COMP 3270 Introduction to Algorithms

Homework 2

1. (20pts) Compare the following pairs of functions in terms of order of magnitude. In each case, say whether $f(n) = O(g(n)), f(n) = \Theta(g(n))$, or $f(n) = \Omega(g(n))$.

	$f(n)$	$g(n)$	
a.	$100n + \log n$	$n + (\log n)^2$	$f(n) = \Theta(g(n))$
b.	$\log n$	$\log(n^2)$	$f(n) = \Theta(g(n))$
c.	$\frac{n^2}{\log n}$	$n(\log n)^2$	$f(n) = \Omega(g(n))$
d.	$n^{\frac{1}{2}}$	$\log n^5$	$f(n) = \Omega(g(n))$
e.	$n2^n$	3^n	$f(n) = O(g(n))$

2. (30 pts) Use the Master Method to solve the following three recurrence relations and state the complexity orders of the corresponding recursive algorithms.

(a) $T(n) = 2T(99n/100) + 100n$

T(n) = 198T(n/100)+100n
Using Masters theorem, we have a = 198, b = 100, c = 1, f(n) = 100n
loga base b = log198 base 100 => 1
c = 1, since c < log a base b, T(n) = theta(n^loga base b) = theta(n)
therefore T(n) = theta(n)

(b) $T(n) = 16T(n/2) + n^3 lgn$

Using Masters theorem, we have a = 16, b= 2, c = 3, k = 1, loga base b = log 16 base 2 = 4
c < log a base be, therefore T(n) = theta(n^log a base b) = theta(n^4)

(c) $T(n) = 16T(n/4) + n^2$

Using Masters Theorem, we have a =16, b = 4, c = 2, log a base b = log 16 base 4 = 2
c = log a base b, therefore T(n) = theta(n^clog^k+1n) = theta(n^2logn)
Therefore, T(n) = theta(n^2logn)

3. (50 pts) Use the Substitution Method to solve the following recurrence relation. Give an exact solution:

$$T(n) = T(n-1) + n/2$$

Using Substitution Method

$$\begin{aligned} T(n) &= T(n-1) + \tfrac{n}{2} \\ &\le C(n-1)^2 + \tfrac{n}{2} \\ &= Cn^2 - 2Cn + C + \tfrac{n}{2} \\ &= Cn^2 - (2Cn - \tfrac{n}{2} - C) \\ &\le Cn^2 \text{ if } 2Cn \ge \tfrac{n}{2} + C \end{aligned}$$

$2Cn \ge \frac{n}{2} + C$ for all $C > 1$

So, $T(n) = O(n^2)$