



# Notebook - Maratona de Programação

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# 1 Algoritmos

## 1.1 Mochila

```
1 int val[MAXN], peso[MAXN], dp[MAXN][MAXS]
2
3 int knapsack(int N, int M) // Objetos | Peso max
4 {
5     for(i=0;i<=N;i++)
6     {
7         for(j=0;j<=M;j++)
8         {
9             if (i==0 || j==0)
10                dp[i][j] = 0;
11             else if (peso[i-1] <= j)
12                dp[i][j] = max(val[i-1]+dp[i-1][j-
13                    peso[i-1]], dp[i-1][j]);
14             else
15                dp[i][j] = dp[i-1][j];
16         }
17     }
18     return dp[N][M];
19 }
```

## 1.2 Kadane-DP

```
1 // Largest Sum Contiguous Subarray
2
3 int maxSubArraySum(vector<int> a)
4 {
5     int size = a.size();
6     int max_so_far = a[0];
7     int curr_max = a[0];
8
9     for (int i=1;i<size;i++)
10     {
11         curr_max = max(a[i], curr_max+a[i]);
12         max_so_far = max(max_so_far, curr_max);
13     }
14     return max_so_far;
15 }
```

## 1.3 Iterative-BS

```
1 int main()
2 {
3     int l=1, r=N;
4     int res=-1;
5
6     while(l <= r)
7     {
8         int m = (l + r)/2;
9         if(!ver(m))
10         {
11             l = m+1;
12         }
13         else
14         {
15             res = m;
16             r = m-1;
17         }
18     }
19     cout << res << endl;
20
21     return 0;
22 }
```

# 2 Grafos

## 2.1 BFS

```
1 //BFS (Breadth First Search) O(V+A)
2
3 vector<vector<int>> adj; // adjacency list
4 // representation
5 int n; // number of nodes
6 int s; // source vertex
7
8 queue<int> q;
9 vector<int> d(n, INF);
10 d[s]=0;
11
12 q.push(s);
13 used[s] = true;
14 while (!q.empty()) {
15     int v = q.front();
16     q.pop();
17     for (int u : adj[v]) {
18         if (d[u] > d[v] + 1) {
19             q.push(u);
20             d[u] = d[v] + 1;
21         }
22     }
23 }
```

## 2.2 Find-bridges

```
1 #define vi vector<int>
2
3 vector< vector<int> > grafo;
4 vector<bool> visited;
5 vi t, low;
6 int timer=0;
7
8 void find_bridges(int v, int p=-1)
9 {
10     visited[v] = true;
11     t[v] = low[v] = timer++;
12     for(int i=0;i<(int)grafo[v].size();i++)
13     {
14         int vert = grafo[v][i];
15         if(vert == p)
16             continue;
17         if(visited[vert])
18             low[v] = min(low[v], t[vert]);
19         else
20         {
21             find_bridges(vert, v);
22             low[v] = min(low[v], low[vert]);
23             if(low[tol] > t[v])
24                 IS_BRIDGE(v, vert);
25         }
26     }
27 }
28
29 int main()
30 {
31     timer = 0;
32     visited.assign(N+1, false);
33     t.assign(N+1, 0);
34     low.assign(N+1, 0);
35
36     for(int i=0;i<N;i++)
37         if(!visited[i])
38             find_bridges(i);
39
40     return 0;
41 }
```

## 2.3 Dijkstra

```
1 // Dijkstra - Shortest Path
2
```

```

3 #define pii pair<int, int>
4 #define vi vector<int>
5 #define vii vector< pair<int,int> >
6 #define INF 0x3f3f3f3f
7
8 vector<vii> grafo(N+1, vii());
9 vi distancia(N+1, INF);
10 priority_queue< pii, vii, greater<pii> > fila;
11
12 void dijkstra(int k)
13 {
14     int dist, vert, aux;
15     distancia[k]=0;
16
17     fila.push(mp(k, 0));
18
19     while(!fila.empty())
20     {
21         aux=fila.top().f;
22         fila.pop();
23
24         for(auto v: grafo[aux])
25         {
26             vert=v.f;
27             dist=v.s;
28             if(distancia[vert]>distancia[aux]+dist)
29             {
30                 distancia[vert]=distancia[aux]+dist;
31                 fila.push(mp(vert, distancia[vert]));
32             }
33         }
34     }
35 }
36
37 int main()
38 {
39     for(int i=0; i<M; i++)
40     {
41         cin >> a >> b >> p;
42         grafo[a].pb(mp(b, p));
43         grafo[b].pb(mp(a, p));
44     }
45 }

```

## 2.4 LCA

```

1 const int K = 100;
2 int logv[MAX+1];
3 int st[MAX][K];
4 vector<vi> grafo(200010, vi());
5
6 void make(){
7     logv[1] = 0; // pre-computar tabela de log
8     for (int i = 2; i <= MAX; i++)
9         logv[i] = logv[i/2] + 1;
10 }
11
12 void precompute(int N, int array[]) { //
13     for (int i = 0; i < N; i++)
14         st[i][0] = array[i];
15
16     int k = logv[N];
17     for (int j = 1; j <= k; j++)
18         for (int i = 0; i + (1 << j) <= N; i++)
19             st[i][j] = min(st[i][j-1], st[i + (1 << (j
20             - 1))][j - 1]);
21 }
22
23 int query(int L, int R) {
24     int j = logv[R - L + 1];
25     int minimum = min(st[L][j], st[R - (1 << j) + 1][
26     j]);

```

```

25
26     return minimum;
27 }
28
29 int start[MAX+1], dfs_time;
30 int tour[2*MAX+1], id[2*MAX+1];
31
32 void dfs(int u, int pai=-1){
33     start[u] = dfs_time;
34     id[dfs_time] = u;
35     tour[dfs_time++] = start[u];
36     for(int v : grafo[u]){
37         if(v==pai)
38             continue;
39         dfs(v, u);
40         id[dfs_time] = u;
41         tour[dfs_time++] = start[u];
42     }
43 }
44
45 int LCA(int u, int v)
46 {
47     if(start[u] > start[v])
48         swap(u, v);
49     return id[query(start[u], start[v])];
50 }
51
52 int main()
53 {
54     int N, k, a, b;
55     cin >> N;
56
57     for(int i=0; i<N-1; i++)
58     {
59         cin >> a >> b;
60         grafo[a].pb(b);
61         grafo[b].pb(a);
62     }
63     dfs(1);
64
65     make();
66     precompute(2*N, tour);
67
68
69     cin >> k;
70     for(int i=0; i<k; i++)
71     {
72         cin >> a >> b;
73         cout << LCA(a, b) << endl;
74     }
75
76     return 0;
77 }

