

## Notebook - Maratona de Programação

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Contents					5.4	Hungarian	
1	Algoritmos		2		5.5	Kosaraju	10
1	_	<del>-</del>		6	Math 1		10
	1.1 Ternary Se	alcii	2	Ü	6.1	Bigmod	
2	DP		2		6.2	Division Trick	11
	2.1 Dp Digitos		2		6.3	Inverso Mult	
			2		6.4	Linear Diophantine Equation	
	1		$\overline{2}$		6.5	Matrix Exponentiation	
					6.6	Totient	11
3	$\mathbf{ED}$		<b>2</b>		0.0	Toucht	11
	3.1 Minqueue		2	7	Mis	$\mathbf{c}$	<b>12</b>
	3.2 Segtree Im	plicita Lazy	3		7.1	Bitwise	12
		ble	3				
			8 5		$\mathbf{Stri}$	ngs	12
4	${f Geometria}$	netria			8.1	Aho Corasick	12
	4.1   2d		4		8.2	Edit Distance	12
	4.2   3d		6		8.3	Hash	12
	4.3 Convex Hu	ıll	7		8.4	Kmp	13
	4.4 Inside Poly	gon	7		8.5	Lcs	13
	4.5 Intersect P	$\operatorname{tolygon}$	7		8.6	Lcsubseq	13
	4.6 Linear Tra	$\operatorname{nsformation} \ldots \ldots \ldots \ldots \ldots$	7				
	4.7 Mindistpai	r	8				
	4.8 Polygon A	rea	8				
	4.9 Sort By A	$_{ m ngle}$	8				
	4.10 Voronoi .		8				
5	Grafos	Frafos					
	5.1 Dfs Tree .		9				
	5.2 Dinic		9				
	5.3 Ford		9				

# Algoritmos

#### Ternary Search

```
1 // Ternary
2 \text{ ld l} = -1\text{e4}, r = 1\text{e4};
3 int iter = 100;
4 while(iter - -) {
      1d m1 = (2*1 + r) / 3;
       1d m2 = (1 + 2*r) / 3;
       if(check(m1) > check(m2))
          1 = m1;
       else
10
           r = m2;
11 }
```

## DP

## 2.1 Dp Digitos

```
_{1} // dp de quantidade de numeros <= r com ate qt
      digitos diferentes de 0
_2 11 dp(int idx, string& r, bool menor, int qt, vector< ^{30}
                                                         3.1
      vector<vi>>% tab) {
      if(qt > 3) return 0;
      if(idx >= r.size()) {
          return 1;
      if(tab[idx][menor][qt] != -1)
         return tab[idx][menor][qt];
      11 res = 0;
10
      for(int i = 0; i <= 9; i++) {
11
          if(menor or i <= r[idx]-'0') {
              res += dp(idx+1, r, menor or i < (r[idx] - ^4
           , qt+(i>0), tab);
14
15
      return tab[idx][menor][qt] = res;
1.7
                                                         10
```

### 2.2 Knapsack

```
1 // Caso base, como i == n
                                                           1.4
2 dp[0][0] = 0;
                                                           1.5
4 // Itera por todos os estados
                                                           16
5 for(int i = 1; i <= n; ++i)
      for(int P = 0; P <= w; ++P){</pre>
                                                           18
          int &temp = dp[i][P];
          // Primeira possibilidade, ãno pega i
                                                          2.0
          temp = dp[i - 1][P];
                                                           21
10
          // Segunda possibilidade, se puder, pega o
                                                           23
          if(P - p[i] >= 0)
              temp = max(temp, dp[i - 1][P - p[i]] + v[^{25}
13
      il):
          ans = max(ans, temp);
                                                           28
                                                           29
```

#### 2.3 Lis

```
1 multiset < int > S;
2 for(int i=0;i<n;i++){</pre>
      auto it = S.upper_bound(vet[i]); // low for inc
      if(it != S.end())
          S.erase(it);
```

```
S.insert(vet[i]);
7 }
8 // size of the lis
9 int ans = S.size();
11 vi LIS(const vi &elements) {
       auto compare = [&](int x, int y) {
          return elements[x] < elements[y];</pre>
      set < int, decltype(compare) > S(compare);
       vi previous ( elements.size(), -1 );
      for(int i=0; i<int( elements.size() ); ++i){</pre>
           auto it = S.insert(i).first;
           if(it != S.begin())
               previous[i] = *prev(it);
           if(*it == i and next(it) != S.end())
               S.erase(next(it));
      vi answer;
      answer.push_back( *S.rbegin() );
      while ( previous[answer.back()] != -1 )
          answer.push_back( previous[answer.back()] );
      reverse( answer.begin(), answer.end() );
       return answer;
32 }
       ED
  3
```

12

1.3

14

15

16

17

18

19

2.0

21 22

23 2.5

26

27

28

11

12 13

#### 3.1 Minqueue

```
struct MinQ {
    stack<pair<11,11>> in;
     stack<pair<11,11>> out;
     void add(ll val) {
         11 minimum = in.empty() ? val : min(val, in.
     top().ss);
          in.push({val, minimum});
     11 pop() {
         if(out.empty()) {
             while(!in.empty()) {
                  ll val = in.top().ff;
                  in.pop();
                 11 minimum = out.empty() ? val : min(
     val, out.top().ss);
                 out.push({val, minimum});
         11 res = out.top().ff;
          out.pop();
          return res;
     ll minn() {
         11 minimum = LLINF;
          if(in.empty() || out.empty())
             minimum = in.empty() ? (11)out.top().ss :
       (11) in.top().ss;
              minimum = min((11)in.top().ss, (11)out.
     top().ss);
          return minimum;
     11 size() {
         return in.size() + out.size();
```

