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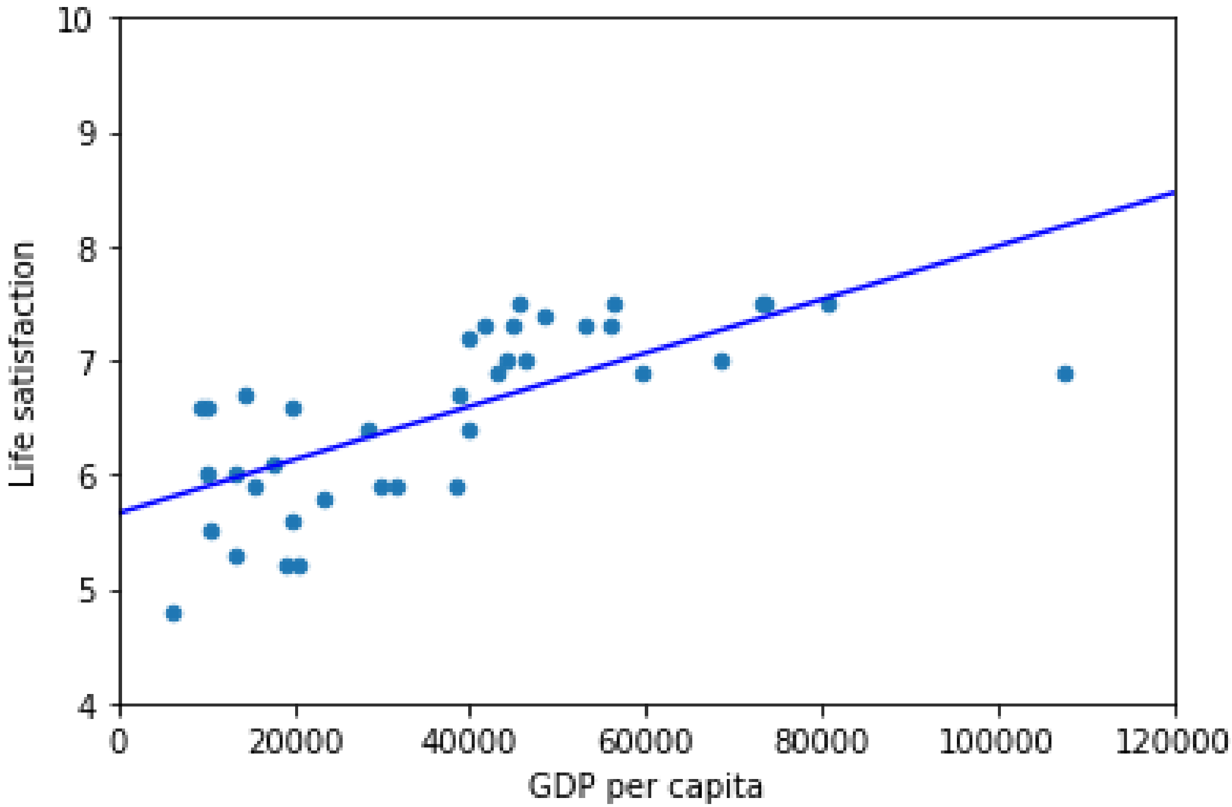
Training Machine Learning Models

Andrey Sozykin
Andrey.Sozykin@urfu.ru

Linear Regression



$y = a * x + b,$

a, b – parameters of the model

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What is training?

Training a model means setting its parameters so that the model best fits the training set.

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Model Metrics

Training a model means setting its parameters so that the model **best fits** the training set.

