Statistical Method

Homework 3

Due data: 17:00, November 01, 2022

1. Use the following distribution to fit the data (Set1.csv and Set2.csv) by maximum likelihood estimates (MLEs). You can use either the theoretical derivation or numerical methods to obtain the estimates.

   (a) Normal distribution with μ and σ^2.

   (b) Exponential distribution with rate λ.

   (c) Laplace distribution with location parameter μ and scale parameter b.

   (d) Gamma distribution with shape parameter α and rate parameter λ.

   i. Use tables summarize the estimates with its corresponding value of log-likelihood function.

Distributions	Parameter 1	Parameter 2	log-likelihood
Normal	$\hat{\mu} =$	$\hat{\sigma^2} =$	
Exponential	$\hat{\lambda} =$	—	
Laplace	$\hat{\mu} =$	$\hat{b} =$	
Gamma	$\hat{\alpha} =$	$\hat{\lambda} =$	

Table 1: Set1

Distributions	Parameter 1	Parameter 2	log-likelihood
Normal	$\hat{\mu} =$	$\hat{\sigma^2} =$	
Exponential	$\hat{\lambda} =$	—	
Laplace	$\hat{\mu} =$	$\hat{b} =$	
Gamma	$\hat{\alpha} =$	$\hat{\lambda} =$	

Table 2: Set2

   ii. For each datasets, please provide the graphical description including histogram, density estimation, and the empirical cumulative distribution function. Add the probability density functions and the cumulative distribution functions of fitted distributions (a)-(d) with the estimates.

   iii. Provide the necessary evidence for the four candidate fitted distributions by Q-Q plot and the hypothesis testing to choose suitable fitted distributions.

iiv. Use the criteria, the maximum value of log of likelihood and the minimum value of the Akaike information criterion (AIC), to select the most appropriate fitted distributions, where

$$\text{AIC} = -2 \text{ log-likelihood value} + 2 \times k,$$

and k is the number of unknown parameters.

2. By using numerical optimization to obtain the estimates, what is your strategies to assign the initial values to the optimization algorithm?