31

3.5

#### 37 }; 1 int logv[N+1]; void make\_log() { 3.2Segtree Implicita Lazy logv[1] = 0; // pre-computar tabela de log for (int i = 2; i <= N; i++)</pre> logv[i] = logv[i/2] + 1;1 struct node { 6 } pll val; 7 struct Sparse { ll lazy; int n; 11 1, r; 4 vector < vector < int >> st; node(){ 9 10 l = -1; r = -1; $val = \{0, 0\}$ ; lazy = 0; 6 Sparse(vector<int>& v) { 11 12 n = v.size();8 }; int k = logv[n]; 1.3 st.assign(n+1, vector < int > (k+1, 0)); 14 10 node tree[40\*MAX]; 11 int id = 2; 15 for (int i=0;i<n;i++) {</pre> 16 12 11 N=1e9+10; 17 st[i][0] = v[i]; 13 18 14 pll merge(pll A, pll B){ if(A.ff==B.ff) return {A.ff, A.ss+B.ss}; 15 for(int j = 1; j <= k; j++) { return (A.ff<B.ff ? A:B);</pre> 20 16 for(int i = 0; i + (1 << j) <= n; i++) { 21 17 } 22 st[i][j] = f(st[i][j-1], st[i + (1 <<18 (j-1))][j-1]); 19 void prop(11 1, 11 r, int no){ 11 mid = (1+r)/2;23 } 20 } 24 **if**(1!=r){ 21 25 if(tree[no].l==-1){ 26 tree[no].1 = id++; 23 int f(int a, int b) { 27 24 tree[tree[no].1].val = {0, mid-1+1}; return min(a, b); 28 2.5 if(tree[no].r==-1){ 29 26 30 tree[no].r = id++;int query(int 1, int r) { tree[tree[no].r].val = $\{0, r-(mid+1)+1\};$ 31 28 int k = logv[r-l+1];32 29 tree[tree[no].1].lazy += tree[no].lazy; 33 return f(st[l][k], st[r - (1 << k) + 1][k]); 3.0 tree[tree[no].r].lazy += tree[no].lazy; 34 3.1 35 }; 32 36 tree[no].val.ff += tree[no].lazy; 33 37 tree[no].lazy=0; 34 38 struct Sparse2d { 35 int n, m; 39 36 $_{37}$ void update(int a, int b, int x, 11 1=0, 11 r=2\*N, 11 $^{40}$ vector < vector < int >>> st; no=1){ 41 Sparse2d(vector<vector<int>> mat) { prop(1, r, no); 43 n = mat.size(); if(a<=1 and r<=b){ 39 m = mat[0].size(); 44 tree[no].lazy += x; 45 int k = logv[min(n, m)]; 41 prop(1, r, no); return; 46 42 st.assign(n+1, vector < vector < int >> (m+1, } 43 vector < int > (k+1))); if(r<a or b<1) return: 44 for(int i = 0; i < n; i++)</pre> 48 int m = (1+r)/2; for(int j = 0; j < m; j++) 49 update(a, b, x, 1, m, tree[no].1); 46 st[i][j][0] = mat[i][j]; update(a, b, x, m+1, r, tree[no].r); 50 48 for(int $j = 1; j \le k; j++) {$ ${\tt tree[no].val = merge(tree[tree[no].1].val, tree[} \begin{subarray}{c} 52 \\ \hline \end{subarray}$ 49 for (int x1 = 0; x1 < n; x1++) { tree[no].r].val); for(int y1 = 0; y1 < m; y1++) { 50 } 5.4 int delta = (1 << (j-1));</pre> 51 if(x1+delta >= n or y1+delta >= m $_{52}$ pll query(int a, int b, int l=0, int r=2\*N, int no=1) $^{56}$ ) continue: 53 prop(1, r, no); st[x1][y1][j] = st[x1][y1][j-1]; 5.8 if(a<=l and r<=b) return tree[no].val;</pre> 54 st[x1][y1][j] = f(st[x1][y1][j], 59 if(r<a or b<1) return {INF, 0};</pre> st[x1+delta][y1][j-1]); int m = (1+r)/2; 56 st[x1][y1][j] = f(st[x1][y1][j], int left = tree[no].1, right = tree[no].r; st[x1][y1+delta][j-1]); 5.8 st[x1][y1][j] = f(st[x1][y1][j],return tree[no].val = merge(query(a, b, 1, m, 59 st[x1+delta][y1+delta][j-1]); left), } query(a, b, m+1, r, } right)); 63 } 64 61 } 65 3.3 Sparse Table 66 67 // so funciona para quadrados

```
int query(int x1, int y1, int x2, int y2) {
                                                          41 bool nulo(point a){
68
           assert(x2-x1+1 == y2-y1+1);
69
                                                          return (eq(a.x, 0) and eq(a.y, 0));
                                                          43 }
           int k = logv[x2-x1+1];
7.0
           int delta = (1 << k);</pre>
                                                          44 point rotccw(point p, ld a){
                                                                // a = PI*a/180; // graus
           int res = st[x1][y1][k];
                                                                 return point((p.x*cos(a)-p.y*sin(a)), (p.y*cos(a)
73
                                                           46
           res = f(res, st[x2 - delta+1][y1][k]);
                                                                 +p.x*sin(a)));
          res = f(res, st[x1][y2 - delta+1][k]);
                                                          47 }
7.5
          res = f(res, st[x2 - delta+1][y2 - delta+1][k.48 point rot90cw(point a) { return point(a.y, -a.x); };
      1);
                                                          49 point rot90ccw(point a) { return point(-a.y, a.x); };
          return res:
                                                          5.0
                                                          51 ld proj(point a, point b){ // a sobre b
7.9
                                                          5.2
                                                                 return a*b/norm(b);
      int f(int a, int b) {
                                                          53 }
80
                                                          14 ld angle(point a, point b) { // em radianos
81
          return a | b;
                                                                 ld ang = a*b / norm(a) / norm(b);
82
                                                          55
83
                                                          56
                                                                 return acos(max(min(ang, (ld)1), (ld)-1));
84 };
                                                          57 }
                                                          58 ld angle_vec(point v){
                                                                // return 180/PI*atan2(v.x, v.y); // graus
       Geometria
  4
                                                          5.9
                                                          60
                                                                 return atan2(v.x, v.y);
                                                          61 }
  4.1 2d
                                                          62 ld order_angle(point a, point b){ // from a to b ccw
                                                                 (a in front of b)
                                                                 ld aux = angle(a,b)*180/PI;
                                                          63
1 #define vp vector<point>
                                                          64
                                                                 return ((a^b) <=0 ? aux:360-aux);
2 #define ld long double
                                                          65 }
s const ld EPS = 1e-6;
                                                          66 bool angle_less(point a1, point b1, point a2, point
4 const ld PI = acos(-1);
                                                                 b2){ // ang(a1,b1) <= ang(a2,b2)
                                                                 point p1((a1*b1), abs((a1^b1)));
6 typedef ld T;
                                                          6.7
                                                          68
                                                                 point p2((a2*b2), abs((a2^b2)));
7 bool eq(T a, T b){ return abs(a - b) <= EPS; }</pre>
                                                                 return (p1^p2) <= 0;
                                                          69
                                                          70 }
9 struct point{
     Тх, у;
                                                          7.1
1.0
                                                          72 ld area(vp &p){ // (points sorted)
      int id;
                                                                 ld ret = 0;
      point (T x=0, T y=0): x(x), y(y) {}
                                                          73
12
                                                                 for(int i=2;i<(int)p.size();i++)</pre>
                                                          7.4
13
      point operator+(const point &o) const{ return {x 75
                                                                   ret += (p[i]-p[0])^(p[i-1]-p[0]);
14
                                                                 return abs(ret/2);
      + o.x, y + o.y; }
      point operator - (const point &o) const{ return {x 77 }
                                                          78 ld areaT(point &a, point &b, point &c){
       -o.x, y - o.y; }
                                                                 return abs((b-a)^(c-a))/2.0;
      point operator*(T t) const{ return {x * t, y * t 79}
      }; }
      point operator/(T t) const{ return \{x \ / \ t, \ y \ / \ t \ ^{81}
                                                          82 point center(vp &A){
      }; }
                                                               point c = point();
      T operator*(const point &o) const{ return x * o.x83
18
                                                                 int len = A.size();
       + y * o.y; }
                                                          84
                                                                 for(int i=0;i<len;i++)</pre>
      T operator^(const point &o) const{ return x * o.y 85
19
                                                                   c=c+A[i];
       - y * o.x; }
                                                          86
                                                          87
                                                                 return c/len:
      bool operator<(const point &o) const{</pre>
20
                                                          88 }
          return (eq(x, o.x) ? y < o.y : x < o.x);
21
                                                          89
      bool operator == (const point &o) const{
                                                          90 point forca_mod(point p, ld m){
23
                                                          91 ld cm = norm(p);
          return eq(x, o.x) and eq(y, o.y);
24
                                                          92
                                                                 if(cm<EPS) return point();</pre>
      }
2.5
      friend ostream& operator<<(ostream& os, point p) 93</pre>
                                                                 return point(p.x*m/cm,p.y*m/cm);
26
          return os << "(" << p.x << "," << p.y << ")"; 95
       }
                                                          96 ld param(point a, point b, point v){
                                                                 // v = t*(b-a) + a // return t;
                                                          97
28 };
                                                                 // assert(line(a, b).inside_seg(v));
                                                          9.8
29
                                                          99
                                                                 return ((v-a) * (b-a)) / ((b-a) * (b-a));
30 int ccw(point a, point b, point e){ // -1=dir; 0=
      collinear; 1=esq;
                                                          100 }
      T \text{ tmp} = (b-a)^{(e-a)}; // \text{ vector from a to b}
                                                          102 bool simetric(vp &a){ //ordered
      return (tmp > EPS) - (tmp < -EPS);</pre>
32
                                                                int n = a.size():
33 }
                                                          103
                                                                 point c = center(a);
                                                          104
34
35 ld norm(point a){ // Modulo
                                                          105
                                                                 if(n&1) return false;
                                                          106
                                                                 for(int i=0;i<n/2;i++)
      return sqrt(a * a);
                                                          107
                                                                     if(ccw(a[i], a[i+n/2], c) != 0)
37
                                                                         return false:
                                                          108
38 T norm2(point a){
                                                                 return true;
39
     return a * a;
                                                          110 }
40 }
```

```
182 bool seg_has_inter(line l1, line l2){
point mirror(point m1, point m2, point p){
                                                                  return ccw(11.p1, 11.p2, 12.p1) * ccw(11.p1, 11.
                                                                  p2, 12.p2) < 0 and
113
       // mirror point p around segment m1m2
       point seg = m2-m1;
                                                                         ccw(12.p1, 12.p2, 11.p1) * ccw(12.p1, 12.
114
       1d t0 = ((p-m1)*seg) / (seg*seg);
                                                                  p2, 11.p2) < 0;
       point ort = m1 + seg*t0;
                                                           185
116
       point pm = ort-(p-ort);
                                                           186
                                                           187 ld dist_seg(point p, point a, point b){ // point -
118
       return pm;
119
                                                                  if((p-a)*(b-a) < EPS) return norm(p-a);
120
                                                           188
                                                                  if((p-b)*(a-b) < EPS) return norm(p-b);
                                                           189
122 ///////////
                                                           190
                                                                  return abs((p-a)^(b-a)) / norm(b-a);
123 // Line //
                                                           191 }
124 ///////////
