CSCI 305, Homework # 7

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Due date: Midnight, Tuesday, May 29

Quadratic probing. This is problem 11-3 in the book.

Suppose that we are given a key k to search for in a hash table with positions $0, 1, ..., m-1$, and suppose that we have a hash function h mapping the key space into the set $\{0, 1, ..., m-1\}$. The search scheme is as follows:

1. Compute the value $j = h(k)$ and set $i = 0$.
2. Probe in position j for the desired key k. If you find it, or if this position is empty, terminate the search.
3. Set $i = i+1$. If i now equals m, the table is full, so terminate the search. Otherwise, set $j = (i+j) \mod m$ and return to step 2.

Assume that m is a power of 2.

a. Show that this scheme is an instance of the general "quadratic probing" scheme by exhibiting the appropriate constants c_1 and c_2 for equation (11.5).

The equation above can be written as

$$
\begin{aligned}
h(k,0) &= j \\
h(k,1) &= j + 1 \mod m \\
h(k,2) &= j + 1 + 2 \mod m \\
h(k,3) &= j + 1 + 2 + 3 \mod m \\
h(k,i) &= j + \sum_{q=1}^{i} q \mod m \\
h(k,i) &= j + \frac{i(i+1)}{2} \mod m \\
h(k,i) &= j + \frac{i^2}{2} + \frac{i}{2} \mod m
\end{aligned}
$$

The general form of the equation is

$$h(k,i) = (h(k) + c_1 i + c_2 i^2) \mod m$$

Choose $c_1 = 1/2$ and $c_2 = 1/2$

$$h(k,i) = (h(k) + i/2 + i^2/2) \mod m$$
$$h(k,i) = j + \frac{i}{2} + \frac{i^2}{2} \mod m$$

b. Prove that this algorithm examines every table position in the worst case.

There are 3 ways to end a search. 1. We find the key. 2. We find an empty entry. 3. i = m. In the worst case we don't find the key and we have a full table. So case 1 and 2 are never met. Leaving only case 3. Case 3 guarentees that we will not look at more than m entries because we stop when i = m.

Now to prove this claim we must show that over the course of those m tries we don't look at the same item twice. In order to visit the same node twice we would have to get two unqiue numbers such that h(k, i) = h(k, q) where both i and q are less then m-1 and greater then 0.

$$j + \frac{i}{2} + \frac{i^2}{2} \mod m = j + \frac{q}{2} + \frac{q^2}{2} \mod m$$
$$\frac{i}{2} + \frac{i^2}{2} \mod m = \frac{q}{2} + \frac{q^2}{2} \mod m$$
$$\frac{i}{2} + \frac{i^2}{2} - (\frac{q}{2} + \frac{q^2}{2}) \mod m = 0$$
$$\frac{(i+i^2) - (q+q^2)}{2}) \mod m = 0$$
$$\frac{i(1+i) - q(1-q)}{2} \mod m = 0$$
$$\frac{(i-q)(i+q+1)}{2} \mod m = 0$$

To get mod m to equal 0, (i - q) must equal m, (i + q + 1) must equal m or (i - q)(i + q + 1) must equal m.

Both i and q are less then m, so i - q is diffently not m.

(i + q + 1) could equal m, but if it does then (i - q) is forced to be odd. Because of the divide by two can't be taken out of the odd, it will force (i + q + 1) to be equal to m/2 , not m.

(i + q + 1) can't equal 2m because of i and q are less then m -1.

(i - q)(i + q + 1) can't equal m because one will be odd and the other even, so can't make a power of two.