Калбанов Дмитрий 6301-020302D

Задание 3. Задача о ранце

$\delta^i = \sum_{j=1}^{n} c_j y_j + \frac{c_{s+1}}{a_{s+1}}(b - \sum_{j=1}^{n} a_j y_j)$

$z = 10x_1' + 11x_2' + 12x_3' \to max$

$11x_1' + 9x_2' + 10x_3' \le 47$

$\frac{10}{11}$ $\frac{11}{9}$ $\frac{12}{10}$

$z = 11x_1 + 12x_2 + 10x_3 \to max$

$9x_1 + 10x_2 + 11x_3 \le 47$

0) $\bar{x}^0 = (5,0,0)$ $z = 55 = z^*$

$\bar{y}^0 = (4,0,0)$, $S = 1$, $\delta^0 = 44 + \frac{12}{10}\cdot(47-36) = 57\frac{2}{10} > z^*$

1) $\bar{x}^1 = \bar{x}^1 = (4,1,0)$, $z = 56 = z^{**}$

$\bar{y}^1 = (4,0,0)$, $S = 2$, $\delta^1 = 44 + \frac{10}{11}\cdot(47-36) = 54 < z^{**}$

2) $\bar{x}^2 = \bar{y}^1 = (4,0,0)$

$\bar{y}^2 = (3,0,0)$, $S = 1$, $\delta^2 = 33 + \frac{12}{10}\cdot(47-27) = 57 > z^{**}$

3) $\bar{x}^3 = \bar{x}^3 = (3,2,0)$ $z = 57 = z^{***}$

$\bar{y}^3 = (3,1,0)$, $S = 2$, $\delta^3 = 45 + \frac{10}{11}\cdot(47-37) = 54\frac{1}{11} < z^{***}$

4) $\bar{x}^4 = \bar{y}^3 = (3,1,0)$

$\bar{y}^4 = (3,0,0)$, $S = 2$, $\delta^4 = 33 + \frac{10}{11}\cdot(47-27) < z^{***}$

5) $\bar{x}^5 = \bar{y}^4 = (3,0,0)$

$\bar{y}^5 = (2,0,0)$, $S = 1$, $\delta^5 = 22 + \frac{12}{10}\cdot(47-18) = 56\frac{4}{5} < z^{***}$

6) $\bar{x}^6 = \bar{y}^5 = (2,0,0)$

$\bar{y}^6 = (1,0,0)$, $S = 1$, $\delta^6 = 11 + \frac{12}{10}(47-9) = 56\frac{3}{5} < z^{***}$

7) $\bar{x}^7 = \bar{y}^6 = (1,0,0)$

$\bar{y}^7 = (0,0,0)$, $S = 1$, $\delta^7 = \frac{12}{10}\cdot\frac{47}{1} = 56\frac{2}{5} < z^{***}$ —

Ответ: $\bar{x} = (3,2,0)$, $z = 57$

Задание 4. Целочисленное программирование.

$L = 2x_1 + x_2 \to min$

$5x_1 + 4x_2 \ge 10$

$(x_1, x_2) \ge 0$

$x_1, x_2 \in Z$

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1) $L = 0 - (-2x_1 - x_2) \to min$ $\quad y_1 = -10 - (-5x_1 - 4x_2)$

2)

	1	x_1	x_2
L	0; 4	-2; $-\frac{2}{5}$	-1; $\frac{8}{5}$
y_1	$\boxed{-10}$; 2	-5; $-\frac{1}{5}$	$\boxed{-4}$; $\frac{4}{5}$

$\to$

	1	y_1	x_2
L	4; $-\frac{3}{2}$	$-\frac{2}{5}$; $\frac{2}{20}$	$\frac{3}{5}$; $-\frac{3}{4}$
x_1	$\boxed{2}$; $\frac{5}{2}$	$\boxed{-\frac{1}{5}}$; $-\frac{1}{4}$	$\frac{4}{5}$; $\frac{5}{4}$

$\to$

	1	y_1	x_1
L	$\frac{5}{2}$; $\frac{1}{6}$	$-\frac{1}{4}$; $-\frac{1}{3}$	$-\frac{3}{4}$; $\frac{1}{12}$
x_2	$\frac{5}{2}$; $\frac{1}{6}$	$-\frac{1}{4}$; $-\frac{1}{3}$	$\frac{5}{4}$; $\frac{1}{12}$
y_2	$\boxed{-\frac{1}{2}}$; $\frac{2}{3}$	$-\frac{3}{4}$; $-\frac{4}{3}$	$\boxed{-\frac{1}{4}}$; $\frac{1}{3}$

$\to$ $\{\frac{5}{2}\} = \frac{1}{2}$; $\{-\frac{1}{4}\} = \frac{3}{4}$; $\{\frac{5}{4}\} = \frac{1}{4}$ $\quad y_2 = -\frac{1}{2} - (-\frac{3}{4}y_1 - \frac{1}{4}x_1)$.