                                                           193 ld dist_line(point p, line l){ // point - line
   struct line {
                                                                  return abs(l.eval(p))/sqrt(l.a*l.a + l.b*l.b);
126
                                                           194
127
       point p1, p2;
                                                           195
       T \ a, b, c; // ax+by+c = 0;
128
                                                           196
       // y-y1 = ((y2-y1)/(x2-x1))(x-x1)
                                                           197 line bisector(point a, point b){
       line(point p1=0, point p2=0): p1(p1), p2(p2){
                                                           198
                                                                  point d = (b-a)*2;
                                                                  return line(d.x, d.y, a*a - b*b);
           a = p1.y - p2.y;
                                                           199
           b = p2.x - p1.x;
                                                           200 }
           c = p1 ^p2;
                                                           201
                                                           202 line perpendicular(line 1, point p){ // passes
       line(T a=0, T b=0, T c=0): a(a), b(b), c(c){
                                                                  through p
           // Gera os pontos p1 p2 dados os coeficientes203
                                                                  return line(1.b, -1.a, -1.b*p.x + 1.a*p.y);
136
           // isso aqui eh um lixo mas quebra um galho 204 }
137
       kkkkkk
                                                           205
           if(b==0){
                                                          207 ///////////
               p1 = point(1, -c/a);
139
               p2 = point(0, -c/a);
                                                           208 // Circle /
140
                                                           209 ///////////
           }else{
141
               p1 = point(1, (-c-a*1)/b);
                                                           210
142
               p2 = point(0, -c/b);
                                                           211 struct circle{
           }
                                                                  point c; T r;
                                                           212
144
       }
                                                                  circle() : c(0, 0), r(0){}
145
                                                           213
                                                                  circle(const point o) : c(o), r(0){}
146
                                                           214
147
       T eval(point p){
                                                           215
                                                                  circle(const point a, const point b){
           return a*p.x+b*p.y+c;
                                                                      c = (a+b)/2;
148
                                                           216
                                                                      r = norm(a-c);
                                                           217
149
       bool inside(point p){
                                                           218
           return eq(eval(p), 0);
                                                           219
                                                                  circle(const point a, const point b, const point
       point normal(){
                                                           220
                                                                      assert(ccw(a, b, cc) != 0);
           return point(a, b);
                                                                      c = inter_line(bisector(a, b), bisector(b, cc
154
                                                           221
                                                                  ))[0];
155
                                                                      r = norm(a-c);
156
                                                           222
       bool inside_seg(point p){
           return (
                                                                  bool inside(const point &a) const{
158
                                                           224
                ((p1-p) ^ (p2-p)) == 0 and
                                                           225
                                                                      return norm(a - c) <= r + EPS;</pre>
                ((p1-p) * (p2-p)) <= 0
                                                           226
           );
                                                           227 };
                                                           228
                                                           229 pair < point , point > tangent_points(circle cr, point p)
163
164 };
165
                                                                  1d d1 = norm(p-cr.c), theta = asin(cr.r/d1);
166 // be careful with precision error
                                                                  point p1 = rotccw(cr.c-p, -theta);
                                                           231
vp inter_line(line l1, line l2){
                                                                  point p2 = rotccw(cr.c-p, theta);
                                                           232
       1d det = 11.a*12.b - 11.b*12.a:
168
                                                           233
                                                                  assert(d1 >= cr.r);
       if(det==0) return {};
                                                           234
                                                                  p1 = p1 * (sqrt(d1*d1-cr.r*cr.r) / d1) + p;
       ld x = (l1.b*l2.c - l1.c*l2.b)/det;
                                                                  p2 = p2 * (sqrt(d1*d1-cr.r*cr.r) / d1) + p;
170
                                                           235
       1d y = (11.c*12.a - 11.a*12.c)/det;
                                                                  return {p1, p2};
                                                           236
       return {point(x, y)};
                                                           237 }
172
173
                                                           238
                                                           239
175 // segments not collinear
                                                           240 circle incircle(point p1, point p2, point p3){
176 vp inter_seg(line l1, line l2){
                                                                  1d m1 = norm(p2-p3);
                                                           241
       vp ans = inter_line(11, 12);
                                                                  1d m2 = norm(p1-p3);
177
                                                                  1d m3 = norm(p1-p2);
       if(ans.empty() or !11.inside_seg(ans[0]) or !12. 243
178
       inside_seg(ans[0]))
                                                                  point c = (p1*m1 + p2*m2 + p3*m3)*(1/(m1+m2+m3));
                                                           244
          return {}:
                                                                  1d s = 0.5*(m1+m2+m3);
179
                                                           245
                                                                  ld r = sqrt(s*(s-m1)*(s-m2)*(s-m3)) / s;
180
       return ans;
181 }
                                                           247
                                                                  return circle(c, r);
```

```
248
                                                            1.3
                                                                   point operator+(const point &o) const {
249
                                                            14
250 circle circumcircle(point a, point b, point c) {
                                                            15
                                                                       return {x+o.x, y+o.y, z+o.z};
       circle ans;
                                                            16
       point u = point((b-a).y, -(b-a).x);
                                                                   point operator - (const point &o) const {
       point v = point((c-a).y, -(c-a).x);
                                                                       return {x-o.x, y-o.y, z-o.z};
253
                                                            18
       point n = (c-b)*0.5;
254
                                                            19
       1d t = (u^n)/(v^u);
                                                                   point operator*(cod t) const {
                                                            2.0
       ans.c = ((a+c)*0.5) + (v*t);
                                                                       return {x*t, y*t, z*t};
256
                                                            21
       ans.r = norm(ans.c-a);
257
                                                            22
                                                                   point operator/(cod t) const {
       return ans:
258
                                                            23
259 }
                                                            24
                                                                       return \{x/t, y/t, z/t\};
260
                                                            2.5
261 vp inter_circle_line(circle C, line L){
                                                                   bool operator == (const point &o) const {
                                                            26
       point ab = L.p2 - L.p1, p = L.p1 + ab * ((C.c-L. 27))
                                                                       return eq(x, o.x) and eq(y, o.y) and eq(z, o.
       p1)*(ab) / (ab*ab));
       ld s = (L.p2-L.p1)^(C.c-L.p1), h2 = C.r*C.r - s*s_{28}
                                                                   }
                                                                   \verb|cod| operator*(const| point| \&o) | const| \{ \ // \ \texttt{dot} \\
        / (ab*ab):
                                                            29
       if (h2 < -EPS) return {};</pre>
                                                                        return x*o.x + y*o.y + z*o.z;
265
       if (eq(h2, 0)) return {p};
                                                            3.1
       point h = (ab/norm(ab)) * sqrt(h2);
                                                                   point operator^(const point &o) const { // cross
                                                            32
266
       return {p - h, p + h};
                                                                        return point(y*o.z - z*o.y,
267
                                                            33
                                                                                     Z*0.X - X*0.Z,
268
                                                            3.4
                                                                                      x*o.y - y*o.x);
270 vp inter_circle(circle c1, circle c2){
                                                            3.6
       if (c1.c == c2.c) { assert(c1.r != c2.r); return 37 };
271
       {}; }
       point vec = c2.c - c1.c;
                                                            39 ld norm(point a) { // Modulo
       1d d2 = vec * vec, sum = c1.r + c2.r, dif = c1.r 40
                                                                   return sqrt(a * a);
       - c2.r;
                                                            41 }
       ld p = (d2 + c1.r * c1.r - c2.r * c2.r) / (2 * d2.42 cod norm2(point a) {
274
       ):
                                                            43
                                                                   return a * a;
       1d h2 = c1.r * c1.r - p * p * d2;
                                                            44 }
       if (sum * sum < d2 or dif * dif > d2) return {}; 45 bool nulo(point a) {
       point mid = c1.c + vec * p, per = point(-vec.y, ^{46}
                                                                  return (eq(a.x, 0) \text{ and } eq(a.y, 0) \text{ and } eq(a.z, 0))
277
       vec.x) * sqrt(fmax(0, h2) / d2);
       if (eq(per.x, 0) and eq(per.y, 0)) return {mid}; 47
279
       return {mid + per, mid - per};
                                                            48 ld proj(point a, point b) { // a sobre b
280 }
                                                            49
                                                                   return (a*b)/norm(b);
                                                            50 }
281
282 // minimum circle cover O(n) amortizado
                                                            51 ld angle(point a, point b) { // em radianos
283 circle min_circle_cover(vp v){
                                                                   return acos((a*b) / norm(a) / norm(b));
                                                            52
       random_shuffle(v.begin(), v.end());
                                                            53
284
285
       circle ans;
                                                            5.4
       int n = v.size();
                                                            55 cod triple(point a, point b, point c) {
286
       for(int i=0;i<n;i++) if(!ans.inside(v[i])){</pre>
