

Finite Differences

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Task 3

The code implements three Finite Difference Methods (FDM) — explicit, implicit, and Crank-Nicolson — to approximate the price of a European put option. The objective is to compare the accuracy of each method when varying the number of space steps, N , and to evaluate their performance against the Black-Scholes model.

The `blackscholesput` function calculates the theoretical option price using the Black-Scholes formula, while the `finite_difference_methods` function computes option prices using explicit, implicit, and Crank-Nicolson methods. Each method propagates option values backward from maturity to the present, relying on different numerical techniques for stability and accuracy. Errors are computed as the absolute difference between the approximated FDM values and the Black-Scholes prices for a range of stock prices and volatilities. The code iterates over varying N values, from 300 to 700, and calculates errors at each step.

The graph generated by the code illustrates the errors for the explicit, implicit, and Crank-Nicolson methods across different stock prices and N values. The results show that the explicit method generally produces higher errors, especially at lower N values, but improves with larger space steps. The implicit method performs better, with errors decreasing significantly as N increases. The Crank-Nicolson method stands out, consistently offering the most accurate results across all values of N , with the smallest errors.

In conclusion, the Crank-Nicolson method is the most effective in terms of accuracy and stability, especially as the number of space steps increases. While the explicit method is computationally simpler, it struggles with accuracy, particularly at lower N . Ultimately, the Crank-Nicolson method provides a strong balance between precision and computational effort, making it the preferred choice for this type of numerical approximation.