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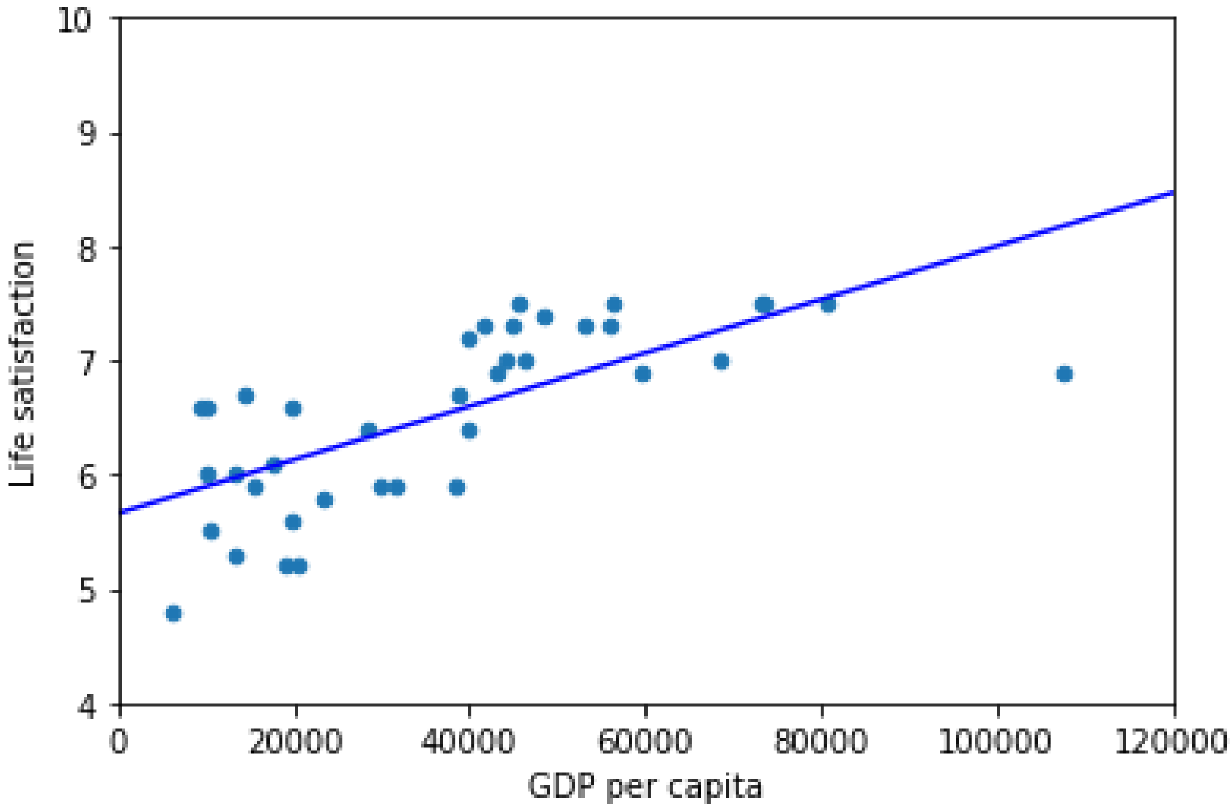
Mean Square Error

$$MSE = \frac{1}{m}\sum_{i=1}^{m}(y_i - c_i)^2$$

Linear Regression



y = *a* * x + *b*,

a, *b* – parameters of the model

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MSE for Linear Regression

$$MSE(X) = \frac{1}{m}\sum_{i=1}^{m}(a \cdot x_i + b - c_i)^2$$

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MSE for Linear Regression

$$y = w_0 \cdot 1 + w_1 \cdot x_1 + \cdots + w_n \cdot x_m$$

$$y = \boldsymbol{w} \cdot \boldsymbol{x}$$

$$y = \boldsymbol{w}^T \boldsymbol{x}$$

$$MSE(X) = \frac{1}{m} \sum_{i=1}^{m} (\boldsymbol{w}^T \boldsymbol{x}_{\boldsymbol{i}} - \ c_i)^2$$

How to find **w**?

$$MSE(X) = \frac{1}{m}\sum_{i=1}^{m}(\boldsymbol{w}^T \boldsymbol{x_i} - c_i)^2$$