                                                            56
                                                                   return (a * (b^c)); // Area do paralelepipedo
287
                                                            57 }
            ans = circle(v[i]);
288
            for(int j = 0; j < i; j ++) if(!ans.inside(v[j])){</pre>
                ans = circle(v[i], v[j]);
                                                            59 point normilize(point a) {
                for(int k=0;k<j;k++) if(!ans.inside(v[k]) 60</pre>
                                                                  return a/norm(a);
291
       ) {
                                                            61 }
                    ans = circle(v[i], v[j], v[k]);
292
                                                            62
                                                            63 struct plane {
                }
           }
                                                                   cod a, b, c, d;
294
                                                            64
                                                                   point p1, p2, p3;
296
       return ans;
                                                                   plane(point p1=0, point p2=0, point p3=0): p1(p1)
                                                                   , p2(p2), p3(p3) {
297 }
                                                                        point aux = (p1-p3)^(p2-p3);
                                                            67
       3d
   4.2
                                                                        a = aux.x; b = aux.y; c = aux.z;
                                                            68
                                                                        d = -a*p1.x - b*p1.y - c*p1.z;
                                                            69
                                                            70
 1 // typedef ll cod;
                                                                   plane(point p, point normal) {
 2 // bool eq(cod a, cod b) { return (a==b); }
                                                                       normal = normilize(normal);
                                                            72
                                                                       a = normal.x; b = normal.y; c = normal.z;
 4 \text{ const} 1d EPS = 1e-6;
                                                            73
                                                                        d = -(p*normal);
 5 #define vp vector<point>
 6 typedef ld cod;
 7 bool eq(cod a, cod b){ return fabs(a - b) <= EPS; }</pre>
                                                            76
                                                                   // ax+by+cz+d = 0;
                                                            77
                                                                   cod eval(point &p) {
                                                            78
 9 struct point
                                                                        return a*p.x + b*p.y + c*p.z + d;
                                                            79
10 €
                                                            80
       cod x, y, z;
       point(cod x=0, cod y=0, cod z=0): x(x), y(y), z(z^{81});
12
       ) {}
```

```
ss cod dist(plane pl, point p) {
                                                                  while(1<r){
                                                           12
       return fabs(pl.a*p.x + pl.b*p.y + pl.c*p.z + pl.d 13
                                                                      int mid = (1+r)/2;
84
       ) / sqrt(pl.a*pl.a + pl.b*pl.b + pl.c*pl.c);
                                                                      if(ccw(p[0], p[mid], e) == 1)
                                                          14
85
                                                           1.5
                                                                         1 = m i d + 1;
                                                                      else{
87 point rotate(point v, point k, ld theta) {
                                                                          r=mid:
                                                           17
       // Rotaciona o vetor v theta graus em torno do
                                                                      }
                                                           18
       eixo k
                                                           19
                                                                  // bordo
       // theta *= PI/180; // graus
89
                                                           20
       return (
                                                                  // if(r==(int)p.size()-1 and ccw(p[0], p[r], e)
90
           v*cos(theta)) +
                                                                  ==0) return false;
91
92
           ((k^v)*sin(theta)) +
                                                                  // if (r==2 and ccw(p[0], p[1], e)==0) return
           (k*(k*v))*(1-cos(theta)
93
                                                                  false;
                                                                  // if(ccw(p[r], p[r-1], e) == 0) return false;
94
                                                           23
95 }
                                                           24
                                                                  return insideT(p[0], p[r-1], p[r], e);
                                                           25 }
96
97 // 3d line inter / mindistance
_{98} cod d(point p1, point p2, point p3, point p4) {
                                                           27
       return (p2-p1) * (p4-p3);
99
                                                           28 // Any O(n)
100 }
                                                           29
vector < point > inter3d(point p1, point p2, point p3,
                                                           30 int inside(vp &p, point pp){
                                                                  // 1 - inside / 0 - boundary / -1 - outside
       point p4) {
                                                           31
                                                                  int n = p.size();
       cod mua = ( d(p1, p3, p4, p3) * d(p4, p3, p2, p1) 32
        - d(p1, p3, p2, p1) * d(p4, p3, p4, p3))
                                                                  for(int i=0;i<n;i++){</pre>
              / ( d(p2, p1, p2, p1) * d(p4, p3, p4, p3) 34
                                                                      int j = (i+1)%n;
       - d(p4, p3, p2, p1) * d(p4, p3, p2, p1));
                                                                      if(line({p[i], p[j]}).inside_seg(pp))
                                                           3.5
       cod mub = (d(p1, p3, p4, p3) + mua * d(p4, p3,
                                                           36
                                                                          return 0;
       p2, p1) ) / d(p4, p3, p4, p3);
                                                           37
       point pa = p1 + (p2-p1) * mua;
                                                                  int inter = 0;
                                                           38
       point pb = p3 + (p4-p3) * mub;
                                                                  for(int i=0;i<n;i++){</pre>
106
                                                           3.9
       if (pa == pb) return {pa};
                                                           40
                                                                      int j = (i+1)%n;
       return {};
                                                                      if(p[i].x \le pp.x and pp.x \le p[j].x and ccw(p
108
                                                           4.1
109 }
                                                                  [i], p[j], pp)==1)
                                                           42
                                                                          inter++; // up
   4.3 Convex Hull
                                                                      else if(p[j].x \le pp.x and pp.x \le p[i].x and
                                                           43
                                                                  ccw(p[i], p[j], pp) == -1)
                                                                          inter++; // down
                                                           44
 vp convex_hull(vp P)
                                                           45
                                                           46
       sort(P.begin(), P.end());
                                                                  if(inter%2==0) return -1; // outside
                                                           47
       vp L, U;
 4
                                                                  else return 1; // inside
       for(auto p: P){
          while(L.size()>=2 and ccw(L.end()[-2], L.back
       (), p)!=1)
                                                                   Intersect Polygon
               L.pop_back();
           L.push_back(p);
 8
                                                            1 bool intersect(vector<point> A, vector<point> B) //
       reverse(P.begin(), P.end());
                                                                  Ordered ccw
1.0
       for(auto p: P){
           while(U.size()>=2 and ccw(U.end()[-2], U.back 3
                                                                  for(auto a: A)
12
                                                                      if(inside(B, a))
       (), p)!=1)
              U.pop_back();
                                                                          return true;
13
           U.push_back(p);
                                                                  for(auto b: B)
14
                                                                      if(inside(A, b))
16
       L.pop_back();
                                                                          return true;
1.7
       L.insert(L.end(), U.begin(), U.end()-1);
       return L:
18
                                                           10
                                                                  if(inside(B, center(A)))
19 }
                                                                      return true:
                                                           11
   4.4 Inside Polygon
                                                                  return false:
                                                           13
 1 // Convex O(logn)
                                                              4.6
                                                                   Linear Transformation
 3 bool insideT(point a, point b, point c, point e){
       int x = ccw(a, b, e);
                                                            1 // Apply linear transformation (p -> q) to r.
       int y = ccw(b, c, e);
                                                            2 point linear_transformation(point p0, point p1, point
       int z = ccw(c, a, e);
                                                                   q0, point q1, point r) {
       return !((x==1 \text{ or } y==1 \text{ or } z==1) \text{ and } (x==-1 \text{ or } y
                                                                  point dp = p1-p0, dq = q1-q0, num((dp^dq), (dp^dq)
                                                                  ));
       ==-1 \text{ or } z==-1));
 8 }
                                                                  return q0 + point((r-p0)^(num), (r-p0)*(num))/(dp
                                                            4
                                                                  *dp);
10 bool inside(vp &p, point e){ // ccw
                                                            5 }
      int 1=2, r=(int)p.size()-1;
```

```
4.7 Mindistpair
                                                                for(auto ps : p) {
                                                          3
                                                                     long double z = seg.eval(ps);
                                                          4
                                                                     1 = \max(1, z);
                                                          5
1 ll MinDistPair(vp &vet){
                                                          6
                                                                     r = min(r, z);
      int n = vet.size();
                                                                 }
      sort(vet.begin(), vet.end());
                                                                 return 1 - r > EPS;
      set <point > s;
                                                          8
                                                          9 }
                                                          1.0
      ll best_dist = LLINF;
                                                          11 int w, h;
      int j=0;
      for(int i=0;i<n;i++){
                                                          13 line getBisector(point a, point b) {
          ll d = ceil(sqrt(best_dist));
                                                               line ans(a, b);
                                                          14
          while(j < n and vet[i].x-vet[j].x >= d){
              s.erase(point(vet[j].y, vet[j].x));
