Chapter 1

Introduction

If Arithmetic is the science that studies numbers and operations with numbers, then Logic is the science that studies *propositions* and operations with propositions.

For example, if in Arithmetic we notice that the sum of two even numbers is an even number, then in Logic we could notice that the disjunction of two true sentences is also true.

Logic sits at the intersection of philosophy, mathematics and computer science and has experienced its greatest development starting with the 1950s, because of its numerous applications in Computer Science.

In this course, we will study at an introductory level *propositional logic* and *first-order logic*.

Propositional logic is extremely simple, but the concepts that we study, the methods that we learn and the issues that we face in propositional logic generalize to other more complex logics. Moreover, propositional logic corresponds intimally to the internal organization of computers at an abstract level, in the sense that electronic circuits can be modeled as formulae in propositional logic. Propositional logic has a rich and mathematically interesting theory (examples: compactness theorem, Craig's interpolation theorem). The *satisfiability problem* for propositional logic has many applications in computer science. It is especially important both from a theoretical viewpoint (being the canonical NP-complete problem) and from a practical viewpoint as well (with applications in program verification, circuit verification, combinatorial optimization, and others).

First-order logic is an extension of propositional logic and also has numerous application in computer science, but also in mathematics. For example, all of the math that you have studied in highschool is based on a so-called first-order logic theory called **ZFC** (the **Z**ermelo–**F**raenkel set theory, together with the Axiom of **C**hoice). In Computer Science, applications of first-order logic

appear in the fields of descriptive complexity, relational databases, software and hardware verification, and others. Additionally, several other logics (for example, higher-order logics) have applications in programming languages, the fundamentals of mathematics, type theory, etc.