Monte Carlo Simulation Assignment

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Report

The provided Python code is designed to price both European and digital call options using two different numerical schemes: the **Euler** and **Milstein** methods. It compares the prices obtained from these Monte Carlo simulations with the analytical prices computed using the **Black-Scholes formula**. The underlying parameters for the option pricing problem are as follows: the initial stock price (S_0) is 5, the volatility (σ) is 30%, the risk-free rate (r) is 6%, and the strike price (K) is equal to S_0 (an at-the-money option). The time to maturity (T) is 1 year, and the time step size for the Monte Carlo simulations is set to $dt = \frac{1}{1000}$ with $m = 1000$ steps. By varying the number of generated paths, N, between 100 and 100,000, the code computes the prices using both numerical schemes and compares them to the analytical Black-Scholes price. The results are plotted and saved as PDF files for both the European and digital call options.

Monte Carlo Simulations and Analytical Pricing

The Monte Carlo simulation generates multiple asset price paths for both the **Euler** and **Milstein** schemes. The Euler scheme uses an approximation for the stochastic differential equation driving the asset price with only the first-order stochastic term, while the Milstein scheme includes a higher-order correction, making it slightly more accurate.

The **Black-Scholes formula** is used to calculate the theoretical price for both the European call option and the digital call option. This formula assumes continuous trading and a lognormal distribution of asset prices. The resulting theoretical prices serve as a benchmark against which the Monte Carlo estimates from both schemes are compared.

Results Analysis: European Call Option

The graph generated for the **European call option** shows the difference between the prices computed using the Euler and Milstein schemes and the Black-Scholes price for various numbers of paths (N) as shown below.

As the number of paths increases, both schemes' prices converge towards the Black-Scholes price, as expected in a Monte Carlo simulation. Initially, for smaller values of N (e.g., 100),

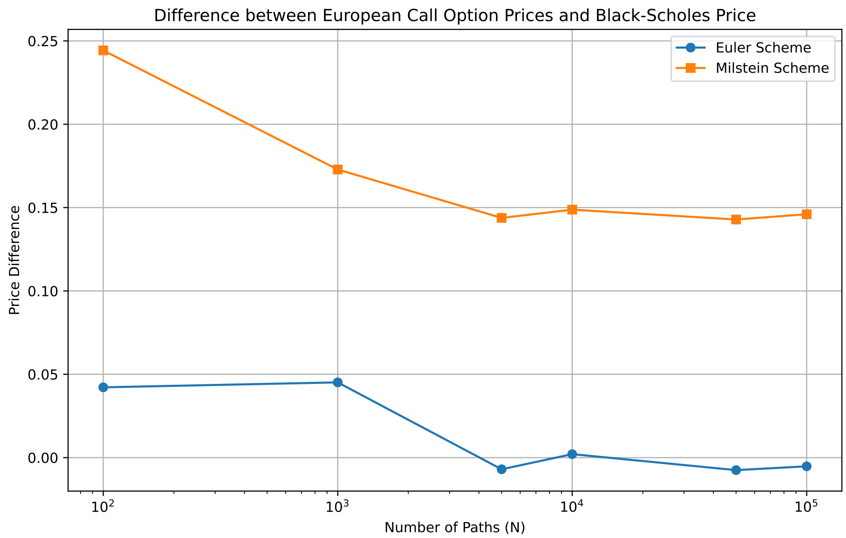

Figure 1: Difference between European Call Option Prices and Black-Scholes Price.

the differences between the Euler scheme and the Black-Scholes price are more significant, indicating that the Euler scheme requires a larger number of paths to reach a higher level of accuracy. The Milstein scheme, on the other hand, performs better even with a smaller number of paths, showing smaller deviations from the Black-Scholes price.

As N increases, the price differences between both schemes and the Black-Scholes formula reduce, with the Milstein scheme consistently showing better convergence. The graph demonstrates that, for a large number of paths, the Milstein scheme is generally more accurate and reliable than the Euler scheme. The Milstein scheme's inclusion of higher-order terms makes it converge faster and more accurately to the theoretical price, as evidenced by the smaller differences across various values of N.

Results Analysis: Digital Call Option

The graph for the **digital call option** presents a different scenario, as shown below.

Here, the differences between the Euler and Milstein schemes are smaller overall compared to the European call option. This is because digital options have a payoff structure that is less sensitive to small changes in the underlying asset price. The payoff is binary (either 1 or 0 depending on whether the asset price exceeds the strike price), which reduces the impact of numerical errors from the simulation on the final price.

For smaller numbers of paths (e.g., $N = 100$), the price difference between the Euler scheme and the Black-Scholes price is slightly larger than that for the Milstein scheme. However, as the number of paths increases, both schemes show very similar convergence behavior. The difference between the two schemes is minimal for large N, and both approach the Black-Scholes price closely. This suggests that for pricing digital options, the choice between the

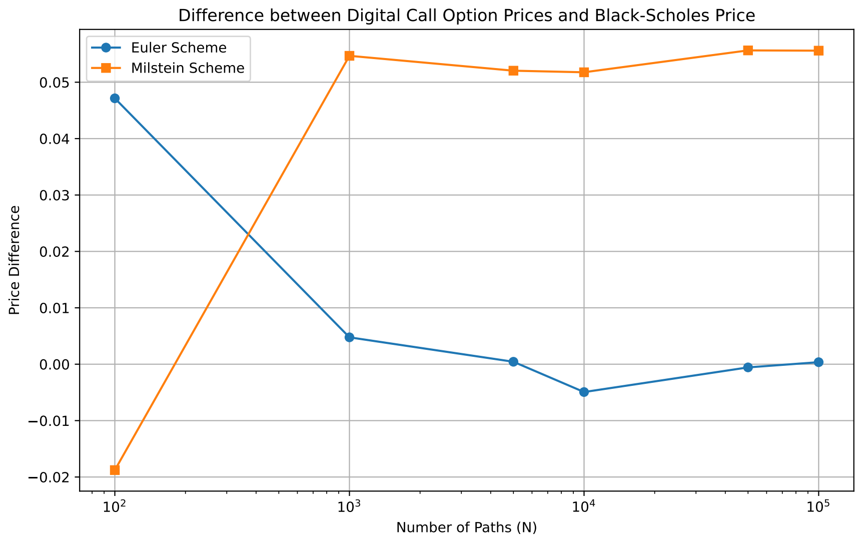

Figure 2: Difference between Digital Call Option Prices and Black-Scholes Price.

Euler and Milstein schemes is less critical than for European call options, since both schemes converge to the same value with sufficient paths.

Conclusion

The code successfully compares the performance of the Euler and Milstein schemes for pricing both European and digital call options. The results show that the **Milstein scheme** generally converges to the Black-Scholes price more quickly and accurately than the Euler scheme, particularly for the European call option, where the payoff is more sensitive to variations in the underlying asset price. For the digital call option, both schemes perform similarly, though the Milstein scheme still shows slightly better accuracy for smaller numbers of paths.

The differences between the two schemes become less pronounced as the number of Monte Carlo paths increases. In general, the **Milstein scheme** is preferable for option pricing, especially when higher accuracy is required with fewer simulations. However, for simpler payoff structures like digital options, the Euler scheme may still provide satisfactory results with minimal computational overhead.