                                                          15
                                                                swap(ans.a, ans.b);
                                                               ans.b *= -1;
                                                          16
          }
                                                                ans.c = ans.a * (a.x + b.x) * 0.5 + ans.b * (a.y)
13
                                                                 + b.y) * 0.5;
14
                                                                 return ans;
          auto it1 = s.lower_bound({vet[i].y - d, vet[i]}^{18}
      ].x});
          auto it2 = s.upper_bound({vet[i].y + d, vet[i<sup>20</sup>
                                                          21 vp cutPolygon(vp poly, line seg) {
      ].x});
                                                                int n = (int) poly.size();
                                                          22
                                                                 vp ans;
                                                          23
          for(auto it=it1; it!=it2; it++){
18
                                                                for(int i = 0; i < n; i++) {
                                                         2.4
              11 dx = vet[i].x - it->y;
19
                                                                     double z = seg.eval(poly[i]);
                                                         25
               11 dy = vet[i].y - it->x;
                                                                     if(z > -EPS)  {
                                                         26
               if(best_dist > dx*dx + dy*dy){
21
                                                         27
                                                                         ans.push_back(poly[i]);
                   best_dist = dx*dx + dy*dy;
                                                         28
                   // vet[i] e inv(it)
23
                                                                     double z2 = seg.eval(poly[(i + 1) % n]);
                                                         29
               }
24
                                                                     if((z > EPS \&\& z2 < -EPS) || (z < -EPS \&\& z2
                                                         30
          }
                                                                 > EPS)) {
                                                                         ans.push_back(inter_line(seg, line(poly[i
                                                          31
          s.insert(point(vet[i].y, vet[i].x));
                                                                 ], poly[(i + 1) % n]))[0]);
28
      }
                                                          32
29
      return best_dist;
                                                          33
                                                                 }
30 }
                                                                 return ans;
                                                          34
  4.8 Polygon Area
                                                          35 }
                                                          3.6
                                                         37 // BE CAREFUL!
1 ll area = 0;
                                                          38 // the first point may be any point
                                                          39 // O(N^3)
3 for(int i = 0; i < n - 1; ++i){</pre>
                                                          40 vp getCell(vp pts, int i) {
      area += pontos[i].x*pontos[i+1].y - pontos[i+1].x 41
                                                                vp ans;
      *pontos[i].y;
                                                                ans.emplace_back(0, 0);
                                                          42
                                                                ans.emplace_back(1e6, 0);
                                                          43
6 area += pontos[n-1].x*pontos[0].y - pontos[0].x*
                                                                 ans.emplace_back(1e6, 1e6);
                                                          44
      pontos[n-1].y;
                                                                 ans.emplace_back(0, 1e6);
                                                          45
                                                                 for(int j = 0; j < (int) pts.size(); j++) {</pre>
                                                          46
8 area = abs(area):
                                                                   if(j != i) {
                                                                         ans = cutPolygon(ans, getBisector(pts[i],
  4.9 Sort By Angle
                                                                  pts[j]));
                                                                    }
1 // Comparator funcion for sorting points by angle
                                                          50
                                                                 return ans;
3 int ret[2][2] = {{3, 2},{4, 1}};
                                                          52 }
4 inline int quad(point p) {
                                                          5.3
      return ret[p.x >= 0][p.y >= 0];
                                                          _{54} // O(N^2) expected time
6 }
                                                         55 vector < vp > getVoronoi(vp pts) {
                                                                 // assert(pts.size() > 0);
                                                         56
                                                                 int n = (int) pts.size();
8 bool comp(point a, point b) { // ccw
                                                         57
     int qa = quad(a), qb = quad(b);
                                                          58
                                                                 vector < int > p(n, 0);
      return (qa == qb ? (a ^ b) > 0 : qa < qb);
                                                                 for(int i = 0; i < n; i++) {
                                                         59
                                                                    p[i] = i;
11 }
                                                         6.0
                                                         61
^{13} // only vectors in range [x+0, x+180)
                                                                 shuffle(p.begin(), p.end(), rng);
                                                         62
14 bool comp(point a, point b){
                                                          63
                                                                 vector < vp > ans(n);
     return (a ^ b) > 0; // ccw
                                                                 ans[0].emplace_back(0, 0);
                                                         64
      // return (a \hat{} b) < 0; // cw
                                                         65
                                                                 ans[0].emplace_back(w, 0);
                                                         66
                                                                 ans[0].emplace_back(w, h);
                                                          67
                                                                 ans[0].emplace_back(0, h);
  4.10 Voronoi
                                                                 for(int i = 1; i < n; i++) {
                                                          68
                                                                     ans[i] = ans[0];
                                                          69
                                                         7.0
1 bool polygonIntersection(line &seg, vp &p) {
                                                         7.1
                                                                 for(auto i : p) {
      long double 1 = -1e18, r = 1e18;
```

```
for(auto j : p) {
                                                                        11 tmp = run(v.to, sink,min(minE, v.cap-v
73
               if(j == i) break;
                                                                .flow));
               auto bi = getBisector(pts[j], pts[i]);
                                                                         v.flow += tmp, rev.flow -= tmp;
7.4
                                                                        ans += tmp, minE -= tmp;
               if(!polygonIntersection(bi, ans[j]))
                                                         26
                                                                        if(minE == 0) break;
              ans[j] = cutPolygon(ans[j], getBisector( 28
      pts[j], pts[i]));
                                                                    return ans;
              ans[i] = cutPolygon(ans[i], getBisector( 30
                                                                bool bfs(int source, int sink) {
      pts[i], pts[j]));
                                                         31
          }
                                                                    qt = 0;
78
                                                                    qu[qt++] = source;
79
                                                         33
80
      return ans;
                                                         34
                                                                    lvl[source] = 1;
                                                                    vis[source] = ++pass;
81
                                                         3.5
                                                                    for(int i = 0; i < qt; i++) {</pre>
                                                         36
                                                                        int u = qu[i];
       Grafos
                                                         3.7
                                                                        px[u] = 0;
                                                         38
                                                                         if(u == sink) return true;
                                                         39
      Dfs Tree
                                                                         for(auto& ed : g[u]) {
  5.1
                                                         40
                                                                             auto v = edge[ed];
                                                                             if(v.flow >= v.cap || vis[v.to] ==
int desce[N], sobe[N], vis[N], h[N];
                                                                pass)
1 int backedges[N], pai[N];
                                                                                 continue; // v.cap - v.flow < lim</pre>
                                                                             vis[v.to] = pass;
4 // backedges[u] = backedges que comecam embaixo de (
                                                                             lvl[v.to] = lvl[u]+1;
      ou =) u e sobem pra cima de u; backedges[u] == 0
                                                                             qu[qt++] = v.to;
      => u eh ponte
5 void dfs(int u, int p) {
                                                                    }
                                                         48
      if(vis[u]) return;
                                                                    return false;
                                                         49
      pai[u] = p;
                                                         50
      h[u] = h[p]+1;
                                                                11 flow(int source, int sink) {
                                                         5.1
      vis[u] = 1;
9
                                                         52
                                                                    reset_flow();
10
                                                                    ll ans = 0;
                                                         5.3
      for(auto v : g[u]) {
11
                                                                    //for(lim = (1LL << 62); lim >= 1; lim /= 2)
                                                         54
          if(p == v or vis[v]) continue;
                                                         55
                                                                    while(bfs(source, sink))
           dfs(v, u);
                                                                        ans += run(source, sink, LLINF);
                                                         56
          backedges[u] += backedges[v];
14
                                                         57
                                                                    return ans;
                                                         5.8
      for(auto v : g[u]) {
16
                                                                void addEdge(int u, int v, ll c, ll rc) {
                                                         59
          if(h[v] > h[u]+1)
17
                                                         60
                                                                    Edge e = \{u, v, 0, c\};
              desce[u]++;
                                                                    edge.pb(e);
