // Binary Indexed tree with
76     // initially 0 at all places.
77     int BIT[n + 1];
78
79     // initially 0
80     memset(BIT, 0, sizeof(BIT));
81
82     // For storing answers for each query( 1-based
83     // indexing ).
84     int ans[q + 1];
85
86     // traverse for numbers and query
87     for (int i = 1; i <= n + q; ++i){
88         if (a[i].pos != 0) {
89             // call function to returns answer for
90             // each query
91             int cnt = query(BIT, a[i].r) - query(BIT,
92             a[i].l - 1);
93
94             // This will ensure that answer of each
95             // query
96             // are stored in order it was initially
97             // asked.
98             ans[a[i].pos] = cnt;
99         }
100         else{
101             // a[i].r contains the position of the
102             // element in the original array.
103             update(BIT, n, a[i].r);
104         }
105     }
106
107     // Output the answer array
108     for (int i = 1; i <= q; ++i){
109         cout << ans[i] << endl;
110     }
111 }
112
113 // Driver Code
114 int main()
115 {
116     int arr[] = { 7, 3, 9, 13, 5, 4 };
117     int n = sizeof(arr) / sizeof(arr[0]);
118 }

```



```

114 // 1-based indexing
115 int QueryL[] = { 1, 2 };
116 int QueryR[] = { 4, 6 };
117
118 // k for each query
119 int QueryK[] = { 6, 8 };
120
121 // number of queries
122 int q = sizeof(QueryL) / sizeof(QueryL[0]);
123
124 // Function call to get
125 solveQuery(arr, n, QueryL, QueryR, QueryK, q);
126
127 return 0;
128 }

```

## 4.2 Iterative-SegTree

```

1 // Segment Tree Iterativa - Range maximum query
2
3 #define N 100010
4
5 struct Segtree{
6     int t[2*N]={0};
7
8     void build(){
9         for(int i=N-1; i>0; i--){
10             t[i]=max(t[i<<1], t[i<<1|1]);
11         }
12
13     int query(int l, int r){
14         int ans=0;
15         for(i+=N, r+=N; l<r; l>>=1, r>>=1)
16         {
17             if(l&1)
18                 ans=max(ans, t[l++]);
19             if(r&1)
20                 ans=max(ans, t[--r]);
21         }
22
23         return ans;
24     }
25
26     void update(int p, int value){
27         for(t[p+=N]=value; p>1; p>>=1)
28             t[p>>1]= max(t[p], t[p^1]);
29     }
30
31 };
32
33 int main()
34 {
35     Segtree st;
36
37     for(int i=0; i<n; i++){
38         cin >> aux;
39         st.t[N+i]=aux; //Leaves are stored in
40         continuous nodes with indices starting with N
41     }
42
43     st.build();
44     x = st.query(inicio, fim);
45     st.update(ind, value);
46 }

```

## 4.3 Recursive-SegTree

```

1 // Segment Tree Recursiva - Range maximum query
2
3 vector<int> val(MAX, 0);
4 vector<int> vet(N);

```

```

5
6 void monta(int i, int j, int no){
7     if(i==j){
8         val[no]=vet[i];
9         return;
10    }
11
12    int esq = 2*no;
13    int dir = 2*no+1;
14    int meio = (i+j)/2;
15
16    monta(i, meio, esq);
17    monta(meio+1, j, dir);
18
19    val[no]=max(val[esq], val[dir]);
20 }
21
22 void atualiza(int no, int i, int j, int pos, int
23     novo_valor){
24     if(i==j){
25         val[no]=novo_valor;
26     }else{
27         int esq = 2*no;
28         int dir = 2*no+1;
29         int meio = (i+j)/2;
30
31         if(pos<=meio)
32             atualiza(esq, i, meio, pos, novo_valor);
33         else
34             atualiza(dir, meio+1, j, pos, novo_valor)
35
36         ;
37
38         if(val[esq]>val[dir])
39             val[no]=val[esq];
40         else
41             val[no]=val[dir];
42     }
43 }
44
45 int consulta(int no, int i, int j, int A, int B){
46     if(i>B || j<A)
47         return -1;
48     if(i>=A and j<=B)
49         return val[no];
50
51     int esq = 2*no;
52     int dir = 2*no+1;
53     int meio = (i+j)/2;
54
55     int resp_esq = consulta(esq, i, meio, A, B);
56     int resp_dir = consulta(dir, meio+1, j, A, B);
57
58     if(resp_dir==-1)
59         return resp_esq;
60     if(resp_esq==-1)
61         return resp_dir;
62
63     if(resp_esq>resp_dir)
64         return resp_esq;
65     else
66         return resp_dir;
67 }
68
69 int main()
70 {
71     monta(1, N, 1);
72     atualiza(1, 1, N, pos, valor);
73     x = consulta(1, 1, N, inicio, fim);
74 }

```

## 4.4 Delta-Encoding