How to find a vector ***w*** that minimize *MSE(x)?*

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The Normal Equation

$$w = (X^T X)^{-1} X^T c$$

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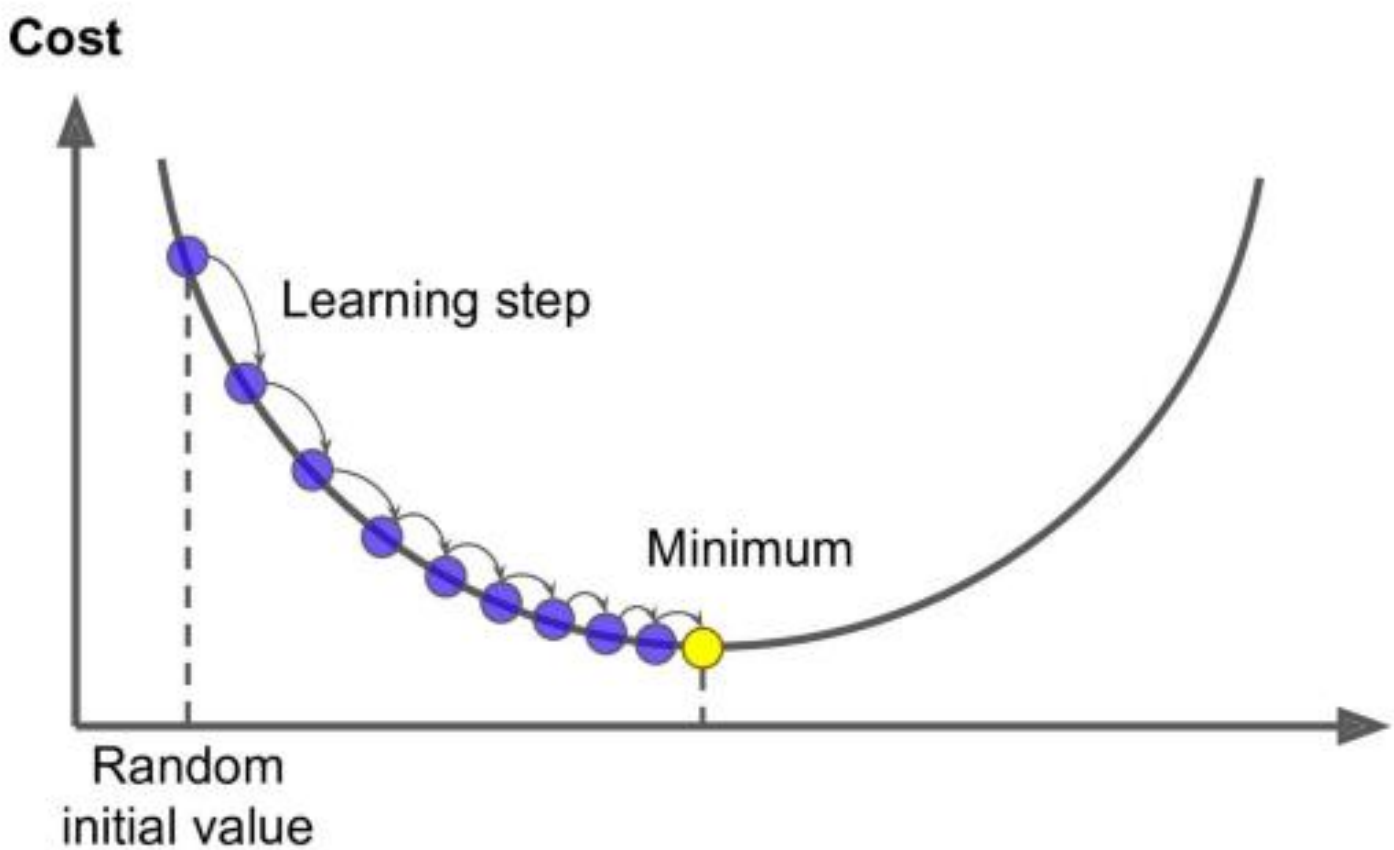
Gradient Descend



