

Euler and Milstein schemes for stochastic differential equations

Problem Statement

Write down the Euler and Milstein schemes for the following SDEs.

(a) $dX_t = (\mu X_t + \eta)dt + \gamma X_t dW_t$, with $X_0 = 1$

(b) $dr_t = \gamma(\bar{r} - r_t)dt + \beta dW_t$

Solution

Part (a)

Euler Scheme

1. Discretize time:

$$t_n = n\Delta t$$

2. Discretize X_t equation using Euler scheme:

$$X_{n+1} = X_n + (\mu X_n + \eta)\Delta t + \gamma X_n \Delta W_n$$

Milstein Scheme

1. Discretize time:

$$t_n = n\Delta t$$

2. Discretize X_t equation using Milstein scheme:

$$X_{n+1} = X_n + (\mu X_n + \eta)\Delta t + \gamma X_n \Delta W_n + \frac{1}{2}\gamma^2 X_n(\Delta W_n^2 - \Delta t)$$

Part (b)

Euler Scheme

1. Discretize time:

$$t_n = n\Delta t$$

2. Discretize r_t equation using Euler scheme:

$$r_{n+1} = r_n + \gamma(\bar{r} - r_n)\Delta t + \beta \Delta W_n$$

Milstein Scheme

1. Discretize time:

$$t_n = n\Delta t$$

2. Discretize r_t equation using Milstein scheme:

$$r_{n+1} = r_n + \gamma(\bar{r} - r_n)\Delta t + \beta \Delta W_n$$