```

1 // Delta encoding

```

```

2
3 for(int i=0;i<q;i++){
4     int l,r,x;
5     cin >> l >> r >> x;
6     delta[l] += x;
7     delta[r+1] -= x;
8 }
9
10 int atual = 0;
11
12 for(int i=0;i<n;i++){
13     atual += delta[i];
14     v[i] += atual;
15 }

```

## 4.5 Seg-Tree-Farao

```

1 typedef struct
2 {
3     pii prefix, sufix, total, maximo;
4 } no;
5
6 int noleft[MAX], noright[MAX]; //Guarda os valores
7                                 dos nos para que nao sejam calculados novamente
8                                 nas queries
9
10 int v[MAX];
11 no arvore[MAX];
12
13 pii somar(pii a, pii b) // une pairs
14 {
15     return mp(a.f+b.f, a.s+b.s);
16 }
17
18 no une(no l, no r)
19 {
20     if(l.total.s==0)
21         return r;
22     if(r.total.s==0)
23         return l;
24
25     no m;
26
27     m.prefix = max(l.prefix, somar(l.total, r.prefix));
28     //prefixo
29     m.sufix = max(r.sufix, somar(r.total, l.sufix));
30     //sufixo
31     m.total = somar(l.total, r.total); //Soma de
32     //todos os elementos da subarvore
33     m.maximo = max(max(l.maximo, r.maximo), somar(l.sufix, r.prefix)); //Resultado para cada
34     //subarvore
35
36     return m;
37 }
38
39 no makenozero()
40 {
41     no m;
42     m.prefix=m.sufix=m.total=m.maximo=mp(0,0);
43     return m;
44 }
45
46 void monta(int n)
47 {
48     if(noleft[n]==noright[n])
49     {

```

```

50         arvore[n]=makeno(v[noleft[n]]);
51         return;
52     }
53
54     int mid = (noleft[n]+noright[n])/2;
55     noleft[2*n]=noleft[n]; noright[2*n]=mid;
56     noleft[2*n+1]=mid+1; noright[2*n+1]=noright[n];
57
58     monta(2*n);
59     monta(2*n+1);
60
61     arvore[n]=une(arvore[2*n], arvore[2*n+1]);
62 }
63
64 no busca(int n, int esq, int dir)
65 {
66     if(noleft[n]>=esq and noright[n]<=dir)
67         return arvore[n];
68     if(noright[n]<esq or noleft[n]>dir)
69         return makenozero();
70
71     return une(busca(2*n, esq, dir),busca(2*n+1, esq,
72         dir));
73 }
74
75 int main()
76 {
77     int T, N, Q, A, B;
78     no aux;
79
80     scanf("%d", &T);
81
82     while(T--)
83     {
84         scanf("%d", &N);
85         for(int i=1;i<=N;i++)
86             scanf("%d", &v[i]); //Elementos da arvore
87
88         noleft[1]=1;noright[1]=N;
89         monta(1);
90
91         cin >> Q;
92         while(Q--)
93         {
94             scanf("%d%d", &A, &B); //Intervalo da
95             //query
96             aux = busca(1, A, B);
97             printf("%d %d\n", aux.maximo.f, aux.
98                 maximo.s);
99         }
100     }
101
102     return 0;

```

## 4.6 BIT-2D

```

1 // BIT 2D
2
3 int bit[MAX][MAX];
4
5 int sum(int x, int y)
6 {
7     int resp=0;
8
9     for(int i=x;i>0;i-=i&-i)
10         for(int j=y;j>0;j-=j&-j)
11             resp+=bit[i][j];
12
13     return resp;
14 }
15

```

```

16 void update(int x, int y, int delta)
17 {
18     for(int i=x; i<MAX; i+=i&-i)
19         for(int j=y; j<MAX; j+=j&-j)
20             bit[i][j] += delta;
21 }
22
23 int query(int x1, y1, x2, y2)
24 {
25     return sum(x2, y2) - sum(x2, y1) - sum(x1, y2) + sum
26         (x1, y1);
27 }

```

## 4.7 BIT

```

1 struct FT {
2     vector<int> bit; // indexado em 1
3     int n;
4
5     FT(int n) {
6         this->n = n + 1;
7         bit.assign(n + 1, 0);
8     }
9
10    int sum(int idx) {
11        int ret = 0;
12        for (++idx; idx > 0; idx -= idx & -idx)
13            ret += bit[idx];
14        return ret;
15    }
16
17    int sum(int l, int r) {
18        return sum(r) - sum(l - 1);
19    }
20
21    void add(int idx, int delta) {
22        for (++idx; idx <= n; idx += idx & -idx)
23            bit[idx] += delta;
24    }
25 };

```

## 4.8 Sparse-Table