```

## 2.5 Floyd-Warshall

```

1 // Floyd Warshall
2
3 int dist[MAX][MAX];
4
5 void Floydwarshall()
6 {
7     for(int k = 1; k <= n; k++)
8         for(int i = 1; i <= n; i++)
9             for(int j = 1; j <= n; j++)
10                 dist[i][j] = min(dist[i][j], dist[i][
11                 k] + dist[k][j]);

```

## 2.6 Kruskal

```

1 // deve-se ter dsu codada com as funcoes make_set,
2 find_set e union_sets

```

```

2 struct Edge {
3     int u, v, weight;
4     bool operator<(Edge const& other) {
5         return weight < other.weight;
6     }
7 };
8
9 int n;
10 vector<Edge> edges;
11
12 int cost = 0;
13 vector<Edge> result;
14 for (int i = 0; i < n; i++)
15     make_set(i);
16
17 sort(edges.begin(), edges.end());
18
19 for (Edge e : edges) {
20     if (find_set(e.u) != find_set(e.v)) {
21         cost += e.weight;
22         result.push_back(e); // vector com as arestas
23         da MST
24         union_sets(e.u, e.v);
25     }
26 }

```

## 2.7 DFS

```

1 //DFS (Depth First Search) O(V+A)
2
3 void DFS(int x)
4 {
5     for(int i=0; i<(int)vizinhos[x].size(); i++)
6     {
7         int v = vizinhos[x][i];
8         if(componente[v] == -1)
9         {
10             componente[v] = componente[x];
11             DFS(v);
12         }
13     }
14 }

```

## 2.8 Kosaraju

```

1 // KOSARAJU - O(V+E) - encontra componentes
2 // fortemente conexos
3 // g -> grafo, gt -> grafo tempo
4 // vis -> visitado, cor -> componente fortemente
5 // conexo ordenado topologicamente
6 vector<int> g[N], gt[N], S; int vis[N], cor[N];
7 void dfs(int u){
8     vis[u] = 1; for(int v : g[u]) if(!vis[v]) dfs(v);
9     S.push_back(u);
10 }
11 void dfst(int u, int e){
12     cor[u] = e;
13     for(int v : gt[u]) if(!cor[v]) dfst(v, e);
14 }
15 void kosaraju(){
16     for(int i = 1; i <= n; i++) if(!vis[i]) dfs(i);
17     for(int i = 1; i <= n; i++) for(int j : g[i])
18         gt[j].push_back(i);
19     int e = 0; reverse(S.begin(), S.end());
20     for(int u : S) if(!cor[u]) dfst(u, ++e);
21 }

```

## 2.9 Represent

```

1 // Grafos
2
3 // List of edges

```

```

4
5 vector< pair<int, int> > arestas;
6 arestas.push_back(make_pair(1, 2));
7 arestas.push_back(make_pair(1, 3));
8
9 // Adjacency Matrix
10
11 int grafo[10][10];
12
13 grafo[1][2] = grafo[2][1] = 1;
14 grafo[1][3] = grafo[3][1] = 2;
15
16 // Adjacency List
17
18 vector<int> vizinhos[10];
19
20 vizinhos[1].push_back(2);
21 vizinhos[1].push_back(3);

```

## 2.10 Centroid

```

1 vi g[MAX];
2 int size[MAX];
3 bool erased[MAX]; // vetor dos vertices apagados na
4 // decomp.
5
6 int sz(int u, int p) {
7     int s = 1;
8     for(auto prox : g[u]) {
9         if(prox != p and !erased[prox])
10             s += sz(prox, u);
11     }
12     return size[u] = s;
13 }
14
15 int centroid(int u, int p, int n) {
16     // chamar funcao sz antes, n = size[u]
17     for(auto prox : g[u]) {
18         if(prox != p and !erased[prox]) {
19             if(size[prox] > n/2) {
20                 return centroid(prox, u, n);
21             }
22         }
23     }
24     return u;
25 }

```

## 2.11 Prim

```

1 // Prim Algorithm
2 #define MAXN 10100
3 #define INFINITO 999999999
4
5 int n, m;
6 int distancia[MAXN];
7 int processado[MAXN];
8 vector<pii> vizinhos[MAXN];
9
10 int Prim()
11 {
12     for(int i = 2; i <= n; i++) distancia[i] = INFINITO;
13     distancia[1] = 0;
14
15     priority_queue< pii, vector<pii>, greater<pii> >
16     fila;
17     fila.push( pii(distancia[1], 1) );
18
19     while(1)
20     {
21         int davez = -1;

```

```

22     while(!fila.empty())
23     {
24         int atual = fila.top().second;
25         fila.pop();
26
27         if(!processado[atual])
28         {
29             davez = atual;
30             break;
31         }
32     }
33
34     if(davez == -1)
35         break;
36
37     processado[davez] = true;
38
39     for(int i = 0; i < (int)vizinhos[davez].size(); i++)
40     {
41
42         int dist = vizinhos[davez][i].first;
43         int atual = vizinhos[davez][i].second;
44
45         if( distancia[atual] > dist && !
processado[atual])
46         {
47             distancia[atual] = dist;
48             fila.push( pii(distancia[atual],
atual) );
49         }
50     }
51 }
52
53 int custo_arvore = 0;
54 for(int i = 1; i <= n; i++)
55     custo_arvore += distancia[i];
56
57 return custo_arvore;
58 }
59
60 int main(){
61
62     cin >> n >> m;
63
64     for(int i = 1; i <= m; i++){
65
66         int x, y, tempo;
67         cin >> x >> y >> tempo;
68
69         vizinhos[x].pb( pii(tempo, y) );
70         vizinhos[y].pb( pii(tempo, x) );
71     }
72
73     cout << Prim() << endl;
74
75     return 0;
76 }

```

## 3 Geometria

### 3.1 Inter-Retas

```

1 // Intersection between lines
2
3 typedef struct
4 {
5     int x, y;
6 } pnt;
7
8 bool collinear(pnt p, pnt q, pnt r)
9 {

```

```

10     if(q.x<=max(p.x,r.x) && q.x>=min(p.x,r.x) && q.y
<=max(p.y,r.y) && q.y>=min(p.y,r.y))
11         return true;
12
13     return false;
14 }
15
16 int orientation(pnt p, pnt q, pnt r)
17 {
18     int val=(q.y-p.y)*(r.x-q.x)-(q.x-p.x)*(r.y-q.y);
19
20     if(val==0)
21         return 0;
22     else if(val>0)
23         return 1;
24     else
25         return 2;
26 }
27
28 bool intersect(pnt p1, pnt q1, pnt p2, pnt q2)
29 {
30     int o1 = orientation(p1, q1, p2);
31     int o2 = orientation(p1, q1, q2);
32     int o3 = orientation(p2, q2, p1);
33     int o4 = orientation(p2, q2, q1);
34
35     if(o1!=o2 and o3!=o4)
36         return true;
37
38     if(o1==0 && collinear(p1, p2, q1))
39         return true;
40
41     if(o2==0 && collinear(p1, q2, q1))
42         return true;
43
44     if(o3==0 && collinear(p2, p1, q2))
45         return true;
46
47     if(o4==0 && collinear(p2, q1, q2))
48         return true;
49
50     return false;
51 }
52 }