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Derivative of MSE

$$MSE'(X) = \frac{1}{m}\sum_{i=1}^{m}(a \cdot x_i + b \ - \ c_i)^2$$

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Chain Rule

$$MSE'(X) = \frac{1}{m}\sum_{i=1}^{m}(a \cdot x_i + b - c_i)^2$$

$$f'(g(x)) = f'(g(x))g'(x)$$

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Derivative of MSE

$$MSE'(X) = \frac{1}{m}\sum_{i=1}^{m}(a \cdot x_i + b \; - \; c_i)^2$$

$$f'(g(x)) = f'(g(x))g'(x)$$

$$MSE'(X) = \frac{2}{m}\sum_{i=1}^{m}(a \cdot x_i + b \; - \; c_i)x_i$$

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Gradient Descent

$$w_{new} = w - \eta \cdot MSE'(x)$$

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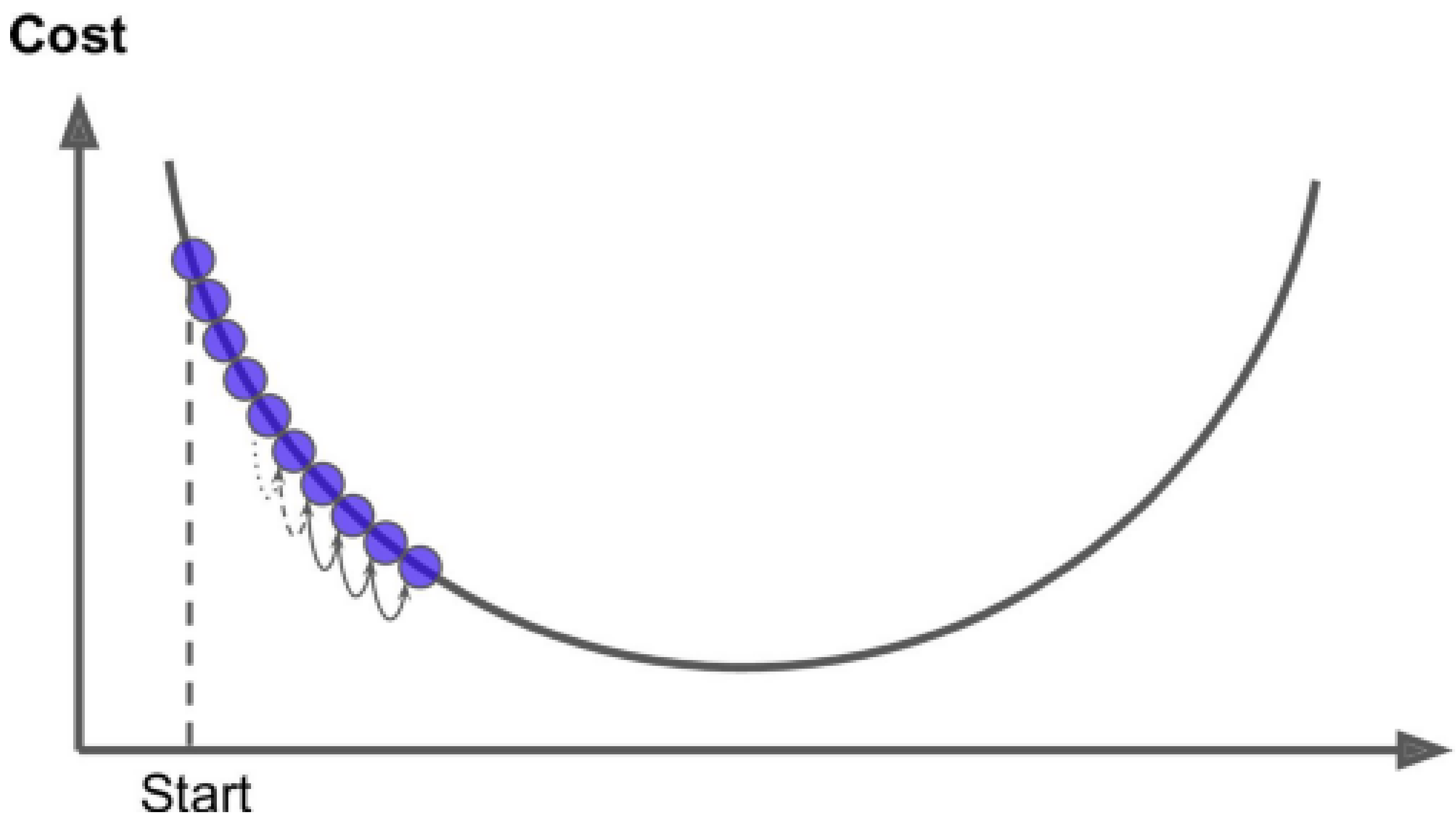
Learning Rate