```

1 int logv[MAX+1];
2 void make_log() {
3     logv[1] = 0; // pre-computar tabela de log
4     for (int i = 2; i <= MAX; i++)
5         logv[i] = logv[i/2] + 1;
6 }
7
8 struct Sparse {
9     int n;
10    vector<vector<int>> st;
11
12    Sparse(int n, vi array) {
13        this->n = n;
14        int k = logv[n];
15        st.assign(n+1, vector<int>(k+1, 0));
16
17        for (int i = 0; i < n; i++)
18            st[i][0] = array[i];
19
20        for (int j = 1; j <= k; j++)
21            for (int i = 0; i + (1 << j) <= n; i++)
22                st[i][j] = f(st[i][j-1], st[i + (1 <<
23                    (j - 1))][j - 1]);
24    }
25
26    int f(int a, int b) {
27        return min(a, b);
28    }
29 }

```

```

29 int query(int L, int R) {
30     int j = logv[R - L + 1];
31     int res = f(st[L][j], st[R - (1 << j) + 1][j]);
32     return res;
33 }
34 };

```

## 4.9 Union-Find

```

1 struct DSU {
2     int n;
3     vector<int> parent, size;
4
5     DSU(int n) {
6         this->n = n;
7         parent.assign(n+1, 0);
8         size.assign(n+1, 1);
9
10        for(int i = 0; i <= n; i++)
11            parent[i] = i;
12    }
13
14    int find(int v) {
15        if(v == parent[v])
16            return v;
17        return find(parent[v]);
18    }
19
20    void join(int a, int b) {
21        a = find(a);
22        b = find(b);
23
24        if(a != b) {
25            if(size[a] < size[b])
26                swap(a, b);
27
28            parent[b] = a;
29            size[a] += size[b];
30        }
31    }
32 };

```

## 4.10 Mo

```

1 const int BLK = 500; // tamanho do bloco, algo entre
2     300 e 500 e nice
3
4 struct Query {
5     int l, r, idx;
6     bool operator<(Query other) const
7     {
8         return make_pair(l / BLK, r) <
9             make_pair(other.l / BLK, other.r);
10    };
11
12    void add(); void remove() // implementar operacoes de
13        acordo com o problema, cuidado com TLE ao
14        utilizar MAP
15
16    vector<pair<int, ll>> mo() {
17        vector<pair<int, ll>> res;
18        sort(queries.begin(), queries.end());
19
20        int l = 0, r = -1;
21        for(Query q : queries) {
22            while(l > q.l) {
23                l--;
24                add(l);
25            }
26            while(r < q.r) {
27                r++;
28                add(r);
29            }
30        }
31    }
32 };

```

```

25         r++;
26         add(r);
27     }
28     while(l < q.l) {
29         remove(l);
30         l++;
31     }
32     while(r > q.r) {
33         remove(r);
34         r--;
35     }
36     res.pb(mp(q.idx, RESPOSTA)); // adicionar
37     resposta de acordo com o problema
38     return res; // ordenar o vetor pelo indice e
39     responder queries na ordem

```

## 5 Math

### 5.1 Totient

```

1 // phi(p^k) = (p^(k-1))*(p-1) com p primo
2 // 0(sqrt(m))
3 ll phi(ll m) {
4     ll res = m;
5     for(ll d = 2; d*d <= m; d++) {
6         if(m % d == 0) {
7             res = (res/d) * (d-1);
8             while(m % d == 0) {
9                 m /= d;
10            }
11        }
12    }
13    if(m > 1) {
14        res /= m;
15        res *= (m-1);
16    }
17    return res;
18 }
19 }
20
21 // modificacao do crivo, 0(n*log(log(n)))
22 vector<ll> phi_to_n(ll n) {
23     vector<bool> isprime(n+1, true);
24     vector<ll> tot(n+1);
25     tot[0] = 0; tot[1] = 1;
26     for(ll i = 1; i <= n; i++) {
27         tot[i] = i;
28     }
29
30     for(ll p = 2; p <= n; p++) {
31         if(isprime[p]) {
32             tot[p] = p-1;
33             for(ll i = p+p; i <= n; i += p) {
34                 isprime[i] = false;
35                 tot[i] = (tot[i]/p)*(p-1);
36             }
37         }
38     }
39
40     return tot;
41 }

```

### 5.2 Sqrt-BigInt

```

1 public static BigInteger isqrtNewton(BigInteger n) {
2     BigInteger a = BigInteger.ONE.shiftLeft(n.
3         bitLength() / 2);
4     boolean p_dec = false;
5     for (;;) {

```

```

5         BigInteger b = n.divide(a).add(a).shiftRight
6         (1);
7         if (a.compareTo(b) == 0 || a.compareTo(b) < 0
8             && p_dec)
9             break;
10        p_dec = a.compareTo(b) > 0;
11        a = b;
12    }
13    return a;

```

### 5.3 Linear-Diophantine-Equation