```

### 3.2 Rotation

```

1 // Rotate clockwise 90 degree
2 (x, y) => (y, -x)
3
4 // Rotate counterclockwise 90 degree
5 (x, y) => (-y, x)

```

### 3.3 Inter-Retangulos

```

1 typedef struct
2 {
3     int x, y;
4 } Point;
5
6 bool doOverlap(Point l1, Point r1, Point l2, Point r2)
7 {
8     if (l1.x>r2.x or l2.x>r1.x or l1.y<r2.y or l2.y<
r1.y)
9         return false;
10     return true;
11 }

```

### 3.4 Analytic-Geometry

```

1 struct point
2 {
3     double x, y;
4     point(double _x=0, double _y=0){
5         x=_x;y=_y;
6     }
7
8     void show(){
9         cout << "x = " << x << endl;
10        cout << "y = " << y << endl;
11    }
12
13    point operator+(const point &o) const{
14        return {x + o.x, y + o.y};
15    }
16    point operator-(const point &o) const{
17        return {x - o.x, y - o.y};
18    }
19    bool operator==(const point &o) const{
20        return (x == o.x and y == o.y);
21    }
22
23 };
24
25 struct line
26 {
27     point fp, sp;
28     line(point _fp=0, point _sp=0){
29         fp=_fp;sp=_sp;
30     }
31
32     //a=y1-y2;
33     //b=x2-x1;
34     //c=x2*y1-y2*x1;
35
36 };
37
38 // Produto Escalar
39 double dot(point a, point b){
40     return a.x*b.x + a.y*b.y;
41 }
42
43 // Produto Vetorial
44 double cross(point a, point b){
45     return a.x*b.y - a.y*b.x;
46 }
47
48 // Dist entre dois pontos
49 double dist(point a, point b){
50     point c = a - b;
51     return sqrt(c.x*c.x + c.y*c.y);
52 }
53
54 // Colinearidade entre 3 pontos
55 bool collinear(point a, point b, point c){
56     return ((c.y-b.y)*(b.x-a.x)==(b.y-a.y)*(c.x-b.x))
57     ;
58     // return (a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b
59     .y)); // Triangle area
60     // No caso de pontos tridimensionais, usar
61     produto vetorial.
62 }
63
64 // Dist entre ponto e reta
65 double distr(point a, line b){
66     double crs = cross(point(a - b.fp), point(b.sp -
67     b.fp));
68     return abs(crs/dist(b.fp, b.sp));
69 }
70
71 void esq(point a, point b, point ext)
72 { // Esquerda = 1; Direita = -1; Collinear = 0;
73     ll v = a.x*b.y+b.x*ext.y+ext.x*a.y - (a.y*b.x+b.y
74

```

```

75     *ext.x+ext.y*a.x);
76     if(v>0) return 1;
77     if(v==0) return 0;
78     return -1;
79 }
80
81 // Area de um poligono (pontos ordenados por
82     adjacencia)
83 double area(vector <point> p){
84     double ret = 0;
85     for(int i=2;i<(int)p.size();i++)
86         ret += cross(p[i] - p[0], p[i-1] - p[0])/2;
87     return abs(ret);
88 }
89
90 // Concavo ou Convexo
91 double ccw(point a, point b, point c){
92     double ret = cross(b - a, c - b);
93     return ret < 0;
94 }

```

## 4 ED

### 4.1 Range-query-bigger-than-k-BIT

```

1 // C++ program to print the number of elements
2 // greater than k in a subarray of range L-R.
3 #include <bits/stdc++.h>
4 using namespace std;
5
6 // Structure which will store both
7 // array elements and queries.
8 struct node {
9     int pos;
10    int l;
11    int r;
12    int val;
13 };
14
15 // Boolean comparator that will be used
16 // for sorting the structural array.
17 bool comp(node a, node b)
18 {
19     // If 2 values are equal the query will
20     // occur first then array element
21     if (a.val == b.val)
22         return a.l > b.l;
23
24     // Otherwise sorted in descending order.
25     return a.val > b.val;
26 }
27
28 // Updates the node of BIT array by adding
29 // 1 to it and its ancestors.
30 void update(int* BIT, int n, int idx)
31 {
32     while (idx <= n) {
33         BIT[idx]++;
34         idx += idx & (-idx);
35     }
36 }
37
38 // Returns the count of numbers of elements
39 // present from starting till idx.
40 int query(int* BIT, int idx)
41 {
42     int ans = 0;
43     while (idx) {
44         ans += BIT[idx];
45         idx -= idx & (-idx);
46     }
47     return ans;
48 }

```

```

49 // Function to solve the queries offline
50 void solveQuery(int arr[], int n, int QueryL[],
51               int QueryR[], int QueryK[], int q)
52 {
53     // create node to store the elements
54     // and the queries
55     node a[n + q + 1];
56     // 1-based indexing.
57
58     // traverse for all array numbers
59     for (int i = 1; i <= n; ++i) {
60         a[i].val = arr[i - 1];
61         a[i].pos = 0;
62         a[i].l = 0;
63         a[i].r = i;
64     }
65
66     // iterate for all queries
67     for (int i = n + 1; i <= n + q; ++i) {
68         a[i].pos = i - n;
69         a[i].val = QueryK[i - n - 1];
70         a[i].l = QueryL[i - n - 1];
71         a[i].r = QueryR[i - n - 1];
72     }
73
74     // In-built sort function used to
75     // sort node array using comp function.
76     sort(a + 1, a + n + q + 1, comp);
77
78     // Binary Indexed tree with
79     // initially 0 at all places.
80     int BIT[n + 1];
81
82     // initially 0
83     memset(BIT, 0, sizeof(BIT));
84
85     // For storing answers for each query( 1-based
86     // indexing ).
87     int ans[q + 1];
88
89     // traverse for numbers and query
90     for (int i = 1; i <= n + q; ++i) {
91         if (a[i].pos != 0) {
92             // call function to returns answer for
93             // each query
94             int cnt = query(BIT, a[i].r) - query(BIT,
95             a[i].l - 1);
96
97             // This will ensure that answer of each
98             // query
99             // are stored in order it was initially
100             // asked.
101             ans[a[i].pos] = cnt;
102         }
103         else {
104             // a[i].r contains the position of the
105             // element in the original array.
106             update(BIT, n, a[i].r);
107         }
108     }
109
110     // Output the answer array
111     for (int i = 1; i <= q; ++i) {
112         cout << ans[i] << endl;
113     }
114 }
115
116 // Driver Code
117 int main()
118 {
119     int arr[] = { 7, 3, 9, 13, 5, 4 };
120     int n = sizeof(arr) / sizeof(arr[0]);