                                                         61
           else if (h[v] < h[u]-1)
19
                                                         62
                                                                    g[u].push_back(ne++);
20
               sobe[u]++;
                                                         63
21
                                                                    e = {v, u, 0, rc};
                                                         64
      backedges[u] += sobe[u] - desce[u];
                                                         6.5
                                                                    edge.pb(e);
23 }
                                                         66
                                                                    g[v].push_back(ne++);
                                                         67
  5.2 Dinic
                                                                void reset_flow() {
                                                         68
                                                                    for(int i = 0; i < ne; i++)</pre>
                                                                       edge[i].flow = 0;
1 const int N = 300;
                                                         7.0
                                                         7.1
                                                                    memset(lvl, 0, sizeof(lvl));
                                                                    memset(vis, 0, sizeof(vis));
                                                         72
s struct Dinic {
                                                                    memset(qu, 0, sizeof(qu));
                                                         73
      struct Edge{
                                                                    memset(px, 0, sizeof(px));
          int from, to; 11 flow, cap;
                                                         74
                                                                    qt = 0; pass = 0;
                                                         7.5
                                                         7.6
      vector < Edge > edge;
                                                                vector<pair<int, int>> cut() {
                                                         7.7
                                                                    vector < pair < int , int >> cuts;
                                                         7.8
      vector < int > g[N];
9
                                                         79
                                                                    for (auto [from, to, flow, cap]: edge) {
      int ne = 0;
10
                                                                        if (flow == cap and vis[from] == pass and
      int lvl[N], vis[N], pass;
                                                         80
                                                                 vis[to] < pass and cap > 0) {
12
      int qu[N], px[N], qt;
                                                                             cuts.pb({from, to});
                                                         8.1
13
                                                         82
      ll run(int s, int sink, ll minE) {
14
          if(s == sink) return minE;
                                                                    }
                                                         83
1.5
                                                                    return cuts;
                                                         84
                                                         85
          11 \text{ ans} = 0:
                                                         86 }:
           for(; px[s] < (int)g[s].size(); px[s]++) {</pre>
19
                                                            5.3 Ford
              int e = g[s][ px[s] ];
20
               auto &v = edge[e], &rev = edge[e^1];
               cap)
                                       // v.cap - v.flow 3 struct Ford {
                   continue:
       < 1 i m
                                                               struct Edge {
```

```
int to, f, c;
                                                             2.0
                                                             21
                                                                    pair <T, vector <int>> assignment() {
6
                                                                        for (int i = 1; i <= n; i++) {
                                                                            p[0] = i;
      int vis[N];
                                                                            int j0 = 0;
       vector < int > adj[N];
       vector < Edge > edges;
                                                                             vector <T> minv(m+1, inf);
10
                                                             25
       int cur = 0;
                                                                             vector < int > used(m+1, 0);
                                                                             do {
       void addEdge(int a, int b, int cap, int rcap) {
                                                                                 used[j0] = true;
13
           Edge e;
                                                                                 int i0 = p[j0], j1 = -1;
14
                                                                                 T delta = inf;
           e.to = b; e.c = cap; e.f = 0;
15
                                                             30
16
           edges.pb(e);
                                                             31
                                                                                 for (int j = 1; j \le m; j++) if (!
           adj[a].pb(cur++);
                                                                    used[j]) {
                                                                                     T cur = a[i0-1][j-1] - u[i0] - v[
18
                                                             32
19
           e = Edge();
                                                                    j];
           e.to = a; e.c = rcap; e.f = 0;
                                                                                     if (cur < minv[j]) minv[j] = cur,</pre>
20
                                                             33
21
           edges pb(e);
                                                                     way[j] = j0;
                                                                                     if (minv[j] < delta) delta = minv</pre>
           adj[b].pb(cur++);
                                                             34
                                                                    [j], j1 = j;
24
                                                             3.5
      int dfs(int s, int t, int f, int tempo) {
                                                                                 for (int j = 0; j <= m; j++)
25
                                                                                      if (used[j]) u[p[j]] += delta, v[
           if(s == t)
26
                                                             37
               return f:
                                                                    il -= delta;
27
           vis[s] = tempo;
                                                                                     else minv[j] -= delta;
                                                                                 j0 = j1;
29
                                                             39
           for(int e : adj[s]) {
                                                                             } while (p[j0] != 0);
3.0
                                                             40
               if(vis[edges[e].to] < tempo and (edges[e 41
31
                                                                             do {
      ].c - edges[e].f) > 0) {
                                                                                 int j1 = way[j0];
                   if(int a = dfs(edges[e].to, t, min(f, 43
                                                                                 p[j0] = p[j1];
        edges[e].c-edges[e].f) , tempo)) {
                                                                                 j0 = j1;
                                                            44
                        edges[e].f += a;
                                                                             } while (j0);
                                                             45
                                                                        }
                        edges[e^1].f -= a;
34
                                                             46
                        return a;
                                                                        vector < int > ans(m);
                                                             47
                    }
                                                                        for (int j = 1; j \le n; j++) ans[p[j]-1] = j
               }
                                                                    -1:
37
           }
                                                                        return make_pair(-v[0], ans);
           return 0:
                                                                    }
39
                                                             5.0
                                                             51 };
40
41
                                                               5.5 Kosaraju
      int flow(int s, int t) {
42
43
           int mflow = 0, tempo = 1;
           while(int a = dfs(s, t, INF, tempo)) {
44
                                                             vector < int > g[N], gi[N]; // grafo invertido
               mflow += a;
45
                                                              2 int vis[N], comp[N]; // componente conexo de cada
46
               tempo++;
                                                                   vertice
47
                                                             stack < int > S;
           return mflow;
48
                                                             4
49
                                                             5 void dfs(int u){
50 };
                                                                    vis[u] = 1;
                                                                    for(auto v: g[u]) if(!vis[v]) dfs(v);
  5.4 Hungarian
                                                                    S.push(u);
                                                             9 }
1 // Hungarian Algorithm
                                                             10
2 //
                                                             void scc(int u, int c){
3 // Assignment problem
                                                                   vis[u] = 1; comp[u] = c;
                                                             12
_{\rm 4} // Put the edges in the 'a' matrix (negative or
                                                                    for(auto v: gi[u]) if(!vis[v]) scc(v, c);
                                                             13
      positive)
                                                             14 }
5 // assignment() returns a pair with the min
                                                             15
      assignment,
                                                             16 void kosaraju(int n){
_{6} // and the column choosen by each row
                                                                    for(int i=0;i<n;i++) vis[i] = 0;</pre>
                                                             17
7 // assignment() - O(n^3)
                                                             18
                                                                    for(int i=0;i<n;i++) if(!vis[i]) dfs(i);</pre>
                                                                    for(int i=0;i<n;i++) vis[i] = 0;
                                                             19
9 template < typename T>
                                                                    while(S.size()){
                                                             20
10 struct hungarian {
                                                             21
                                                                        int u = S.top();
      int n, m;
                                                                        S.pop();
                                                             22
      vector < vector < T >> a;
12
                                                                        if(!vis[u]) scc(u, u);
                                                             23
13
      vector < T > u , v;
                                                                    }
                                                             24
      vector < int > p, way;
                                                             25 }
14
15
      T inf;
                                                                     Math
      \label{eq:hungarian} \mbox{ hungarian(int $n_{-}$, int $m_{-}$) : $n(n_{-})$, $m(m_{-})$, $u(m+1)$,}
1.7
       v(m+1), p(m+1), way(m+1) {
                                                                      Bigmod
           a = vector < vector < T >> (n, vector < T > (m));
                                                               6.1
18
           inf = numeric_limits < T > :: max();
19
```