```

1 // Linear Diophantine Equation
2 int gcd(int a, int b, int &x, int &y)
3 {
4     if (a == 0)
5     {
6         x = 0; y = 1;
7         return b;
8     }
9     int x1, y1;
10    int d = gcd(b%a, a, x1, y1);
11    x = y1 - (b / a) * x1;
12    y = x1;
13    return d;
14 }
15
16 bool find_any_solution(int a, int b, int c, int &x0,
17     int &y0, int &g)
18 {
19     g = gcd(abs(a), abs(b), x0, y0);
20     if (c % g)
21         return false;
22
23     x0 *= c / g;
24     y0 *= c / g;
25     if (a < 0) x0 = -x0;
26     if (b < 0) y0 = -y0;
27     return true;
28 }
29
30 // All solutions
31 // x = x0 + k*b/g
32 // y = y0 - k*a/g

```

### 5.4 Sum-n2

Soma dos n primeiros números ao quadrado =  $\frac{(2N^3+3N^2+N)}{6}$

### 5.5 Factorization-sqrt

```

1 // Factorization of a number in sqrt(n)
2
3 int main()
4 {
5     ll N;
6     vector<int> div;
7
8     cin >> N;
9
10    for(ll i=2;i*i<=N;i++)
11    {
12        if(N%i==0)
13        {
14            vet.pb(i);
15            while(N%i==0)
16                N/=i;
17        }
18    }
19    if(N!=1)

```

```

20         vet.pb(N);
21
22     return 0;
23 }

```

## 5.6 Modular-Exponentiation

```

1 ll fexp(ll b, ll e, ll mod) {
2     if(e == 0) return 1LL;
3     ll res = fexp(b, e/2LL, mod);
4     res = (res*res)%mod;
5     if(e%2LL)
6         res = (res*b)%mod;
7
8     return res%mod;
9 }

```

## 5.7 Miller-Habin

```

1 ll llrand()
2 {
3     ll tmp = rand();
4     return (tmp << 31) | rand();
5 }
6
7 ll add(ll a, ll b, ll c)
8 {
9     return (a + b)%c;
10 }
11
12 ll mul(ll a, ll b, ll c)
13 {
14     ll ans = 0;
15     while(b)
16     {
17         if(b & 1)
18             ans = add(ans, a, c);
19         a = add(a, a, c);
20         b /= 2;
21     }
22     return ans;
23 }
24
25 ll fexp(ll a, ll b, ll c)
26 {
27     ll ans = 1;
28     while(b)
29     {
30         if(b & 1)
31             ans = mul(ans, a, c);
32         a = mul(a, a, c);
33         b /= 2;
34     }
35     return ans;
36 }
37
38 bool rabin(ll n)
39 {
40     if(n <= 1)
41         return 1;
42     if(n <= 3)
43         return 1;
44
45     ll s=0, d=n-1;
46     while(d%2==0)
47     {
48         d/=2;
49         s++;
50     }
51
52     for(int k = 0; k < 64*4; k++)
53     {

```

```

54         ll a = (llrand()%(n - 3)) + 2;
55         ll x = fexp(a, d, n);
56         if(x != 1 and x != n-1)
57         {
58             for(int r = 1; r < s; r++)
59             {
60                 x = mul(x, x, n);
61                 if(x == 1)
62                     return 0;
63                 if(x == n-1)
64                     break;
65             }
66             if(x != n-1)
67                 return 0;
68         }
69     }
70
71     return 1;
72 }
73
74
75 int main()
76 {
77
78     ll N;
79     cin >> N;
80
81     cout << rabin(N) << endl;
82
83     return 0;
84 }
85 }

```

## 5.8 Inverso-Mult

```

1 // gcd(a, m) = 1 para existir solucao
2 // ax + my = 1, ou a*x = 1 (mod m)
3 ll inv(ll a, ll m) { // com gcd
4     ll x, y;
5     gcd(a, m, x, y);
6     return ((x % m) + m) % m;
7 }
8
9 ll inv(ll a, ll phim) { // com phi(m), se m for primo
10     entao phi(m) = p-1
11     ll e = phim-1;
12     return fexp(a, e);
13 }

```

## 5.9 Pollard-Rho

```

1 // Pollard Rho Algorithm
2
3 #include <bits/stdc++.h>
4 #define ll long long
5
6 using namespace std;
7
8 ll llrand()
9 {
10     ll tmp = rand();
11     return (tmp << 31) | rand();
12 }
13
14 ll add(ll a, ll b, ll c)
15 {
16     return (a + b)%c;
17 }
18
19 ll mul(ll a, ll b, ll c)
20 {
21     ll ans = 0;

```

```

22 while(b)
23 {
24     if(b & 1)
25         ans = add(ans, a, c);
26     a = add(a, a, c);
27     b /= 2;
28 }
29 return ans;
30 }
31
32 ll rho(ll n)
33 {
34     ll x, c, y, d, k;
35     int i;
36     do{
37         i = 1;
38         x = llrand()%n;
39         c = llrand()%n;
40         y = x, k = 4;
41         do{
42             if(++i == k)
43             {
44                 y = x;
45                 k *= 2;
46             }
47             x = add(mul(x, x, n), c, n);
48             d = __gcd(abs(x - y), n);
49         }
50         while(d == 1);
51     }
52     while(d == n);
53
54     return d;
55 }
56
57 int main()
58 {
59     srand(time(0));
60
61     ll N;
62     cin >> N;
63
64     ll div = rho(N);
65     cout << div << " " << N/div << endl;
66
67     // Finding all divisors
68
69     vector<ll> div;
70
71     while(N>1 and !rabin(N))
72     {
73         ll d = rho(N);
74         div.pb(d);
75         while(N%d==0)
76             N/=d;
77     }
78     if(N!=1)
79         div.pb(N);
80
81     return 0;
82 }

```

## 5.10 Verif-primo