```

```

117 // 1-based indexing
118 int QueryL[] = { 1, 2 };
119 int QueryR[] = { 4, 6 };
120
121 // k for each query
122 int QueryK[] = { 6, 8 };
123
124 // number of queries
125 int q = sizeof(QueryL) / sizeof(QueryL[0]);
126
127 // Function call to get
128 solveQuery(arr, n, QueryL, QueryR, QueryK, q);
129
130 return 0;
131 }
132

```

## 4.2 Iterative-SegTree

```

1 // Segment Tree Iterativa - Range maximum query
2
3 #define N 100010
4
5 struct Segtree
6 {
7     int t[2*N]={0};
8
9     void build()
10     {
11         for(int i=N-1; i>0; i--)
12             t[i]=max(t[i<<1], t[1<<1|1]);
13     }
14
15     int query(int l, int r)
16     {
17         int ans=0;
18         for(i+=N, r+=N; l<r; l>>=1, r>>=1)
19         {
20             if(l&1)
21                 ans=max(ans, t[l++]);
22             if(r&1)
23                 ans=max(ans, t[--r]);
24         }
25
26         return ans;
27     }
28
29     void update(int p, int value)
30     {
31         for(t[p+=N]=value; p>1; p>>=1)
32             t[p>>1]= max(t[p], t[p^1]);
33     }
34 };
35
36 int main()
37 {
38     Segtree st;
39
40     for(int i=0;i<n;i++)
41     {
42         cin >> aux;
43         st.t[N+i]=aux; //Leaves are stored in
44         // continuous nodes with indices starting with N
45     }
46
47     st.build();
48     x = st.query(inicio, fim);
49     st.update(ind, value);
50 }
51

```

## 4.3 Recursive-SegTree

```
1 // Segment Tree Recursiva - Range maximum query
2
3 vector<int> val(MAX, 0);
4 vector<int> vet(N);
5
6 void monta(int i, int j, int no)
7 {
8     if(i==j)
9     {
10         val[no]=vet[i];
11         return;
12     }
13
14     int esq = 2*no;
15     int dir = 2*no+1;
16     int meio = (i+j)/2;
17
18     monta(i, meio, esq);
19     monta(meio+1, j, dir);
20
21     val[no]=max(val[esq], val[dir]);
22 }
23
24 void atualiza(int no, int i, int j, int pos, int
    novo_valor)
25 {
26     if(i==j)
27     {
28         val[no]=novo_valor;
29     }else
30     {
31         int esq = 2*no;
32         int dir = 2*no+1;
33         int meio = (i+j)/2;
34
35         if(pos<=meio)
36             atualiza(esq, i, meio, pos, novo_valor);
37         else
38             atualiza(dir, meio+1, j, pos, novo_valor)
39
40         ;
41
42         if(val[esq]>val[dir])
43             val[no]=val[esq];
44         else
45             val[no]=val[dir];
46     }
47 }
48
49 int consulta(int no, int i, int j, int A, int B)
50 {
51     if(i>B || j<A)
52         return -1;
53     if(i>=A and j<=B)
54         return val[no];
55
56     int esq = 2*no;
57     int dir = 2*no+1;
58     int meio = (i+j)/2;
59
60     int resp_esq = consulta(esq, i, meio, A, B);
61     int resp_dir = consulta(dir, meio+1, j, A, B);
62
63     if(resp_dir==-1)
64         return resp_esq;
65     if(resp_esq==-1)
66         return resp_dir;
67
68     if(resp_esq>resp_dir)
69         return resp_esq;
70     else
71         return resp_dir;
```

```
70 }
71
72 int main()
73 {
74     monta(1, N, 1);
75     atualiza(1, 1, N, pos, valor);
76     x = consulta(1, 1, N, inicio, fim);
77 }
78 }
```

## 4.4 Delta-Encoding

```
1 // Delta encoding
2
3 for(int i=0;i<q;i++)
4 {
5     int l,r,x;
6     cin >> l >> r >> x;
7     delta[l] += x;
8     delta[r+1] -= x;
9 }
10
11 int atual = 0;
12
13 for(int i=0;i<n;i++)
14 {
15     atual += delta[i];
16     v[i] += atual;
17 }
```

## 4.5 Seg-Tree-Farao

```
1 typedef struct
2 {
3     pii prefix, sufix, total, maximo;
4 } no;
5
6 int noleft[MAX], noright[MAX]; //Guarda os valores
    dos nos para que nao sejam calculados novamente
    nas queries
7 int v[MAX];
8 no arvore[MAX];
9
10 pii somar(pii a, pii b) // une pairs
11 {
12     return mp(a.f+b.f, a.s+b.s);
13 }
14
15 no une(no l, no r)
16 {
17     if(l.total.s==0)
18         return r;
19     if(r.total.s==0)
20         return l;
21
22     no m;
23
24     m.prefix = max(l.prefix, somar(l.total, r.prefix)
25 ); //prefixo
26     m.sufix = max(r.sufix, somar(r.total, l.sufix));
27     //sufixo
28     m.total = somar(l.total, r.total); //Soma de
29     todos os elementos da subarvore
30     m.maximo = max(max(l.maximo, r.maximo), somar(l.
31 sufix, r.prefix)); //Resultado para cada
32 subarvore
33
34     return m;
35 }
36
37 no makenozero()
38 {
39 }
```



```

34     no m;
35     m.prefix=m.suffix=m.total=m.maximo=mp(0,0);
36     return m;
37 }
38
39 no makeno(int k)
40 {
41     no m;
42     m.prefix=m.suffix=m.total=m.maximo=mp(k,1);
43     return m;
44 }
45
46 void monta(int n)
47 {
48     if(noleft[n]==noright[n])
49     {
50         arvore[n]=makeno(v[noleft[n]]);
51         return;
52     }
53
54     int mid = (noleft[n]+noright[n])/2;
55     noleft[2*n]=noleft[n]; noright[2*n]=mid;
56     noleft[2*n+1]=mid+1; noright[2*n+1]=noright[n];
57
58     monta(2*n);
59     monta(2*n+1);
60
61     arvore[n]=une(arvore[2*n], arvore[2*n+1]);
62 }
63
64 no busca(int n, int esq, int dir)
65 {
66     if(noleft[n]>=esq and noright[n]<=dir)
67         return arvore[n];
68     if(noright[n]<esq or noleft[n]>dir)
69         return makenozer();
70
71     return une(busca(2*n, esq, dir), busca(2*n+1, esq,
72         dir));
73 }
74
75 int main()
76 {
77     int T, N, Q, A, B;
78     no aux;
79
80     scanf("%d", &T);
81
82     while(T--)
83     {
84         scanf("%d", &N);
85         for(int i=1; i<=N; i++)
86             scanf("%d", &v[i]); //Elementos da arvore
87
88         noleft[1]=1; noright[1]=N;
89         monta(1);
90
91         cin >> Q;
92         while(Q--)
93         {
94             scanf("%d%d", &A, &B); //Intervalo da
95             query
96                 aux = busca(1, A, B);
97                 printf("%d %d\n", aux.maximo.f, aux.
98                 maximo.s);
99         }
100
101     return 0;
102 }

