Taylor's Expansion and Euler Scheme

The provided code initializes parameters for simulating Geometric Brownian Motion (GBM), including initial value (X_0), drift (μ), volatility (σ), and terminal time (T), along with a range of time step values (num_steps_values). It computes the exact GBM solution using the SDE formulation, incorporating drift and diffusion components. The Euler-Maruyama method is then employed to numerically approximate GBM solutions, iterating over specified time steps and computing stochastic increments. Errors between exact and approximate solutions are computed, and their natural logarithms are taken to analyze convergence behavior. A linear regression on these logarithmic errors versus time step sizes assesses convergence rates, with results visualized in a log-log plot illustrating the relationship. This approach provides insights into the method's convergence properties, essential for evaluating its accuracy in stochastic process simulations.