```

1 // prime verification sqrt(N)
2
3 bool eh_primo(long long N)
4 {
5     if(N==2)
6         return true;
7     else if(N==1 or N%2==0)

```

```

8         return false;
9     for(long long i=3;i*i<=N;i+=2)
10         if(N%i==0)
11             return false;
12     return true;
13 }

```

## 5.11 Crivo

```

1 // Sieve of Eratosthenes
2
3 int N;
4 vector<bool> primos(100010, true);
5 cin >> N;
6
7 primos[0]=false;
8 primos[1]=false;
9
10 for(int i=2;i<=N;i++)
11     if(primos[i])
12         for(int j=i+i; j<=N; j+=i)
13             primos[j]=false;

```

## 5.12 Simpson's-formula

```

1 inline ld simpson(ld fl, ld fr, ld fmid, ld l, ld r){
2     return (fl+fr+4*fmid)*(r-l)/6;
3 }
4
5 ld rsimpson(ld slr, ld fl, ld fr, ld fmid, ld l, ld r
6 )
7 {
8     ld mid = (l+r)/2;
9     ld fml = f((l+mid)/2), fmr = f((mid+r)/2);
10    ld slm = simpson(fl,fmid,fml,l,mid);
11    ld smr = simpson(fmid,fr,fmr,mid,r);
12    if(fabs(slr-slm-smr) < EPS) return slm+smr; //
13    aprox. good enough
14    return rsimpson(slm,fl,fmid,fml,l,mid)+rsimpson(
15        smr,fmid,fr,fmr,mid,r);
16 }
17
18 ld integrate(ld l, ld r)
19 {
20     ld mid = (l+r)/2;
21     ld fl = f(l), fr = f(r);
22     ld fmid = f(mid);
23     return rsimpson(simpson(fl,fr,fmid,l,r),fl,fr,
24         fmid,l,r);
25 }

```

## 5.13 FFT

```

1 struct num{
2     double x, y;
3     num() { x = y = 0; }
4     num(double x, double y) : x(x), y(y) {}
5 };
6
7 inline num operator+(num a, num b) { return num(a.x +
8     b.x, a.y + b.y); }
9 inline num operator-(num a, num b) { return num(a.x -
10     b.x, a.y - b.y); }
11 inline num operator*(num a, num b) { return num(a.x *
12     b.x - a.y * b.y, a.x * b.y + a.y * b.x); }
13 inline num conj(num a) { return num(a.x, -a.y); }
14
15 int base = 1;
16 vector<num> roots = {{0, 0}, {1, 0}};
17 vector<int> rev = {0, 1};
18
19 const double PI = acos(-1.0);

```

```

17
18 void ensure_base(int nbase){
19     if(nbase <= base)
20         return;
21
22     rev.resize(1 << nbase);
23     for(int i = 0; i < (1 << nbase); i++)
24         rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (
nbase - 1));
25
26     roots.resize(1 << nbase);
27
28     while(base < nbase){
29         double angle = 2*PI / (1 << (base + 1));
30         for(int i = 1 << (base - 1); i < (1 << base);
i++){
31             roots[i << 1] = roots[i];
32             double angle_i = angle * (2 * i + 1 - (1
<< base));
33             roots[(i << 1) + 1] = num(cos(angle_i),
sin(angle_i));
34         }
35         base++;
36     }
37 }
38
39 void fft(vector<num> &a, int n = -1){
40     if(n == -1)
41         n = a.size();
42
43     assert((n & (n-1)) == 0);
44     int zeros = __builtin_ctz(n);
45     ensure_base(zeros);
46     int shift = base - zeros;
47     for(int i = 0; i < n; i++)
48         if(i < (rev[i] >> shift))
49             swap(a[i], a[rev[i] >> shift]);
50
51     for(int k = 1; k < n; k <= 1)
52         for(int i = 0; i < n; i += 2 * k)
53             for(int j = 0; j < k; j++){
54                 num z = a[i+j+k] * roots[j+k];
55                 a[i+j+k] = a[i+j] - z;
56                 a[i+j] = a[i+j] + z;
57             }
58 }
59
60 vector<num> fa, fb;
61 vector<int> multiply(vector<int> &a, vector<int> &b){
62     int need = a.size() + b.size() - 1;
63     int nbase = 0;
64     while((1 << nbase) < need) nbase++;
65     ensure_base(nbase);
66     int sz = 1 << nbase;
67     if(sz > (int) fa.size())
68         fa.resize(sz);
69
70     for(int i = 0; i < sz; i++){
71         int x = (i < (int) a.size() ? a[i] : 0);
72         int y = (i < (int) b.size() ? b[i] : 0);
73         fa[i] = num(x, y);
74     }
75     fft(fa, sz);
76     num r(0, -0.25 / sz);
77     for(int i = 0; i <= (sz >> 1); i++){
78         int j = (sz - i) & (sz - 1);
79         num z = (fa[j] * fa[j] - conj(fa[i] * fa[i]))
* r;
80         if(i != j) {
81             fa[j] = (fa[i] * fa[i] - conj(fa[j] * fa[
j])) * r;
82         }
83         fa[i] = z;

```