```

## 4.6 BIT-2D

```

1 // BIT 2D
2
3 int bit[MAX][MAX];
4
5 int sum(int x, int y)
6 {
7     int resp=0;
8
9     for(int i=x; i>0; i-=i&-i)
10         for(int j=y; j>0; j-=j&-j)
11             resp+=bit[i][j];
12
13     return resp;
14 }
15
16 void update(int x, int y, int delta)
17 {
18     for(int i=x; i<MAX; i+=i&-i)
19         for(int j=y; j<MAX; j+=j&-j)
20             bit[i][j]+=delta;
21 }
22
23 int query(int x1, y1, x2, y2)
24 {
25     return sum(x2,y2) - sum(x2,y1) - sum(x1,y2) + sum
26         (x1,y1);
27 }

```

## 4.7 BIT

```

1 struct FT {
2     vector<int> bit; // indexado em 1
3     int n;
4
5     FT(int n) {
6         this->n = n + 1;
7         bit.assign(n + 1, 0);
8     }
9
10    int sum(int idx) {
11        int ret = 0;
12        for (++idx; idx > 0; idx -= idx & -idx)
13            ret += bit[idx];
14        return ret;
15    }
16
17    int sum(int l, int r) {
18        return sum(r) - sum(l - 1);
19    }
20
21    void add(int idx, int delta) {
22        for (++idx; idx <= n; idx += idx & -idx)
23            bit[idx] += delta;
24    }
25 };

```

## 4.8 Sparse-Table

```

1 logv[1] = 0; // pre-computar tabela de log
2 for (int i = 2; i <= MAXN; i++)
3     logv[i] = logv[i/2] + 1;
4
5 int logv[MAXN+1];
6 int st[MAXN][K];
7
8 // operacao da sparse table deve ser idempotente ->
9 op(x, x) = x
10 void precompute(int N) { //
11     for (int i = 0; i < N; i++)
12         st[i][0] = array[i];

```

```

12
13 int k = logv[N];
14 for (int j = 1; j <= k; j++)
15     for (int i = 0; i + (1 << j) <= N; i++)
16         st[i][j] = max(st[i][j-1], st[i + (1 << (j
17 - 1))][j - 1]);
18 }
19 int query(int L, int R) {
20     int j = logv[R - L + 1];
21     int maximum = max(st[L][j], st[R - (1 << j) + 1][
22 j]);
23     return maximum;
24 }

```

## 4.9 Union-Find

```

1 // Union-Find Functions
2
3 int pai[MAX], peso[MAX];
4
5 int find(int aux)
6 {
7     if(pai[aux]==aux)
8         return aux;
9     else
10         return pai[aux]=find(pai[aux], pai);
11 }
12
13 void join(int x, int y)
14 {
15     x = find(x);
16     y = find(y);
17
18     if(pesos[x]<pesos[y])
19         pai[x] = y;
20     else if(pesos[x]>pesos[y])
21         pai[y] = x;
22     else if(pesos[x]==pesos[y])
23     {
24         pai[x] = y;
25         pesos[y]++;
26     }
27 }
28
29 int main()
30 {
31     for(int i=1;i<=N;i++)
32         pai[i]=i;
33 }

```

## 4.10 Mo

```

1 const int BLK = 500; // tamanho do bloco, algo entre
   300 e 500 e nice
2
3 struct Query {
4     int l, r, idx;
5     bool operator<(Query other) const
6     {
7         return make_pair(l / BLK, r) <
8         make_pair(other.l / BLK, other.r);
9     }
10 };
11
12 void add(); void remove() // implementar operacoes de
   acordo com o problema, cuidado com TLE ao
   utilizar MAP
13
14 vector<pair<int,ll>> mo() {
15     vector<pair<int,ll>> res;

```

```

16     sort(queries.begin(), queries.end());
17
18     int l = 0, r = -1;
19     for(Query q : queries) {
20         while(l > q.l) {
21             l--;
22             add(l);
23         }
24         while(r < q.r) {
25             r++;
26             add(r);
27         }
28         while(l < q.l) {
29             remove(l);
30             l++;
31         }
32         while(r > q.r) {
33             remove(r);
34             r--;
35         }
36         res.pb(mp(q.idx, RESPOSTA)); // adicionar
   resposta de acordo com o problema
37     }
38     return res; // ordenar o vetor pelo indice e
   responder queries na ordem
39 }

```

## 5 Math

### 5.1 Totient

```

1 // phi(p^k) = (p^(k-1))*(p-1) com p primo
2 // O(sqrt(m))
3 ll phi(ll m) {
4     ll res = m;
5     for(ll d = 2; d*d <= m; d++) {
6         if(m % d == 0) {
7             res = (res/d) * (d-1);
8             while(m % d == 0) {
9                 m /= d;
10             }
11         }
12     }
13     if(m > 1) {
14         res /= m;
15         res *= (m-1);
16     }
17     return res;
18 }
19
20
21 // modificacao do crivo, O(n*log(log(n)))
22 vector<ll> phi_to_n(ll n) {
23     vector<bool> isprime(n+1, true);
24     vector<ll> tot(n+1);
25     tot[0] = 0; tot[1] = 1;
26     for(ll i = 1; i <= n; i++) {
27         tot[i] = i;
28     }
29
30     for(ll p = 2; p <= n; p++) {
31         if(isprime[p]) {
32             tot[p] = p-1;
33             for(ll i = p+p; i <= n; i += p) {
34                 isprime[i] = false;
35                 tot[i] = (tot[i]/p)*(p-1);
36             }
37         }
38     }
39
40     return tot;
41 }

```

## 5.2 Linear-Diophantine-Equation

```
1 // Linear Diophantine Equation
2 int gcd(int a, int b, int &x, int &y)
3 {
4     if (a == 0)
5     {
6         x = 0; y = 1;
7         return b;
8     }
9     int x1, y1;
10    int d = gcd(b%a, a, x1, y1);
11    x = y1 - (b / a) * x1;
12    y = x1;
13    return d;
14 }
15
16 bool find_any_solution(int a, int b, int c, int &x0,
17 int &y0, int &g)
18 {
19     g = gcd(abs(a), abs(b), x0, y0);
20     if (c % g)
21         return false;
22
23     x0 *= c / g;
24     y0 *= c / g;
25     if (a < 0) x0 = -x0;
26     if (b < 0) y0 = -y0;
27     return true;
28 }
29 // All solutions
30 // x = x0 + k*b/g
31 // y = y0 - k*a/g
```

