7.9 Exercise Sheet

Exercise 109. *Give a formal proof of* $(\mathtt{p} \land \mathtt{r})$ *from* $\{((\mathtt{q} \land \mathtt{r}) \land \mathtt{q}), (\mathtt{p} \land \mathtt{p})\}$.

Exercise 110. *Show the validity of the following sequents:*

1. $(\mathtt{p} \land \mathtt{q}), \mathtt{r} \vdash (\mathtt{p} \land (\mathtt{r} \lor \mathtt{r}'))$;
2. $(\mathtt{p} \to (\mathtt{q} \to \mathtt{r})) \vdash ((\mathtt{p} \land \mathtt{q}) \to \mathtt{r})$;
3. $((\mathtt{p} \land \neg\mathtt{r}) \to \mathtt{q}), \neg\mathtt{q}, \mathtt{p} \vdash \mathtt{r}$;

Exercise 111. *Finish the game at* `https://profs.info.uaic.ro/~stefan.ciobaca/lnd.html`. *Do not cheat. It is considered cheating if you change the JavaScript source code, if someone else solves a level for you or if you prove the derived rules using the derived rules themselves.*

Exercise 112. *Prove that the following inference rules are derivable:*

1. $\neg\neg i$;

2. *LEM (law of excluded middle):* $$\textsc{Lem}\ \frac{}{\Gamma \vdash (\varphi \vee \neg\varphi)};$$

3. *PBC (proof by contradiction):* $$\textsc{Pbc}\ \frac{\Gamma, \neg\varphi \vdash \bot}{\Gamma \vdash \varphi};$$

4. *MT (modus tollens):* $$\mathrm{MT}\ \frac{\Gamma \vdash (\varphi \to \varphi') \qquad \Gamma \vdash \neg\varphi'}{\Gamma \vdash \neg\varphi}.$$

Exercise 113. *Prove the soundness theorem for natural deduction (by induction on the number of sequents in the formal proof).*

Exercise 114. *Show that the rule $\neg\neg e$ is derivable using the LEM (i.e., you may use LEM in the derivation, but not $\neg\neg e$).*

Exercise 115. *Prove, using the soundness and completeness theorems, that* $\varphi_1 \dashv\vdash \varphi_2$ *if and only if* $\varphi_1 \equiv \varphi_2$.