```

84     }
85     fft(fa, sz);
86     vector<int> res(need);
87     for(int i = 0; i < need; i++)
88         res[i] = fa[i].x + 0.5;
89
90     return res;
91 }
92
93 int main()
94 {sws;
95
96     //FFT
97     vector<int> fx{1, 2, 3}; // 1+2x+3x^2
98     vector<int> gx{4, 5}; // 4+5x
99     vector<int> res;
100
101     res = multiply(fx,gx); //4 + 13x + 22x^2 + 15x^3
102
103     return 0;
104 }
105 }

```

## 5.14 Next-Permutation

```

1 vector<int> a = {1, 2, 3};
2 int n = a.size();
3 do{
4     display(a, n); // 1,2,3; 1,3,2; 2,1,3; 3,1,2;
5     2,3,1; 3,2,1;
6 }while(next_permutation(a.begin(), a.begin() + n));

```

## 5.15 Fast-Exponentiation

```

1 // Modular exponentiaion - (x^y)%mod in O(log y)
2 ll power(ll x, ll y, ll mod)
3 {
4     ll res = 1;
5     x%=mod;
6
7     while(y)
8     {
9         if(y&1)
10             res=(res*x)%mod;
11
12         y=y>>1;
13         x=(x*x)%mod;
14     }
15     return res;
16 }

```

## 5.16 Recursao-linear

```

1 vector<vl> id(int n) {
2     vector<vl> res(n, vl(n, 0));
3     for(int i = 0; i < n; i++) res[i][i] = 1;
4     return res;
5 }
6
7 vector<vl> mult(vector<vl> a, vector<vl> b, int n) {
8     vector<vl> res(n, vl(n, 0));
9
10    for(int row = 0; row < n; row++) {
11        for(int col = 0; col < n; col++) {
12            ll val = 0;
13            for(int k = 0; k < n; k++) {
14                ll delta = (a[row][k] * b[k][col]) %
MOD;
15                val = (val + delta) % MOD;
16            }
17            res[row][col] = val;
18        }
19    }
20 }

```

```

19     }
20
21     return res;
22 }
23
24 vector<vl> fexp(vector<vl> b, ll e, int n) {
25     if(e == 0) {
26         return id(n);
27     }
28
29     vector<vl> res = fexp(b, e/2, n);
30     res = mult(res, res, n);
31
32     if(e%2)
33         res = mult(res, b, n);
34
35     return res;
36 }
37
38 // k = tamanho da recorrência/matriz, n = n-esimo
39 // termo
40 // f(n) = c1*f(n-1) + c2*f(n-2) + ... + ck*f(n-k)
41 // base -> [f(k-1), f(k-2), ..., f(0)]
42 // coeficientes -> [c1, c2, ..., ck]
43 vl solve(int k, int n, vl base, vl coef) {
44     vector<vl> inicial;
45     inicial.pb(coef);
46     for(int row = 0; row < k-1; row++) {
47         vl tmp;
48         for(int col = 0; col < k; col++) {
49             if(col == row)
50                 tmp.pb(1);
51             else
52                 tmp.pb(0);
53         }
54         inicial.pb(tmp);
55     }
56     vector<vl> matexp = fexp(inicial, max(0, n-k+1),
57 k);
58 vl res(k);
59
60 for(int row = 0; row < k; row++) {
61     ll val = 0;
62     for(int aux = 0; aux < k; aux++) {
63         val += matexp[row][aux]*base[aux];
64     }
65     res[row] = val; // res = (f(n), f(n-1), ...,
66 f(n-k+1))
67 }
68
69 return res;
70 }

```

## 5.17 Raiz-primitiva

```

1 ll fexp(ll b, ll e, ll mod) {
2     if(e == 0) return 1LL;
3     ll res = fexp(b, e/2LL, mod);
4     res = (res*res)%mod;
5     if(e%2LL)
6         res = (res*b)%mod;
7
8     return res%mod;
9 }
10
11 vl fatorar(ll n) { // fatora em primos
12     vl fat;
13     for(int i = 2; i*i <= n; i++) {
14         if(n%i == 0) {
15             fat.pb(i);
16             while(n%i == 0)
17                 n /= i;

```

