Quantum simulation of a space-time atom in loop quantum gravity

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I. INTRODUCTION

With quantum computers becoming more useful in recent years, many problems with large solution spaces or problem states are being tested.

Loop Quantum Gravity (LQG), is a theory based on the quantization of space-time, where entangled quantum tetrahedra give rise to space-time in a way the unifies quantum mechanics and general relativity.

II. LOOP QUANTUM GRAVITY

A. Quantum Tetrahedra

In any n-dimensional space, an n-simplex is the shape with the fewest possible number of faces; a 2-simplex is a triangle, a 3-simplex is a tetrahedron, and so on to higher dimensions. One central tenet of LQG is quantized space-time **CITATION - WHY tetrahedra?**. Therefore, to get discrete space in 3 dimensions we will choose the 3-simplex, or tetrahedron, to be our discrete unit of space. One tetrahedron is defined by the equation

$$\vec{J}_1 + \vec{J}_2 + \vec{J}_3 + \vec{J}_4 = 0 \tag{1}$$

where $\vec{J}_i = (J_x, J_y, J_z)$ is the angular momentum vector of the ith face [2]. We can then define a quantum tetrahedron, or qubit of space, with the following [3], where θ and ϕ are angles on the Bloch sphere

$$|t\rangle = \cos\left(\frac{\theta}{2}\right)|0_L\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1_L\rangle \tag{2}$$

$$|0_L\rangle = \frac{1}{2}(|01\rangle - |10\rangle)(|01\rangle - |10\rangle) \tag{3}$$

$$|1_L\rangle = \frac{1}{\sqrt{3}}[|1100\rangle + |0011\rangle - \frac{1}{2}(|01\rangle + |10\rangle)(|01\rangle + |10\rangle)] \tag{4}$$

This quantum tetrahedron is the discrete unit of 3 dimensional quantum space.

B. Space-time Atom

The 4-simplex is a geometric object used to create discrete 3+1 dimensional space-time [4], so in order to properly simulate LQG spinfoam amplitudes we need to create a 4-simplex with our quantum tetrahedra. An n-simplex is created by gluing $n+1$ simplices from the $n-1$ dimension; a 2 dimension triangle is created by “gluing” 3 lines together. To create the 4-simplex, we will glue 5 quantum tetrahedra together with entanglement between faces.

A collection of connected quantum tetrahedra gives rise to a spin-network graph, where each node in the graph is a tetrahedron and links are formed by gluing adjacent faces [5].

C. Gaussian Constraint

D. Transition Amplitudes

III. QUANTUM SIMULATION

We performed quantum simulations in order to obtain the aforementioned vertex amplitudes. Simulations were intended to solve the equation

$$A(B,S) = \langle B| \, P_G \, |S\rangle \tag{5}$$

where P_G is the Gaussian constraint. There is no guarantee that P_G is unitary, so in order to simulate the vertex amplitude, we must further constrain it to the projection operator $P_G = |T\rangle \langle T|$.

A. Circuit

Template for how circuits are formed. Function of $|0_L\rangle$ state.

B. Topology

Emergent non-homeomorphisms among the spin network.

IV. DIPOLE SPIN NETWORK

V. 4-SIMPLEX SPIN NETWORK

VI. RESULTS AND DISCUSSION

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