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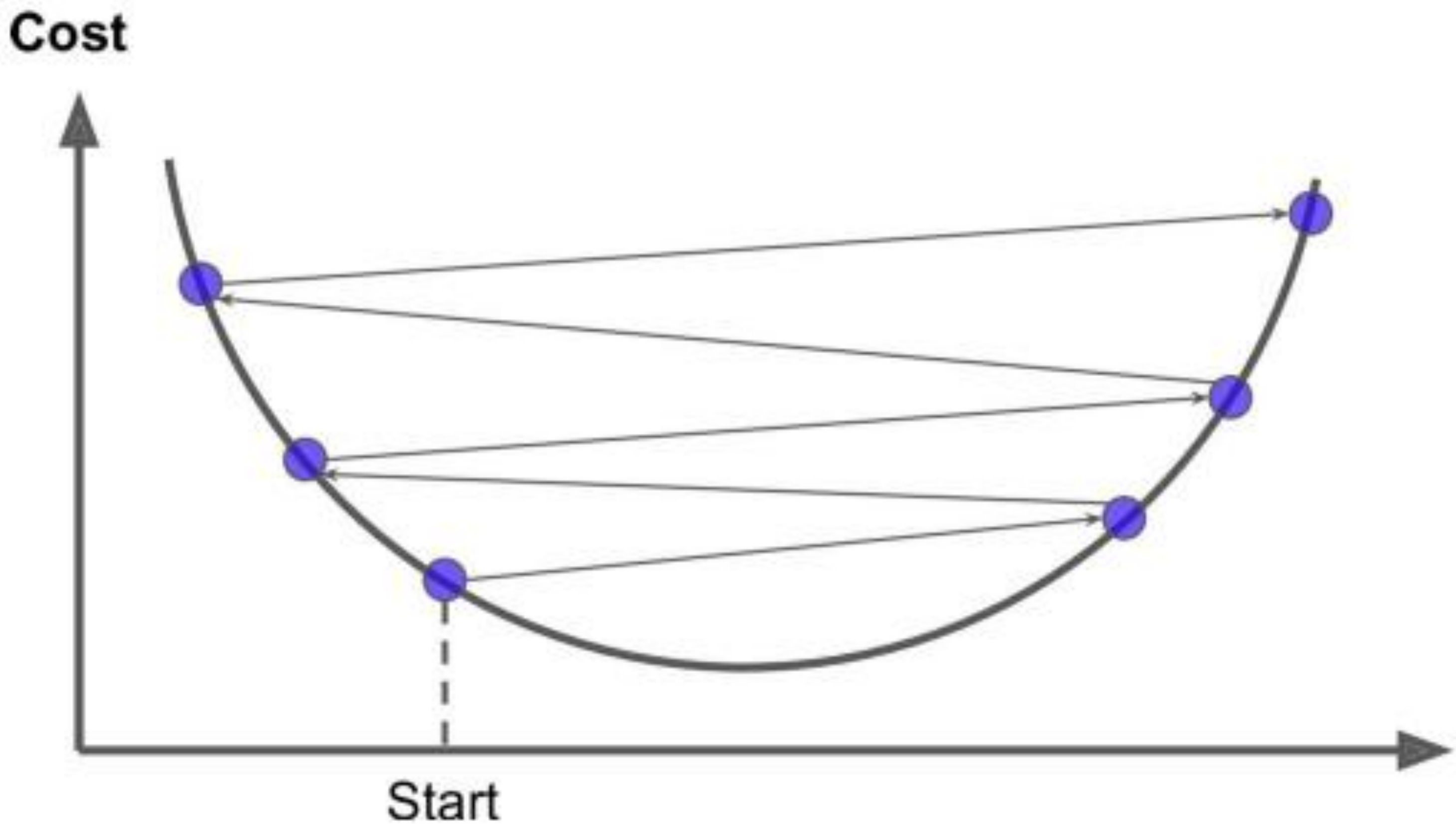
Learning Rate



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Multiple Features

$$\nabla_w MSE(w) = \begin{bmatrix} \frac{\partial}{\partial w_0} MSE(w) \\ \frac{\partial}{\partial w_1} MSE(w) \\ \frac{\partial}{\partial w_2} MSE(w) \\ \dots \\ \frac{\partial}{\partial w_n} MSE(w) \end{bmatrix} = \frac{2}{m} X^T (Xw - c)$$