```

18     }
19 }
20 return fat;
21 }
22
23 //  $O(\log(n)^2)$ 
24 bool raiz_prim(ll a, ll mod, ll phi, vl fat) {
25     if(__gcd(a, mod) != 1 or fexp(a, phi/2, mod) ==
26 1) // phi de euler sempre eh PAR
27         return false;
28
29     for(auto f : fat) {
30         if(fexp(a, phi/f, mod) == 1)
31             return false;
32     }
33
34     return true;
35 }
36
37 // mods com raizes primitivas: 2, 4,  $p^k$ ,  $2*p^k$ , p eh
38 // primo impar, k inteiro ---  $O(n \log^2(n))$ 
39 ll achar_raiz(ll mod, ll phi) {
40     if(mod == 2) return 1;
41     vl fat, elementos;
42     fat = fatorar(phi);
43
44     for(ll i = 2; i <= mod-1; i++) {
45         if(raiz_prim(i, mod, phi, fat))
46             return i;
47     }
48
49     return -1; // retorna -1 se nao existe
50 }
51
52 vl todas_raizes(ll mod, ll phi, ll raiz) {
53     vl raizes;
54     if(raiz == -1) return raizes;
55     ll r = raiz;
56     for(ll i = 1; i <= phi-1; i++) {
57         if(__gcd(i, phi) == 1) {
58             raizes.pb(r);
59         }
60         r = (r * raiz) % mod;
61     }
62
63     return raizes;
64 }

```

## 5.18 Kamenetsky

```

1 // Number of digits in n!  $O(1)$ 
2
3 #define Pi 3.14159265358979311599796346854
4 #define Eul 2.71828182845904509079559829842
5
6 long long findDigits(int n)
7 {
8     double x;
9
10    if (n < 0)
11        return 0;
12    if (n == 1)
13        return 1;
14
15    x = ((n * log10(n / euler) + log10(2 * Pi * n)
16 / 2.0));
17
18    return floor(x) + 1;

```



## 6 Misc

### 6.1 2SAT

```
1 vector<int> g[MAX], gt[MAX], S; int vis[MAX], cor[MAX]
  ];
2
3 int val(int n, bool tvalue) {
4     if(tvalue) return 2*n;
5     return 2*n + 1;
6 }
7
8 void dfs(int u) {
9     vis[u] = 1; for(int v : g[u]) if(!vis[v]) dfs(v);
10    S.push_back(u);
11 }
12
13 void dfst(int u, int e) {
14     cor[u] = e;
15     for(int v : gt[u]) if(!cor[v]) dfst(v, e);
16 }
17
18 void kosaraju(int n) {
19     for(int i = 0; i <= n; i++) if(!vis[i]) dfs(i);
20     for(int i = 0; i <= n; i++) for(int j : g[i])
21         gt[j].push_back(i);
22     int e = 0; reverse(S.begin(), S.end());
23     for(int u : S) if(!cor[u]) dfst(u, ++e);
24 }
25
26 // antes de chamar essa funcao, colocar as arestas do
   grafo
27 bool solve(int n, vi &res) {
28     kosaraju(2*n); // MAX > 2*N
29     vi r;
30
31     forn(i, n) {
32         int t = val(i, true), f = val(i, false);
33         if(cor[t] == cor[f]) {
34             return false;
35         }
36         else {
37             if(cor[t] > cor[f])
38                 r.pb(1);
39             else
40                 r.pb(0);
41         }
42     }
43     swap(r, res);
44     return true;
45 }
```

### 6.2 LIS

```
1 multiset<int> S;
2 for(int i = 0; i < n; i++){
3     auto it = S.upper_bound(vet[i]); // low for inc
4     if(it != S.end())
5         S.erase(it);
6     S.insert(vet[i]);
7 }
8 // size of the lis
9 int ans = S.size();
```

### 6.3 Bitwise

```
1 // Bitwise
2 #pragma GCC target("popcnt")
3 unsigned char a = 5, b = 9; // a = (00000101), b
   = (00001001)
4
```

```
5 AND -          a&b    // The result is 00000001
   (1)
6 OR -          a|b    // The result is 00001101
   (13)
7 XOR -         a^b    // The result is 00001100
   (12)
8 NOT -         ~a     // The result is 11111010
   (250)
9 Left shift -   b<<1  // The result is 00010010
   (18)
10 Right shift - b>>1  // The result is 00000100
   (4)
```

// Exchange two int variables

```
a^=b;
b^=a;
a^=b;
```

// Even or Odd

```
(x & 1)? printf("Odd"): printf("Even");
```

// Turn on the j-th bit

```
int S = 34; //(100010)
int j = 3;

S = S | (1<<j);
```

// Turn off the j-th bit

```
int S = 42; //(101010)
int j = 1;

S &= ~(1<<j)
```

```
S == 40 //(101000)
```

// Check the j-th element

```
int S = 42; //(101010)
int j = 3;

T = S & (1<<j); // T = 0
```

// Least significant bit (lsb)

```
int lsb(int x){ return x&-x; }
```

// Exchange o j-th element

```
S ^= (1<<j)
```

// Position of the first bit on

```
T = (S & (-S))
T -> 4 bit ligado //(1000)
```

// Most significant digit of N