#### 6.5 Matrix Exponentiation 1 ll mod(string a, ll p) { 11 res = 0, b = 1;reverse(all(a)); 1 struct Matrix { vector <vl> m; for(auto c : a) { int r, c; 3 11 tmp = (((11)c-'0')\*b) % p;4 res = (res + tmp) % p; Matrix(vector<vl> mat) { 5 m = mat;6 b = (b \* 10) % p;r = mat.size(); } 10 c = mat[0].size(); 8 11 9 return res; 10 13 Matrix(int row, int col, bool ident=false) { 11 r = row; c = col; 12 6.2 Division Trick 13 m = vector < vl > (r, vl(c, 0));if(ident) { 14 for(int i = 0; i < min(r, c); i++) { 1.5 1 for(int l = 1, r; l <= n; l = r + 1) {</pre> m[i][i] = 1; r = n / (n / 1);// n / i has the same value for 1 <= i <= r 17 } 18 19 2.0 6.3 Inverso Mult Matrix operator\*(const Matrix &o) const { 21 assert(c == o.r); // garantir que da pra 22 1 // gcd(a, m) = 1 para existir solucao multiplicar $\frac{1}{2}$ // ax + my = 1, ou a\*x = 1 (mod m) vector<vl> res(r, vl(o.c, 0)); 23 3 ll inv(ll a, ll m) { // com gcd 24 11 x, y; for(int i = 0; i < r; i++) {</pre> for(int k = 0; k < c; k++) {</pre> gcd(a, m, x, y); 5 26 for(int j = 0; j < o.c; j++) { res[i][j] = (res[i][j] + m[i][k]\* return (((x % m) +m) %m); 27 7 } o.m[k][j]) % MOD; 9 ll inv(ll a, ll phim) { // com phi(m), se m for primo 29 } entao phi(m) = p-1} 11 e = phim - 1;} return fexp(a, e); 3.2 return Matrix(res); 33 34 6.4 Linear Diophantine Equation 35 }: 37 Matrix fexp(Matrix b, int e, int n) { 1 // Linear Diophantine Equation if(e == 0) return Matrix(n, n, true); // 38 2 int gcd(int a, int b, int &x, int &y) identidade 3 -{ Matrix res = fexp(b, e/2, n); 39 **if** (a == 0) res = (res \* res); 40 5 { if(e%2) res = (res \* b); 41 6 x = 0; y = 1;return b; 43 return res; 44 } 9 int x1, y1; 10 int d = gcd(b%a, a, x1, y1); 6.6Totient x = y1 - (b / a) \* x1;y = x1;12 return d; $_{1}$ // phi(p^k) = (p^(k-1))\*(p-1) com p primo 13 2 // O(sqrt(m)) 14 } 1.5 3 ll phi(ll m){ 16 bool find\_any\_solution(int a, int b, int c, int &x0, ll res = m; for(11 d=2; d\*d<=m; d++) { int &y0, int &g) $if(m \% d == 0){$ res = (res/d)\*(d-1);18 g = gcd(abs(a), abs(b), x0, y0);while (m%d == 0)**if** (c % g) 19 8 20 return false; 9 m /= d;} 10 x0 \*= c / g;} y0 \*= c / g; **if**(m > 1) { 23 12 if (a < 0) x0 = -x0;res /= m; 13 if (b < 0) y0 = -y0; res \*= (m-1); 25 14 26 return true; 15 27 } return res; 16 17 } 29 // All solutions $x_0 / / x = x_0 + k*b/g$ 19 // modificacao do crivo, O(n\*log(log(n)))

