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ZAMS Stellar Structure Calculation for a $2M_\odot$ Star

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1. INTRODUCTION

Modeling the internal structure of stars is important for utilizing observations of stars. Most of the internal physics of stars cannot be probed directly through observations, but by comparing them with models based on known properties of the star, they can be very well understood. The zero age main sequence (ZAMS) defines the point where stars leave the protostellar phase and begin fusing helium in their cores. Getting this initial state right is important for models that then evolve the star through its life, which again is necessary for interpreting properties of real stars since most are not on the ZAMS.

This work aims to create a ZAMS model of a $2M_\odot$ star with the same chemical composition of the Sun. The equations describing the energy generation, equation of state, and the four coupled ODEs describing the structure all come from Hansen (2004) and Kippenhahn (2012).

2. STELLAR STRUCTURE MODEL

The most basic stellar structure models use four coupled ordinary differential equations that do a very good job of modeling the interiors of stars. The equations are:

$$\frac{dr}{dM_r} = \frac{1}{4\pi r^4 \rho} \tag{1}$$

$$\frac{dl}{dM_r} = \epsilon = \epsilon_{PP} + \epsilon_{CNO} \tag{2}$$

$$\frac{dP}{dM_r} = -\frac{GM_r^2}{r^4} \tag{3}$$

$$\frac{dT}{dM_r} = -\frac{GM_r T}{4\pi r^4 P}\nabla \tag{4}$$

where M_r is the enclosed mass at a certain radius r and l, T, P, and ρ are the luminosity, temperature, pressure, and density in that shell. For the model used in this analysis, the total energy generation ϵ is the sum of the energy produced in the core by the PP-chain, ϵ_{PP}, and through the CNO cycle, ϵ_{CNO}. Analytic expression for these are given in Equations 5 and 6 come from Kippenhahn (2012). The energy produced through PP-chain reactions is

$$\begin{aligned}
\epsilon_{PP} &= 2.57 \times 10^4 f_{11} g_{11} \rho X^2 T_9^{-2/3} e^{-3.381/T_9^{1/3}} \\
f_{11} &= e^{5.92\times 10^{-3}\rho/(10^2 T_9)^{1/2}} \\
g_{11} &= 1 + 3.82T_9 + 1.51T_9^2 + 0.144T_9^3 - 0.0114T_9^4
\end{aligned} \tag{5}$$

where f_{11} is a weak screening factor accounting for the effects of electrons surrounding the nucleus, which alters it's potential from the potential of an isolated nucleus. Similarly, the energy produced through the CNO cycle is given by

$$\begin{aligned} \epsilon_{CNO} &= 8.24 \times 10^{25} g_{14,1} Z X \rho T_9^{-2/3} e^{-15.231 T_9^{-1/3} - (T_9/0.8)^2} \\ g_{14,1} &= 1 - 2T_9 + 3.41 T_9^2 - 2.43 T_9^3. \end{aligned} \tag{6}$$

The temperature gradient includes ∇ which is the logarithmic slope of temperature versus pressure, $d \ln P / d \ln T$. The value of this gradient determines whether the region is convective or radiative, which is found by comparing the adiabatic del ∇_{ad} with the radiative del ∇_{rad}. For an ideal, monatomic gas, $\nabla_{ad} = 0.4$. To determine if a region is convective, this value is compared to the radiative del,

$$\nabla_{rad} = \frac{3Pl\kappa}{16\pi acGM_r T^4} \tag{7}$$

where κ is the Rosseland mean opacity in the shell. If $\nabla_{ad} > \nabla_{rad}$, the region is convective and $\nabla = \nabla_{ad}$. Otherwise, the region is radiative and $\nabla = \nabla_{rad}$.

The density of each shell is determined using the equation of state for radiation and an ideal, monatomic gas that is completely ionized. Since protostars are fully convective, ZAMS stars can be treated as chemically homogeneous meaning the mean molecular weight, μ, is constant throughout (Hansen 2004). The equation of state, solving for density, is then

$$\rho = (P - \frac{1}{3} a T^4) \frac{\mu}{N_a k T} \tag{8}$$

where $\mu = 4/(3 + 5X)$ for $Z << 1$. All of these are reasonable assumptions for a Sun-like star.

3. MODEL PARAMETERS

The star selected to model has a mass of $2M_\odot$ and a composition $X = 0.7$, $Y = 0.28$, and $Z = 0.02$. The opacity is generated by interpolating a table from the Los Alamos National Laboratory astrophysical opacities website (Magee et al. 1995). The guesses for the core temperature T_c, core pressure P_c, total luminosity $L_\star$, and total radius $R_\star$ are used to solve for the inner and outer boundary conditions of the star, and these must also be given as input to the model. Table 1 shows the initial values input for each of these parameters.

Table 1: Initial guesses for the model input parameters. This are automatically adjusted by the root finding code to achieve a converged solution. However, getting the initial guesses as close as possible to the final solution is crucial for getting the solution to converge without ending up on a bad integration path.