```
double K = log10(N);
K = K - floor(K);
int X = pow(10, K);
```

// Number of digits in N

```
X = floor(log10(N)) + 1;
```

// Power of two

```
bool isPowerOfTwo(int x){ return x && !(x&(x
-1)); }
```

```

71 // Turn off the first bit 1
72 m = m & (m-1);
73
74 // Built-in functions
75
76 // Number of bits 1
77 __builtin_popcount()
78 __builtin_popcountll()
79
80 // Number of leading zeros
81 __builtin_clz()
82 __builtin_clzll()
83
84 // Number of trailing zeros
85 __builtin_ctz()
86 __builtin_ctzll()
87
88 // floor(log2(x))
89
90 int flog2(int x){ return 32-1-__builtin_clz(x); }
91
92 int flog2ll(ll x){ return 64-1-__builtin_clzll(x); }
93

```

## 6.4 Template

```

1 #include <bits/stdc++.h>
2 #define ff first
3 #define ss second
4 #define ll long long
5 #define ld long double
6 #define pb push_back
7 #define eb emplace_back
8 #define mp make_pair
9 #define mt make_tuple
10 #define pii pair<int, int>
11 #define vi vector<int>
12 #define sws ios_base::sync_with_stdio(false);cin.tie(
    NULL)
13 #define endl '\n'
14 #define teto(a, b) (a+b-1)/(b)
15
16 const int MAX = 400010;
17 const int MOD = 1e9+7;
18 const int INF = 0x3f3f3f3f;
19 const ll LLINF = 0x3f3f3f3f3f3f3f3f;
20 const ld EPS = 1e-7;
21
22 using namespace std;

```

## 7 Strings

### 7.1 KMP

```

1 vector<int> prefix_function(const string &s){
2     int n = s.size(); vector<int> b(n+1);
3     b[0] = -1; int i = 0, j = -1;
4     while(i < n){
5         while(j >= 0 && s[i] != s[j]) j = b[j];
6         b[++i] = ++j;
7     }
8     return b;
9 }
10
11 void kmp(const string &t, const string &p){
12     vector<int> b = prefix_function(p);
13     int n = t.size(), m = p.size();
14     int j = 0;
15     for(int i = 0; i < n; i++){

```

```

16         while(j >= 0 && t[i] != p[j]) j = b[j];
17         j++;
18         if(j == m) {
19             j = b[j];
20         }
21     }
22 }

```

### 7.2 LCS

```

1 string LCSSubStr(string X, string Y)
2 {
3     int m = X.size();
4     int n = Y.size();
5
6     int result = 0, end;
7     int len[2][n];
8     int currRow = 0;
9
10    for(int i=0;i<=m;i++){
11        for(int j=0;j<=n;j++){
12            if(i==0 || j==0)
13                len[currRow][j] = 0;
14            else if(X[i-1] == Y[j-1]){
15                len[currRow][j] = len[1-currRow][j-1]
16                + 1;
17                if(len[currRow][j] > result){
18                    result = len[currRow][j];
19                    end = i - 1;
20                }
21            }
22            else
23                len[currRow][j] = 0;
24        }
25        currRow = 1 - currRow;
26    }
27
28    if(result==0)
29        return string();
30
31    return X.substr(end - result + 1, result);
32 }

```

### 7.3 Pal-int

```

1 bool ehpalindromo(ll n) {
2     if(n<0)
3         return false;
4
5     int divisor = 1;
6     while(n/divisor >= 10)
7         divisor *= 10;
8
9     while(n != 0) {
10        int leading = n / divisor;
11        int trailing = n % 10;
12
13        if(leading != trailing)
14            return false;
15
16        n = (n % divisor)/10;
17        divisor = divisor/100;
18    }
19
20    return true;
21 }

```

### 7.4 Z-Func

```

1 vector<int> z_algo(const string &s)

```

```

2 {
3     int n = s.size();
4     int L = 0, R = 0;
5     vector<int> z(n, 0);
6     for(int i = 1; i < n; i++)
7     {
8         if(i <= R)
9             z[i] = min(z[i-L], R - i + 1);
10        while(z[i]+i < n && s[ z[i]+i ] == s[ z[i] ])
11            z[i]++;
12        if(i+z[i]-1 > R)
13        {
14            L = i;
15            R = i + z[i] - 1;
16        }
17    }
18    return z;
19 }

```

## 7.5 Hash

```

1 ll compute_hash(string const& s) {
2     const ll p = 31; // primo, melhor = perto da
3     // quantidade de caracteres
4     const ll m = 1e9 + 9; // maior mod = menor
5     // probabilidade de colisao
6     ll hash_value = 0;
7     ll p_pow = 1;
8     for (char c : s) {
9         hash_value = (hash_value + (c - 'a' + 1) *
10         p_pow) % m;
11         p_pow = (p_pow * p) % m;
12     }
13     return hash_value;
14 }

```