## 5.3 Sum-n2

Soma dos n primeiros números ao quadrado =  $\frac{(2N^3+3N^2+N)}{6}$

## 5.4 Factorization-sqrt

```
1 // Factorization of a number in sqrt(n)
2
3 int main()
4 {
5     ll N;
6     vector<int> div;
7
8     cin >> N;
9
10    for(ll i=2;i*i<=N;i++)
11    {
12        if(N%i==0)
13        {
14            vet.pb(i);
15            while(N%i==0)
16                N/=i;
17        }
18    }
19    if(N!=1)
20        vet.pb(N);
21
22    return 0;
23 }
```

## 5.5 Modular-Exponentiation

```
1 // Modular exponentiaion - (x^y)%mod in O(log y)
2 ll power(ll x, ll y, ll mod)
3 {
4     ll res = 1;
```

```
5     x%=mod;
6
7     while(y)
8     {
9         if(y&1)
10             res=(res*x)%mod;
11
12         y=y>>1;
13         x=(x*x)%mod;
14     }
15     return res;
16 }
```

## 5.6 Miller-Habin

```
1 #include <bits/stdc++.h>
2 #define mod 1000000007
3 #define Pi 3.14159265358979311599796346854
4 #define INF 0x3f3f3f3f
5 #define MAX 1000010
6 #define f first
7 #define s second
8 #define ll long long
9 #define pb push_back
10 #define mp make_pair
11 #define pii pair<int, int>
12 #define vi vector<int>
13 #define vii vector< pii >
14 #define sws ios_base::sync_with_stdio(false);cin.tie(
15     NULL)
16 #define forn(i, n) for(int i=0; i<(int)(n); i++)
17 #define mdc(a, b) (__gcd((a), (b)))
18 #define mmc(a, b) (((a)/__gcd(a, b)) * b)
19 #define endl '\n'
20 #define teto(a, b) (a+b-1)/b
21
22 using namespace std;
23
24 ll llrand()
25 {
26     ll tmp = rand();
27     return (tmp << 31) | rand();
28 }
29
30 ll add(ll a, ll b, ll c)
31 {
32     return (a + b)%c;
33 }
34
35 ll mul(ll a, ll b, ll c)
36 {
37     ll ans = 0;
38     while(b)
39     {
40         if(b & 1)
41             ans = add(ans, a, c);
42         a = add(a, a, c);
43         b /= 2;
44     }
45     return ans;
46 }
47
48 ll fexp(ll a, ll b, ll c)
49 {
50     ll ans = 1;
51     while(b)
52     {
53         if(b & 1)
54             ans = mul(ans, a, c);
55         a = mul(a, a, c);
56         b /= 2;
57     }
58     return ans;
```

```

58 }
59
60 bool rabin(ll n)
61 {
62     if(n <= 1)
63         return 1;
64     if(n <= 3)
65         return 1;
66
67     ll s=0, d=n-1;
68     while(d%2==0)
69     {
70         d/=2;
71         s++;
72     }
73
74     for(int k = 0; k < 64*4; k++)
75     {
76         ll a = (llrand()%(n - 3)) + 2;
77         ll x = fexp(a, d, n);
78         if(x != 1 and x != n-1)
79         {
80             for(int r = 1; r < s; r++)
81             {
82                 x = mul(x, x, n);
83                 if(x == 1)
84                     return 0;
85                 if(x == n-1)
86                     break;
87             }
88             if(x != n-1)
89                 return 0;
90         }
91     }
92
93     return 1;
94 }
95
96 int main()
97 {
98     //sws;
99     //freopen("input.txt", "r", stdin);
100    //freopen("output.txt", "w", stdout);
101
102    ll N;
103    cin >> N;
104
105    cout << rabin(N) << endl;
106
107    return 0;
108 }
109
110 }

```

## 5.7 Inverso-Mult

```

1 // gcd(a, m) = 1 para existir solucao
2 // ax + my = 1, ou a*x = 1 (mod m)
3 ll inv(ll a, ll m) { // com gcd
4     ll x, y;
5     gcd(a, m, x, y);
6     return ((x % m) + m) % m;
7 }
8
9 ll inv(ll a, ll phim) { // com phi(m), se m for primo
10     entao phi(m) = p-1
11     ll e = phim-1;
12     return fexp(a, e);
13 }

```

## 5.8 Pollard-Rho

```

1 // Pollard Rho Algorithm
2
3 #include <bits/stdc++.h>
4 #define ll long long
5
6 using namespace std;
7
8 ll llrand()
9 {
10     ll tmp = rand();
11     return (tmp << 31) | rand();
12 }
13
14 ll add(ll a, ll b, ll c)
15 {
16     return (a + b)%c;
17 }
18
19 ll mul(ll a, ll b, ll c)
20 {
21     ll ans = 0;
22     while(b)
23     {
24         if(b & 1)
25             ans = add(ans, a, c);
26         a = add(a, a, c);
27         b /= 2;
28     }
29     return ans;
30 }
31
32 ll rho(ll n)
33 {
34     ll x, c, y, d, k;
35     int i;
36     do{
37         i = 1;
38         x = llrand()%n;
39         c = llrand()%n;
40         y = x, k = 4;
41         do{
42             if(++i == k)
43             {
44                 y = x;
45                 k *= 2;
46             }
47             x = add(mul(x, x, n), c, n);
48             d = __gcd(abs(x - y), n);
49         }
50         while(d == 1);
51     }
52     while(d == n);
53
54     return d;
55 }
56
57 int main()
58 {
59     srand(time(0));
60
61     ll N;
62     cin >> N;
63
64     ll div = rho(N);
65     cout << div << " " << N/div << endl;
66
67     // Finding all divisors
68
69     vector<ll> div;
70
71     while(N>1 and !rabin(N))
72     {
73

```

```

74         ll d = rho(N);
75         div.pb(d);
76         while(N%d==0)
77             N/=d;
78     }
79     if(N!=1)
80         div.pb(N);
81
82     return 0;
83 }
84 }

```

## 5.9 Verif-primo

```

1  // prime verification sqrt(N)
2
3  bool eh_primo(long long N)
4  {
5      if(N==2)
6          return true;
7      else if(N==1 or N%2==0)
8          return false;
9      for(long long i=3;i*i<=N;i+=2)
10         if(N%i==0)
11             return false;
12     return true;
13 }

```

## 5.10 Crivo

```

1  // Sieve of Eratosthenes
2
3  int N;
4  vector<bool> primos(100010, true);
5  cin >> N;
6
7  primos[0]=false;
8  primos[1]=false;
9
10 for(int i=2;i<=N;i++)
11     if(primos[i])
12         for(int j=i+i; j<=N; j+=i)
13             primos[j]=false;