Parameter	Initial Value
T_c	2×10^7 K
P_c	2.5×10^{17} dyne / cm^2
$L_\star$	5.8×10^{34} erg / s
$R_\star$	8×10^{10} cm

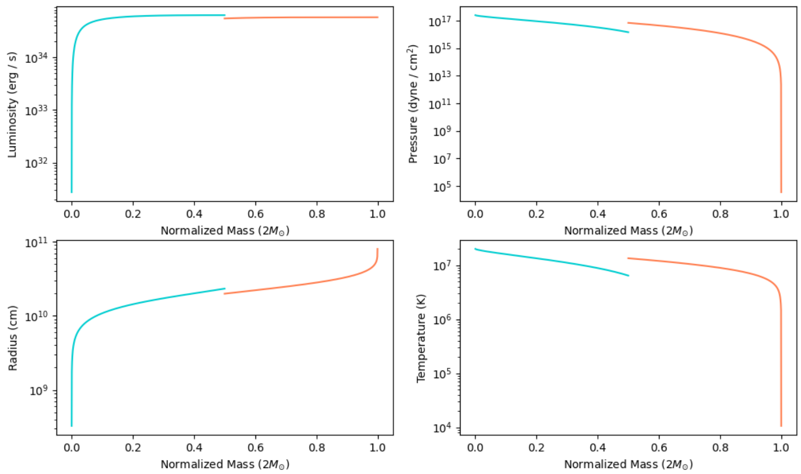

Figure 1: Results of integrating the model from the inner and outer boundaries using the initial parameter guesses. The blue line is the integration from the inner boundary, and the orange line is the integration from the inner boundary. There is a clear difference at the point the two models should meet for all four parameters.

For T_c and P_c, the initial values come from the solar values provided by Williams (2024). For the code to converge they were slightly increased, since they both increase with increasing stellar mass. The input values for $L_\star$ and $R_\star$ also come from the solar values in Williams (2024), which are then scaled using the homology relations in Hansen (2004).

4. NUMERICAL METHODS

The methods used to achieve a converged solution follow the shooting to a fitting point method described in Press (2007). The initial parameter values described in Section 3 are used to load inner and outer boundary values for temperature, pressure, luminosity, and radius. Kippenhahn (2012) describes the equations used to turn the four initial values into the eight boundary values. An important consideration is that there is a singularity at the center of the star where the mass, radius, luminosity, etc. will all go to 0. To avoid this, the inner boundary must be selected as a point slightly outside the center of the star and was set to $10^{-5}M_\star$. The inner boundary values defined using

$$l_i = (\epsilon_{pp} + \epsilon_{cno})M_r \tag{9}$$

$$r_i = M_r^{1/3}\frac{1}{4\pi\rho_c)^{1/3}} \tag{10}$$

$$P_i = P_c - \frac{3G}{8\pi}\left(\frac{4\pi\rho_c}{3}\right)^{4/3} M_r^{2/3} \tag{11}$$

$$T_i = \left(T_c^4 - \frac{1}{2ac}\left(\frac{3}{4\pi}\right)^{2/3}\kappa_c(\epsilon_{pp} + \epsilon_{cno})\rho_c^{4/3}M_r^{2/3}\right)^{1/4} \tag{12}$$

which account for the fact that the inner boundary is not at $r = 0$. The outer boundary conditions use the central opacity to get an initial guess for the pressure. The equations for the outer boundary values are

$$l_o = L_\star \tag{13}$$
$$r_o = R_\star \tag{14}$$
$$P_o = \frac{2GM_r}{3R_\star^2\kappa_c} \tag{15}$$
$$T_o = T_{eff} = \left(\frac{L_\star}{4\pi R_\star^2\sigma}\right)^{1/4}. \tag{16}$$

Using equations 1, 2, 3, and 4 and the SciPy integration library, these four parameters are integrated from each set of boundaries to the midpoint of the star. Figure 1 shows the results of the integration using the initial guesses at the boundary parameters. There is a clear mismatch where the integration from the inner boundary meets the integration from the outer boundary.

The goal is to minimize this difference using a root finding function. A function is defined that returns the difference between the two models at the point they should meet. Then, using the SciPy optimization library, the roots of this function are found by adjusting the initial parameters until a converged solution is reached. The final solutions converge to agreement $\leq 10^{-7}\%$ for all four parameters.

The initial guesses are crucial to get close to the true values for the solution to converge. If they are all very far off, a local minimum may be found instead of the global minimum. If the relative values of parameters are incorrect, especially between the pairs of parameters corresponding to the inner and outer boundary conditions, the integration may lead to nonphysical values such as negative densities.

5. RESULTS

The final optimized parameters are shown in Table 2 and the resulting model is plotted in Figure 2. After obtaining a convergent solution, the convective / radiative boundary was found using ∇_{ad} and ∇_{rad} as described in Section 2. The model was then rerun using the optimized guesses with the heat transfer method boundary used at the new fitting point. It was found to be at $M = 0.14M_\star$, which is consistent with the expectation of a radiative core surrounded by a large convective envelope.

Table 2: Final parameter values for converged solution.