Multiple Features

$$\boldsymbol{w}_{new} = \boldsymbol{w} - \eta \cdot \nabla_w MSE(w)$$

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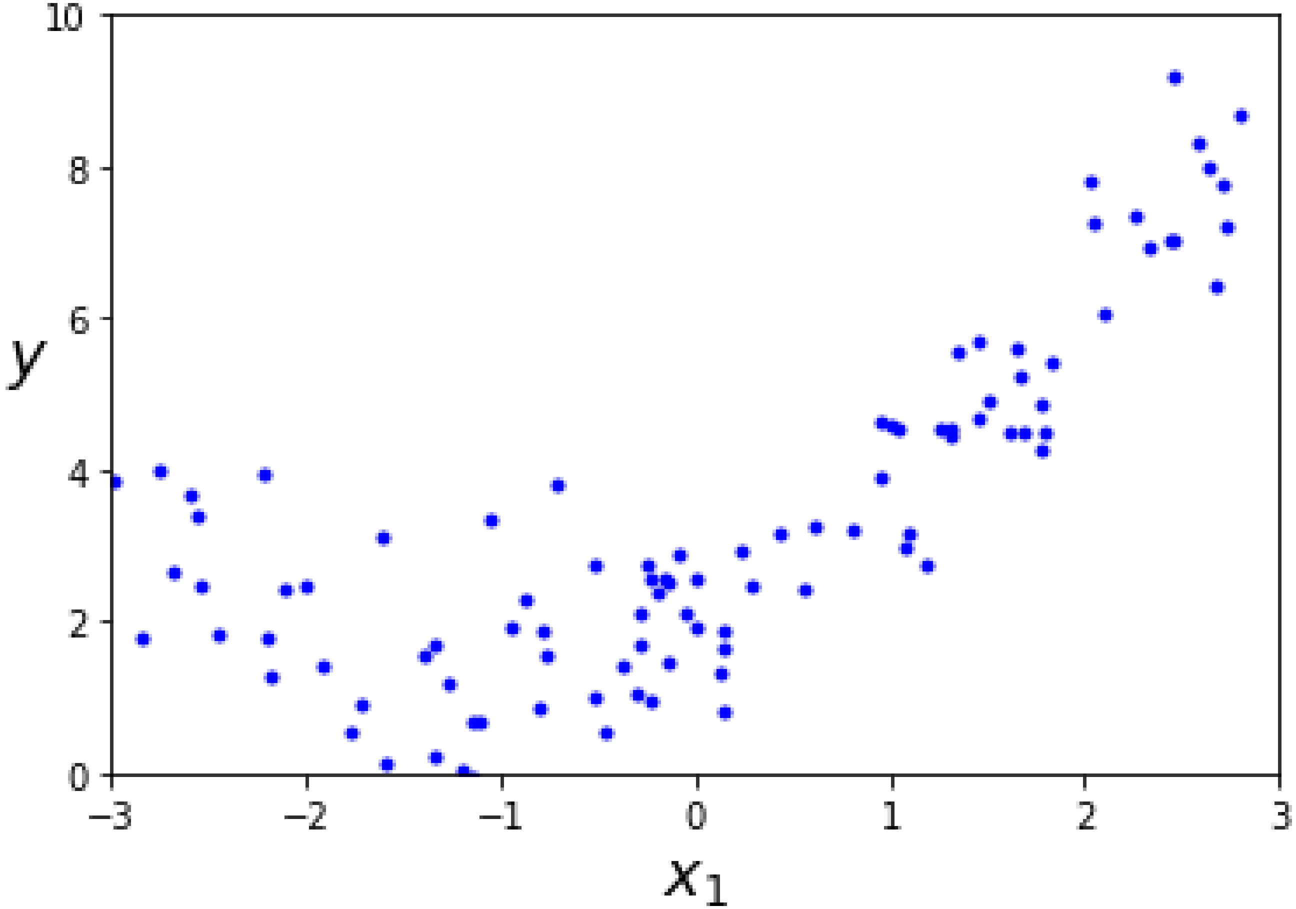
Types of Gradient Descent

Batch Gradient Descent

Stochastic Gradient Descent

Mini-Batch Gradient Descent

How to deal with non-linear data?