```

## 5.11 FFT-golffbot

```

1  #include <bits/stdc++.h>
2
3  using namespace std;
4
5  const int N = (1<<19);
6  const double two_pi = 4 * acos(0);
7
8  struct cpx
9  {
10     cpx(){}
11     cpx(double aa): a(aa){}
12     cpx(double aa,double bb):a(aa),b(bb){}
13     double a;
14     double b;
15     double modsq(void) const
16     {
17         return a*a+b*b;
18     }
19     cpx bar(void) const
20     {
21         return cpx(a,-b);
22     }
23 };
24
25 cpx b[N+100];
26 cpx c[N+100];

```

```

27 cpx B[N+100];
28 cpx C[N+100];
29 int a[N+100];
30 int x[N+100];
31 double coss[N+100], sins[N+100];
32 int n,m,p;
33
34 cpx operator +(cpx a,cpx b)
35 {
36     return cpx(a.a+b.a,a.b+b.b);
37 }
38
39 cpx operator *(cpx a,cpx b)
40 {
41     return cpx(a.a*b.a-a.b*b.b,a.a*b.b+a.b*b.a);
42 }
43
44 cpx operator /(cpx a,cpx b)
45 {
46     cpx r = a*b.bar();
47     return cpx(r.a/b.modsq(),r.b/b.modsq());
48 }
49
50 cpx EXP(int i,int dir)
51 {
52     return cpx(coss[i],sins[i]*dir);
53 }
54
55 void FFT(cpx *in,cpx *out,int step,int size,int dir)
56 {
57     if(size<1) return;
58     if(size==1)
59     {
60         out[0]=in[0];
61         return;
62     }
63     FFT(in,out,step*2,size/2,dir);
64     FFT(in+step,out+size/2,step*2,size/2,dir);
65     for(int i=0;i<size/2;++i)
66     {
67         cpx even=out[i];
68         cpx odd=out[i+size/2];
69         out[i] = even+EXP(i*step,dir)*odd;
70         out[i+size/2]=even+EXP((i+size/2)*step,dir)*
71         odd;
72     }
73 }
74
75 int main()
76 {
77     for(int i=0;i<=N;++i)
78     {
79         coss[i]=cos(two_pi*i/N);
80         sins[i]=sin(two_pi*i/N);
81     }
82     while(cin >> n) // Numero de tacadas possiveis
83     {
84         fill(x,x+N+100,0);
85         fill(a,a+N+100,0);
86         for(int i=0;i<n;++i)
87         {
88             cin >> p; // Distancia das tacadas
89             x[p]=1;
90         }
91         for(int i=0;i<N+100;++i)
92         {
93             b[i]=cpx(x[i],0);
94         }
95         cin >> m; // Querys
96         for(int i=0;i<m;++i)
97         {
98             cin >> a[i]; // Distancia da query

```

```

99     FFT(b,B,1,N,1);
100     for(int i=0;i<N;++i)
101         C[i]=B[i]*B[i];
102     FFT(C,c,1,N,-1);
103     for(int i=0;i<N;++i)
104         c[i]=c[i]/N;
105     int cnt=0;
106     for(int i=0;i<m;++i)
107         if(c[a[i]].a>0.5 || x[a[i]])
108             cnt++;
109     cout << cnt << endl;
110 }
111 return 0;
112 }

```

## 5.12 Modular-Factorial

```

1 // C++ program to compute n! % p using Wilson's
  Theorem
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 int power(int x, unsigned int y, int p)
6 {
7     int res = 1;
8     x = x % p;
9
10    while(y > 0)
11    {
12        if(y & 1)
13            res = (res * x) % p;
14
15        y = y >> 1;
16        x = (x * x) % p;
17    }
18    return res;
19 }
20
21 int modInverse(int a, int p)
22 {
23     return power(a, p-2, p);
24 }
25
26 int modFact(int n, int p)
27 {
28     if (p <= n)
29         return 0;
30
31     int res = (p - 1);
32
33     for(int i = n + 1; i < p; i++)
34         res = (res * modInverse(i, p)) % p;
35     return res;
36 }
37
38 int main()
39 {
40     int n = 25, p = 29;
41     cout << modFact(n, p);
42     return 0;
43 }

```

## 5.13 Recursao-linear

```

1 vector<vl> mult(vector<vl> a, vector<vl> b, int n) {
2     vector<vl> res;
3     for(int i = 0; i < n; i++) {
4         vl tmp;
5         for(int j = 0; j < n; j++) {
6             tmp.pb(0);
7         }
8         res.pb(tmp);

```

```

9     }
10
11    for(int row = 0; row < n; row++) {
12        for(int col = 0; col < n; col++) {
13            ll val = 0;
14            for(int k = 0; k < n; k++) {
15                val += (a[row][k]*b[k][col]);
16            }
17            res[row][col] = val;
18        }
19    }
20
21    return res;
22 }
23
24 vector<vl> fexp(vector<vl> b, ll e, int n) {
25     if(e == 0) {
26         vector<vl> id;
27         for(int i = 0; i < n; i++) {
28             vl tmp;
29             for(int j = 0; j < n; j++) {
30                 if(i == j)
31                     tmp.pb(1);
32                 else
33                     tmp.pb(0);
34             }
35             id.pb(tmp);
36         }
37
38         return id;
39     }
40
41     vector<vl> res = fexp(b, e/2, n);
42     res = mult(res, res, n);
43
44     if(e%2)
45         res = mult(res, b, n);
46
47     return res;
48 }
49
50 // k = tamanho da recorrência/matriz, n = n-esimo
  termo
51 // f(n) = c1*f(n-1) + c2*f(n-2) + ... + ck*f(n-k)
52 // base -> [f(k-1), f(k-2), ..., f(0)]
53 // coeficientes -> [c1, c2, ..., ck]
54 vl solve(int k, int n, vl base, vl coef) {
55     vector<vl> inicial;
56     inicial.pb(coef);
57     for(int row = 0; row < k-1; row++) {
58         vl tmp;
59         for(int col = 0; col < k; col++) {
60             if(col == row)
61                 tmp.pb(1);
62             else
63                 tmp.pb(0);
64         }
65         inicial.pb(tmp);
66     }
67
68     vector<vl> matexp = fexp(inicial, max(0, n-k+1),
69 k);
70     vl res(k);
71
72     for(int row = 0; row < k; row++) {
73         ll val = 0;
74         for(int aux = 0; aux < k; aux++) {
75             val += matexp[row][aux]*base[aux];
76         }
77         res[row] = val; // res = (f(n), f(n-1), ...,
78 f(n-k+1))

```

```

79     return res;
80 }

```

## 5.14 Kamenetsky

```

1 // Number of digits in n! 0(1)
2
3 #define Pi 3.14159265358979311599796346854
4 #define Eul 2.71828182845904509079559829842
5
6 long long findDigits(int n)
7 {
8     double x;
9
10    if (n < 0)
11        return 0;
12    if (n == 1)
13        return 1;
14
15    x = ((n * log10(n / euler) + log10(2 * Pi * n)
16         / 2.0));
17
18    return floor(x) + 1;
19 }

