CSCI 305, Homework # 4

YOUR NAME HERE

Due date: Friday, May 11, Midnight

Explain the reasoning behind each answer.

1. Suppose we shuffle a deck of 10 cards, each bearing a distinct number from 1 to 10, to mix the cards thoroughly. We then remove three cards, one at a time, from the deck. What is the probability that we select the three cards in sorted (increasing) order?

   Choose any 3 cards. There are 3! combinations of ways that you could have drawn. Of the those 6 only 1 is sorted. So 1/6.

2. Use indicator random variables to solve the following problem, known as the **hat-check problem**. Each of n customers gives a hat to a hat-check person at a restaurant. The hat-check person gives the hats back to the customers in a random order. What is the expected number of customers who get back their own hat?

$$
\begin{aligned}
X_h &= I\{\text{Getting the correct hat back}\} \\
X_h &= 1/n \\
E[X] &= E\left[\sum_{k=1}^{n} X_h\right] \\
E[X] &= \sum_{k=1}^{n} E[X_h] \\
E[X] &= \sum_{k=1}^{n} 1/n \\
E[X] &= n/n \\
E[X] &= 1
\end{aligned}
$$

   The expected number of customers who get there hat back is 1 for any n.