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Polynomial Regression

Add powers of existing features as a new features

- One feature: x
- New features: x^2, x^3, x^4, *etc.*

Train a linear model on the extended set of features

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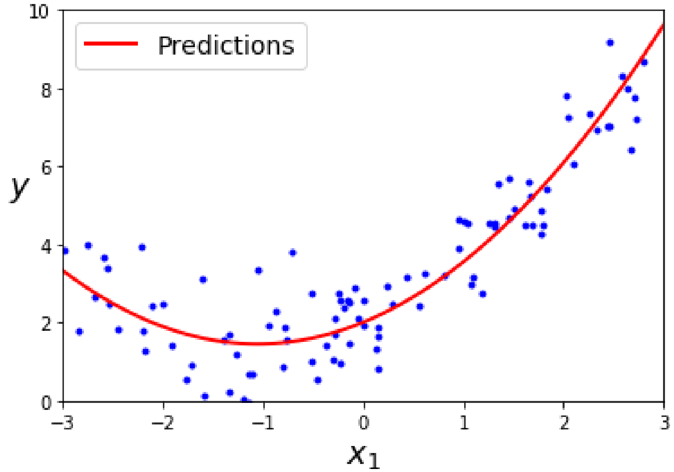
Overfitting



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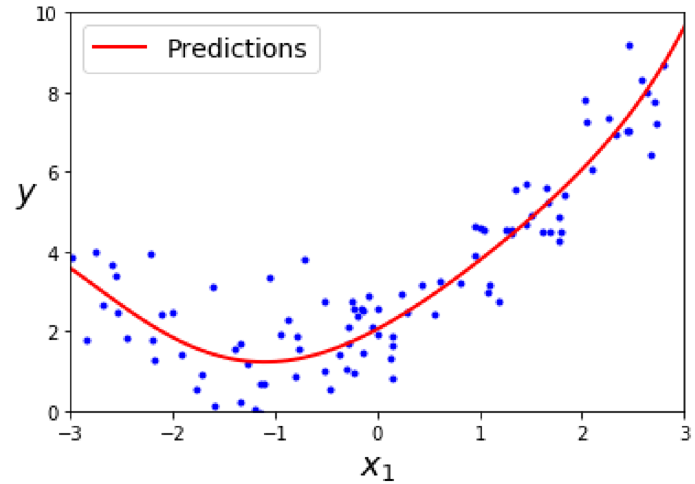

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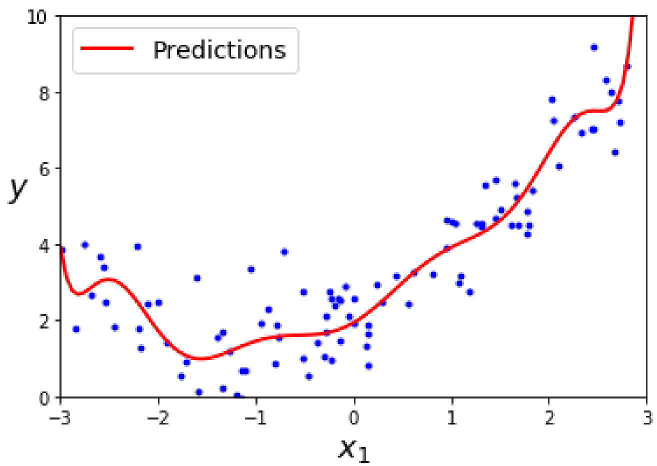

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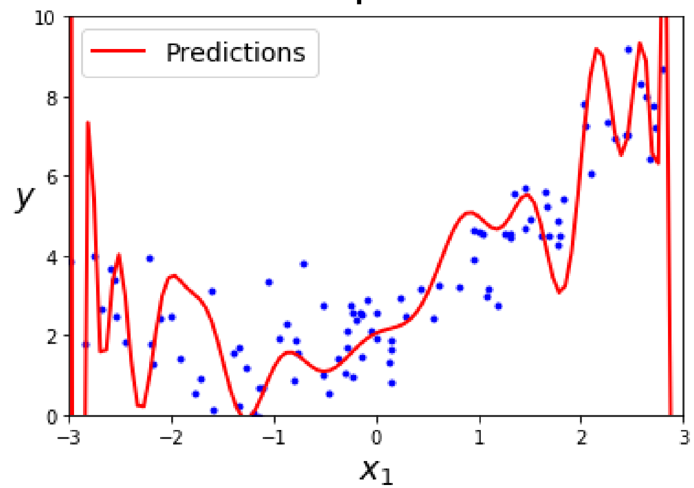

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Overfitting

Overfitting – the models adapts to the training set instead of searching the general patterns in data

Poor generalization – the model works well with data from the training set, but bad with additional data

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Conclusions

Training a model means setting its parameters so that the model best fits the training set.

Metrics (cost functions) are used to determine how good the model fits the data

Overfitting – the models adapts to the training set instead of searching the general patterns in data

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Thank you!