```

## 6 Misc

### 6.1 LIS

```

1 multiset<int> S;
2 for(int i = 0; i < n; i++){
3     auto it = S.upper_bound(vet[i]); // low for inc
4     if(it != S.end())
5         S.erase(it);
6     S.insert(vet[i]);
7 }
8 // size of the lis
9 int ans = S.size();

```

### 6.2 Bitwise

```

1 // Bitwise
2
3 unsigned char a = 5, b = 9; // a = (00000101), b
4 = (00001001)
5
6 AND -      a&b    // The result is 00000001
7 (1)
8 OR -      a|b    // The result is 00001101
9 (13)
10 XOR -     a^b    // The result is 00001100
11 (12)
12 NOT -     ~a     // The result is 11111010
13 (250)
14 Left shift - b<<1 // The result is 00010010
15 (18)
16 Right shift - b>>1 // The result is 00000100
17 (4)
18
19 // Exchange two int variables
20
21     a^=b;
22     b^=a;
23     a^=b;
24
25 // Even or Odd
26
27     (x & 1)? printf("Odd"): printf("Even");
28
29 // Turn on the j-th bit
30

```

```

24     int S = 34; //(100010)
25     int j = 3;
26
27     S = S | (1<<j);
28
29 // Turn off the j-th bit
30
31     int S = 42; //(101010)
32     int j = 1;
33
34     S &= ~(1<<j)
35
36     S == 40 //(101000)
37
38 // Check the j-th element
39
40     int S = 42; //(101010)
41     int j = 3;
42
43     T = S & (1<<j); // T = 0
44
45 // Least significant bit (lsb)
46
47     int lsb(int x){ return x&-x; }
48
49 // Exchange o j-th element
50
51     S ^= (1<<j)
52
53 // Position of the first bit on
54
55     T = (S & (-S))
56     T -> 4 bit ligado //(1000)
57
58 // Most significant digit of N
59
60     double K = log10(N);
61     K = K - floor(K);
62     int X = pow(10, K);
63
64 // Number of digits in N
65
66     X =floor(log10(N)) + 1;
67
68 // Power of two
69
70     bool isPowerOfTwo(int x){ return x && !(x&(x
71 -1))); }
72
73 // Turn off the first bit 1
74     m = m & (m-1);
75
76 // Built-in functions
77
78 // Number of bits 1
79     __builtin_popcount()
80     __builtin_popcountll()
81
82 // Number of leading zeros
83     __builtin_clz()
84     __builtin_clzll()
85
86 // Number of trailing zeros
87     __builtin_ctz()
88     __builtin_ctzll()
89
90 // floor(log2(x))
91
92     int flog2(int x){ return 32-1-__builtin_clz(x
93 ); }
94
95     int flog2ll(ll x){ return 64-1-
96     __builtin_clzll(x); }

```

## 6.3 Template

```
1 #include <bits/stdc++.h>
2 #define ff first
3 #define ss second
4 #define ll long long
5 #define ld long double
6 #define pb push_back
7 #define eb emplace_back
8 #define mp make_pair
9 #define mt make_tuple
10 #define pii pair<int, int>
11 #define vi vector<int>
12 #define sws ios_base::sync_with_stdio(false); cin.tie(
    NULL)
13 #define endl '\n'
14 #define teto(a, b) (a+b-1)/(b)
15
16 const int MAX = 400010;
17 const int MOD = 1e9+7;
18 const int INF = 0x3f3f3f3f;
19 const ll LLINF = 0x3f3f3f3f3f3f3f3f;
20 const ld EPS = 1e-7;
21
22 using namespace std;
```

## 7 Strings

### 7.1 KMP

```
1 vector<int> prefix_function(const string &s){
2     int n = s.size(); vector<int> b(n+1);
3     b[0] = -1; int i = 0, j = -1;
4     while(i < n){
5         while(j >= 0 && s[i] != s[j]) j = b[j];
6         b[++i] = ++j;
7     }
8     return b;
9 }
10 void kmp(const string &t, const string &p){
11     vector<int> b = prefix_function(p);
12     int n = t.size(), m = p.size();
13     int j = 0;
14     for(int i = 0; i < n; i++){
15         while(j >= 0 && t[i] != p[j]) j = b[j];
16         j++;
17         if(j == m){
18
19             j = b[j];
20         }
21     }
22 }
```

### 7.2 LCS

```
1 string LCSSubStr(string X, string Y)
2 {
3     int m = X.size();
4     int n = Y.size();
5
6     int result = 0, end;
7     int len[2][n];
8     int currRow = 0;
9
10    for(int i=0; i<=m; i++){
11        for(int j=0; j<=n; j++){
12            if(i==0 || j==0)
13                len[currRow][j] = 0;
14            else if(X[i-1] == Y[j-1]){
15                len[currRow][j] = len[1-currRow][j-1]
16
17                + 1;
```

```
16                if(len[currRow][j] > result){
17                    result = len[currRow][j];
18                    end = i - 1;
19                }
20            }
21            else
22                len[currRow][j] = 0;
23        }
24        currRow = 1 - currRow;
25    }
26
27    if(result==0)
28        return string();
29
30    return X.substr(end - result + 1, result);
31 }
32 }
```

### 7.3 Pal-int

```
1 bool ehpalindromo(ll n)
2 {
3     if(n<0)
4         return false;
5
6     int divisor = 1;
7     while(n/divisor >= 10)
8         divisor *= 10;
9
10    while(n != 0)
11    {
12        int leading = n / divisor;
13        int trailing = n % 10;
14
15        if(leading != trailing)
16            return false;
17
18        n = (n % divisor)/10;
19
20        divisor = divisor/100;
21    }
22
23    return true;
24 }
```

### 7.4 Z-Func

```
1 vector<int> z_algo(const string &s)
2 {
3     int n = s.size();
4     int L = 0, R = 0;
5     vector<int> z(n, 0);
6     for(int i = 1; i < n; i++){
7         {
8             if(i <= R)
9                 z[i] = min(z[i-L], R - i + 1);
10            while(z[i]+i < n && s[ z[i]+i ] == s[ z[i] ])
11                z[i]++;
12            if(i+z[i]-1 > R)
13            {
14                L = i;
15                R = i + z[i] - 1;
16            }
17        }
18        return z;
19 }
```

### 7.5 Hash

```
1 ll compute_hash(string const& s) {
2     const ll p = 31; // primo, melhor = perto da
3     // quantidade de caracteres
```



<pre> 3  const ll m = 1e9 + 9; // maior mod = menor    probabilidade de colisao 4  ll hash_value = 0; 5  ll p_pow = 1; 6  for (char c : s) { 7      hash_value = (hash_value + (c - 'a' + 1) * </pre>	<pre>       p_pow) % m; 8      p_pow = (p_pow * p) % m; 9  } 10 return hash_value; 11 } </pre>
---	--