Parameter	Final Value
T_c	2.02×10^7 K
P_c	1.54×10^{17} dyne / cm^2
$L_\star$	5.79×10^{34} erg / s
$R_\star$	1.16×10^{11} cm

5.1. *Comparison to MESA*

Modules for Experiments in Stellar Astrophysics (MESA) is the standard state-of-the-art code for calculating stellar models (Paxton 2024). MESA was run for a $2M_\odot$ star with the same composition

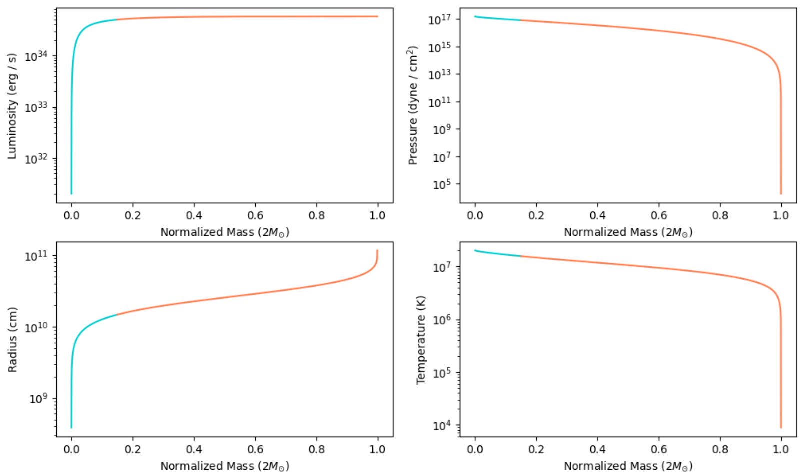

Figure 2: Converged model for a $2M_\odot$ star. All four boundaries are now in good agreement with each other. The transition from the integration starting at the inner boundary (blue) to the integration from the outer boundary (orange) occurs where the stars changes from radiative to convective.

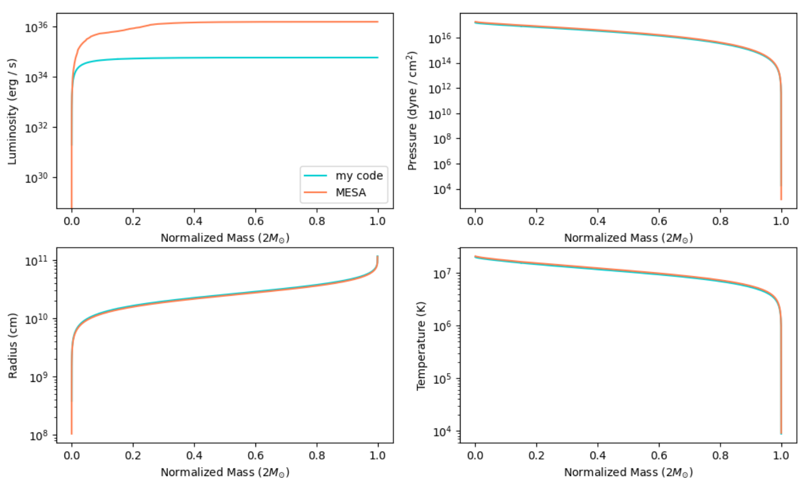

Figure 3: Comparison of the converged model (blue) with the results from MESA (orange). The values are in good agreement with MESA for all parameters expect for the luminosity.

and stopped when the protostar reached the ZAMS. Figure 3 shows a comparison between the simple model described in this work and the output from MESA. All of the parameters agree very well except for the luminosity, which is two orders of magnitude higher from MESA. This is likely because MESA includes a more detailed model of energy generation in the core of the star.

6. CONCLUSIONS

The simple stellar structure model described in this work provides a powerful probe into the inner working of stars. It agrees fairly well with the much more accurate MESA model, which means it can serve very well as a quick first look at the inner structure of lower mass stars. This can be applied to real stars by adjusting the input parameters and even some of the assumptions to make the model slightly more complex until the observed properties of the star are matched. While the model is limited to stellar masses between a solar mass to a few solar masses, in that range it is very effective at modeling their interior structure.

7. ACKNOWLEDGMENTS

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REFERENCES

Hansen, C. J. 2004, Stellar interiors: physical principles, structure, and evolution., 2nd edn., Astronomy and astrophysics library (New York: Springer), doi: 10.1007/978-1-4419-9110-2

Kippenhahn, R. 2012, Stellar Structure and Evolution, 2nd edn., Astronomy and Astrophysics Library (Berlin, Heidelberg: Springer Berlin Heidelberg), doi: 10.1007/978-3-642-30304-3

Magee, N., Jr, A., Clark, R., et al. 1995, 78, 51

Paxton, B. 2024, Modules for Experiments in Stellar Astrophysics (MESA), Zenodo, doi: 10.5281/zenodo.13353788

Press, W. H., ed. 2007, Numerical recipes: the art of scientific computing, 3rd edn. (Cambridge: Cambridge University Press)

Williams, D. R. 2024, Sun Fact Sheet. https://nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html