CSCI 305, Homework # 2

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Due date: Tue, May 1, midnight

In all cases, we require that $f(n)$ and $g(n)$ be positive functions, *i.e.* $f(n) > 0$ and $g(n) > 0$ for all $n > 0$. Prove or disprove each of the following conjectures.

1. $f(n) = O((f(n))^2)$
   $f(n) \leq c(f(n))^2)$
   let c = 1
   $f(n) \leq f(n) * f(n)$

   Provided the result of f(n) is equal to or greater then 1 this is always true

2. $f(n) = \Theta(f(n/2))$
   take $f(n) = 2^n$
   $2^n = \Theta(2^{n/2})$
   for n = 2 f(n) = 4 and f(n/2) = 2 difference = 2
   for n= 4 f(n) = 16 and f(n/2) = 4 difference = 12
   for n= 6 f(n) = 64 and f(n/2) = 8 difference = 46
   the difference between the two functions is increasing meaning as n -¿ infinity the difference between goes to infinity and f(n) is o(n/2) for this equation.

3. $f(n) + o(f(n)) = \Theta(f(n))$
   $cg(n) \leq f(n) \leq dg(n)$ for all $n \geq n_0$
   $f(n) + o(f(n)) \leq df(n)$
   o(f(n)) is always smaller then f(n), so we can subsitute that in.
   $f(n) + f(n) \leq df(n)$
   d can be 2

   $cf(n) \leq f(n) + o(f(n))$
   let c = 1
   $f(n) \leq f(n) + o(f(n))$
   f(n) will always be less then f(n) + something.

4. If $f(n) = O(g(n))$ then $f(n) + g(n) = O(f(n))$.
   Suppose $f(n) = O(n)$ and $g(n) = O(n^2)$
   $f(n) = O(g(n))$ would be true,
   but $f(n) + g(n) = O(f(n))$ would not be true because $g(n) < f(n)$.