20 vector<ll> phi\_to\_n(ll n){

 $_{31}$  // y = y0 - k\*a/g

```
vector < bool > isprime(n+1, true);
                                                         14 void init(){
22
      vector<ll> tot(n+1);
                                                         15
                                                                for(int i = 0; i < ne; i++) fail[i] = 1;
      tot[0] = 0; tot[1] = 1;
                                                                queue < int > q; q.push(1);
23
                                                         16
24
      for(ll i=1;i<=n; i++){
                                                                int u, v;
          tot[i] = i;
                                                         18
                                                                while(!q.empty()){
                                                                    u = q.front(); q.pop();
26
                                                         19
                                                                    for(int i = 0; i < A; i++){</pre>
                                                         20
      for(11 p=2;p<=n;p++){
                                                                         if(to[u][i]){
28
                                                                             v = to[u][i]; q.push(v);
          if(isprime[p]){
29
                                                         22
              tot[p] = p-1;
                                                                             if(u!= 1){
30
              for(ll i=p+p;i<=n;i+=p){</pre>
                                                                                fail[v] = to[ fail[u] ][i];
31
                                                         24
                   isprime[i] = false;
                                                                                 term[v] += term[ fail[v] ];
                   tot[i] = (tot[i]/p)*(p-1);
33
                                                         26
                                                         27
34
                                                                         else if(u != 1) to[u][i] = to[ fail[u] ][
          }
3.5
                                                         28
                                                                i];
36
37
      return tot;
                                                                         else to[u][i] = 1;
38 }
                                                                    }
                                                         3.0
                                                         31
                                                                }
                                                         32 }
       Misc
                                                            8.2 Edit Distance
  7.1 Bitwise
                                                          int edit_distance(int a, int b, string& s, string& t)
1 // Least significant bit (lsb)
                                                                // indexado em 0, transforma s em t \,
      int lsb(int x) { return x&-x; }
                                                                if(a == -1) return b+1;
      int lsb(int x) { return __builtin_ctz(x); } //
                                                                if(b == -1) return a+1;
      bit position
                                                                if(tab[a][b] != -1) return tab[a][b];
4 // Most significant bit (msb)
      int msb(int x) { return 32-1-__builtin_clz(x); }
                                                                int ins = INF, del = INF, mod = INF;
      // bit position
                                                                ins = edit_distance(a-1, b, s, t) + 1;
                                                                del = edit_distance(a, b-1, s, t) + 1;
7 // Power of two
                                                                mod = edit_distance(a-1, b-1, s, t) + (s[a] != t[
      bool isPowerOfTwo(int x) { return x && (!(x&(x-1))
                                                                b]);
      ): }
                                                                return tab[a][b] = min(ins, min(del, mod));
                                                          12
10 // floor(log2(x))
                                                          13 }
int flog2(int x) { return 32-1-_builtin_clz(x); }
12 int flog2ll(ll x) { return 64-1-_builtin_clzll(x); } 8.3 Hash
14 // Built - in functions
                                                          1 // String Hash template
15 // Number of bits 1
                                                          _2 // constructor(s) - O(|s|)
16 __builtin_popcount()
                                                          3 // query(1, r) - returns the hash of the range [1,r]
17 __builtin_popcountll()
                                                                from left to right - O(1)
                                                          4 // query_inv(l, r) from right to left - O(1)
19 // Number of leading zeros
20 __builtin_clz()
                                                          6 struct Hash {
21 __builtin_clzll()
                                                                const 11 P = 31;
                                                                int n; string s;
23 // Number of trailing zeros
                                                                vector<11> h, hi, p;
                                                          9
24 __builtin_ctz()
                                                                Hash() {}
                                                          10
25 __builtin_ctzll()
                                                                Hash(string s): s(s), n(s.size()), h(n), hi(n), p
                                                          11
       Strings
                                                                    for (int i=0;i<n;i++) p[i] = (i ? P*p[i-1]:1)
                                                                 % MOD:
                                                                    for (int i=0;i<n;i++)</pre>
  8.1 Aho Corasick
                                                                        h[i] = (s[i] + (i ? h[i-1]:0) * P) % MOD;
                                                          14
                                                                     for (int i=n-1; i>=0; i--)
1 // https://github.com/joseleite19/icpc-notebook/blob/16
                                                                         hi[i] = (s[i] + (i+1 < n ? hi[i+1]:0) * P)
                                                                % MOD;
      master/code/string/aho_corasick.cpp
2 const int A = 26;
                                                                }
3 int to[N][A];
                                                                int query(int 1, int r) {
                                                          1.8
4 int ne = 2, fail[N], term[N];
                                                          19
                                                                    ll\ hash = (h[r] - (l ? h[l-1]*p[r-l+1]%MOD :
                                                                0));
5 void add_string(string str, int id){
                                                                    return hash < 0 ? hash + MOD : hash;
      int p = 1;
      for(auto c: str){
                                                         21
          int ch = c - 'a'; // !
                                                         22
                                                                int query_inv(int 1, int r) {
          if(!to[p][ch]) to[p][ch] = ne++;
                                                                    11 \text{ hash} = (hi[1] - (r+1 < n ? hi[r+1]*p[r-1]
          p = to[p][ch];
                                                                +1] % MOD : 0));
1.0
                                                                     return hash < 0 ? hash + MOD : hash;
                                                         24
      term[p]++;
12
                                                         2.5
```

26 };

13

```
8.4 Kmp
                                                          26
                                                                if (result == 0)
                                                          28
string p;
                                                                   return string();
                                                          29
1 int neighbor[N];
3 int walk(int u, char c) { // leader after inputting ^{30}
                                                                return X.substr(end - result + 1, result);
      while (u != -1 && (u+1 >= (int)p.size() || p[u + 32 }
      1] != c)) // leader doesn't match
                                                                 Lcsubseq
          u = neighbor[u];
      return p[u + 1] == c ? u+1 : u;
7 }
                                                          1 // Longest Common Subsequence
                                                          2 string lcs(string x, string y){
8 void build() {
                                                                int n = x.size(), m = y.size();
      neighbor[0] = -1; // -1 is the leftmost state
9
                                                                vector < vi > dp(n+1, vi(m+1, 0));
      for (int i = 1; i < (int)p.size(); i++)</pre>
          neighbor[i] = walk(neighbor[i-1], p[i]);
                                                                for(int i=0;i<=n;i++){
                                                                    for(int j=0;j<=m;j++){
                                                                         if(!i or !j)
  8.5 Lcs
                                                                            dp[i][j]=0;
                                                          9
                                                                         else if(x[i-1] == y[j-1])
                                                          10
string LCSubStr(string X, string Y)
                                                          11
                                                                            dp[i][j]=dp[i-1][j-1]+1;
2 {
                                                          12
      int m = X.size();
                                                          13
                                                                             dp[i][j]=max(dp[i-1][j], dp[i][j-1]);
      int n = Y.size();
                                                                    }
4
                                                          14
                                                          15
      int result = 0, end;
                                                          16
      int len[2][n];
                                                                // int len = dp[n][m];
                                                          17
      int currRow = 0;
                                                                string ans="";
                                                          18
9
                                                          1.9
      for(int i=0;i<=m;i++){
10
                                                          20
                                                                // recover string
                                                                int i = n-1, j = m-1;
          for(int j=0;j<=n;j++){
11
                                                          21
               if(i==0 || j==0)
                                                                while(i>=0 and j>=0){
                                                         22
                  len[currRow][j] = 0;
                                                                    if(x[i] == y[j]){
13
                                                         23
              else if(X[i-1] == Y[j-1]){
                                                                        ans.pb(x[i]);
14
                                                          24
                  len[currRow][j] = len[1-currRow][j-1]_{25}
15
                                                                        i - -; j - -;
                                                                    }else if(dp[i][j+1]>dp[i+1][j])
       + 1;
                                                         26
16
                   if(len[currRow][j] > result){
                                                                        i --;
                       result = len[currRow][j];
                                                                     else
                                                         28
                       end = i - 1;
                                                          29
                                                                         j --;
19
                                                          30
              }
20
                                                         31
               else
                                                         32
                                                                reverse(ans.begin(), ans.end());
                  len[currRow][j] = 0;
22
                                                         3.3
          }
                                                         34
                                                                return ans;
                                                         35 }
24
          currRow = 1 - currRow;
```