CSCI 405, Homework # 1

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1.

$$n^2 - 16n = \Theta(n^2)$$
$$0 < c_1 n^2 < n^2 - 16n < c_2 n^2 < \infty$$

Finding c_1

$$c_1 n^2 < n^2 - 16n$$
$$c_1 n^2 < n(n - 16)$$

let n_0 be 17.

$$c_1 17^2 < 17(17 - 16)$$
$$c_1 17 < 1$$
$$c_1 < \frac{1}{17}$$

Finding c_2 is trivial and can be satisifyed by just using 1.
$c_1 = \frac{1}{17}$
$c_2 = 1$
$n_0 = 17$

2.

$$T(n) = 2T(\frac{n}{2}) + n$$

$T(n) = O(n)$ implies $T(n) \leq cn$. Thus we can subsitute $T(n)$ for cn letting c be some constant.

$$cn = 2c(\frac{n}{2}) + n$$
$$cn = cn + n$$
$$cn = n(c + 1)$$
$$cn \neq n(c + 1)$$

There is no value for c that makes this true. Thus the Assumption that $T(n) = O(n)$ is false.

3.

$$f(0) = 7$$
$$f(n) = 2f(n - 1)$$

a.

$$\begin{aligned}
f(n) &= \cancel{2f(n-1)} \\
\cancel{2f(n-1)} &= \cancel{2^2 f(n-2)} \\
\cancel{2^2 f(n-2)} &= \cancel{2^3 f(n-3)} \\
\cancel{2^3 f(n-3)} &= \cancel{2^4 f(n-4)} \\
&\cdots \\
\cancel{2^{k-1} f(n-(k-1)} &= 2^k f(n-k)
\end{aligned}$$

Consider n = k being the last case

$$\begin{aligned}
f(n) &= 2^n f(0) \\
f(n) &= (7)2^n
\end{aligned}$$

b.

$$\begin{aligned}
f(n) &= 2f(n-1) \\
(7)2^n &= 2((7)2^{n-1}) \\
(7)2^n &= 2(1/2(7)2^n) \\
(7)2^n &= (7)2^n
\end{aligned}$$

4. $3 + 7 + 11 + ... + 283$

$$\begin{aligned}
\sum_{i=0}^{n}(4i+3) \\
\sum_{i=0}^{n} 4i + \sum_{i=0}^{n} 3 \\
4\sum_{i=0}^{n} i + 3(n+1) \\
2(n+1)n + 3(n+1) \\
2n^2 + 2n + 3n + 3 \\
2n^2 + 5n + 3
\end{aligned}$$

(283 - 3)/ 4 = 70

$$\begin{aligned}
2(70)^2 + 5(70) + 3 \\
9800 + 350 + 3 \\
10153
\end{aligned}$$

5. $(1+2+...+n)+(2+3+...+n)+(3+4+...n)+...+n$

$$\sum_{i=1}^{n} i^i$$

$$\sum_{i=1}^{n}\sum_{j=i}^{n} j$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} j - \sum_{x=1}^{i-1} x$$

6.

$$\sum_{i=1}^{n}\sum_{j=1}^{n} j - \sum_{x=1}^{i} x + i$$

$$\sum_{i=1}^{n} \frac{n(n+1)}{2} - \frac{i(i+1)}{2} + i$$

$$\sum_{i=1}^{n} \frac{n(n+1)}{2} - \sum_{i=1}^{n} \frac{i(i-1)}{2} + \sum_{i=1}^{n} i$$

$$\frac{n^2(n+1)}{2} - \sum_{i=1}^{n} \frac{i(i+1)}{2} + \frac{n(n+1)}{2}$$

$$\frac{n^2(n+1)}{2} - 1/2(\sum_{i=1}^{n} i^2 + i) + \frac{n(n+1)}{2}$$

$$\frac{n^2(n+1)}{2} - 1/2(\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}) + \frac{n(n+1)}{2}$$