$\to$

	1	y_2	x_1
L	$\frac{8}{3}$; $\frac{1}{3}$	$-\frac{1}{3}$; $-\frac{1}{2}$	$-\frac{2}{3}$; $\frac{1}{6}$
x_2	$\frac{8}{3}$; $\frac{1}{3}$	$-\frac{1}{3}$; $-\frac{1}{2}$	$\frac{4}{3}$; $\frac{1}{6}$
y_1	$\frac{2}{3}$; $\frac{2}{3}$	$-\frac{4}{3}$; -2	$\frac{1}{3}$; $\frac{2}{3}$
y_3	$\boxed{-\frac{2}{3}}$; 1	$-\frac{2}{3}$; $-\frac{3}{2}$	$\boxed{-\frac{1}{3}}$; $\frac{1}{2}$

$\to$

	1	y_3	x_1
L	3; 0	$-\frac{1}{2}$; -1	$-\frac{1}{2}$; $\frac{1}{2}$
x_2	3; 0	$-\frac{1}{2}$; -1	$\frac{3}{2}$; $\frac{1}{2}$
y_1	2; 0	-2; -4	1; 2
y_2	1; 0	$-\frac{3}{2}$; -3	$\frac{1}{2}$; $\frac{3}{2}$
y_4	$\boxed{0}$; 0	$-\frac{1}{2}$; -2	$\boxed{-\frac{1}{2}}$; 1

$\to$

	1	y_4	x_1
L	3	-1	0
x_2	3	-1	2
y_1	2	-4	3
y_2	1	-3	2
y_3	0	-2	1

$\{\frac{8}{3}\} = \frac{2}{3}$; $\{-\frac{1}{3}\} = \frac{2}{3}$; $\{\frac{4}{3}\} = \frac{1}{3}$

$y_3 = -\frac{2}{3} - (-\frac{2}{3}y_2 - \frac{1}{3}x_1)$ $\quad \{3\} = 0$; $\{-\frac{1}{2}\} = \frac{1}{2}$

$y_4 = 0 - (-\frac{1}{2}y_3 - \frac{1}{2}x_1)$

Ответ: $L = 3$; $x_1 = 0$; $x_2 = 3$

Задание 5. Метод ветвей и границ для задачи ЛЦП.

$L = x_1 + 4x_2 \to min$ (1)

$$\left.\begin{aligned} x_2 &\geq \frac{1}{4}x_1 + 2 \\ 4x_1 + 5x_2 &\geq 20 \end{aligned}\right\} (2)$$

$(x_1, x_2) \geq 0$

$x_1, x_2 \in \mathbb{Z}$

$y_1 = \frac{x_1}{-8} + \frac{x_2}{2} = 1$

$y_2 = \frac{x_1}{5} + \frac{x_2}{4} = 1$

$L = 4: \frac{x_1}{4} + \frac{x_2}{1} = 1$

$$\begin{cases} -\frac{1}{4}x_1 + x_2 = 2 \\ 4x_1 + 5x_2 = 20 \end{cases} \Rightarrow x_1 = \frac{40}{21};\ x_2 = \frac{52}{21};\ L = 11\frac{17}{21}.$$

$Б_0 \to Б_1: (x_1 \leq 1)$ и (2) $\quad B(1; \frac{16}{5})$, $б_1 = 13\frac{4}{5}$

$Б_0 \to Б_2: (x_1 \geq 2)$ и (2) $\quad C(2; 2\frac{1}{2})$, $б_2 = 12$

$x_1 = 1$; $x_2 = \frac{16}{5}$, $L = б_1 = \frac{69}{5} = 13\frac{4}{5}$

$x_1 = 2$; $x_2 = 2\frac{1}{2}$, $L = б_2 = 12$.

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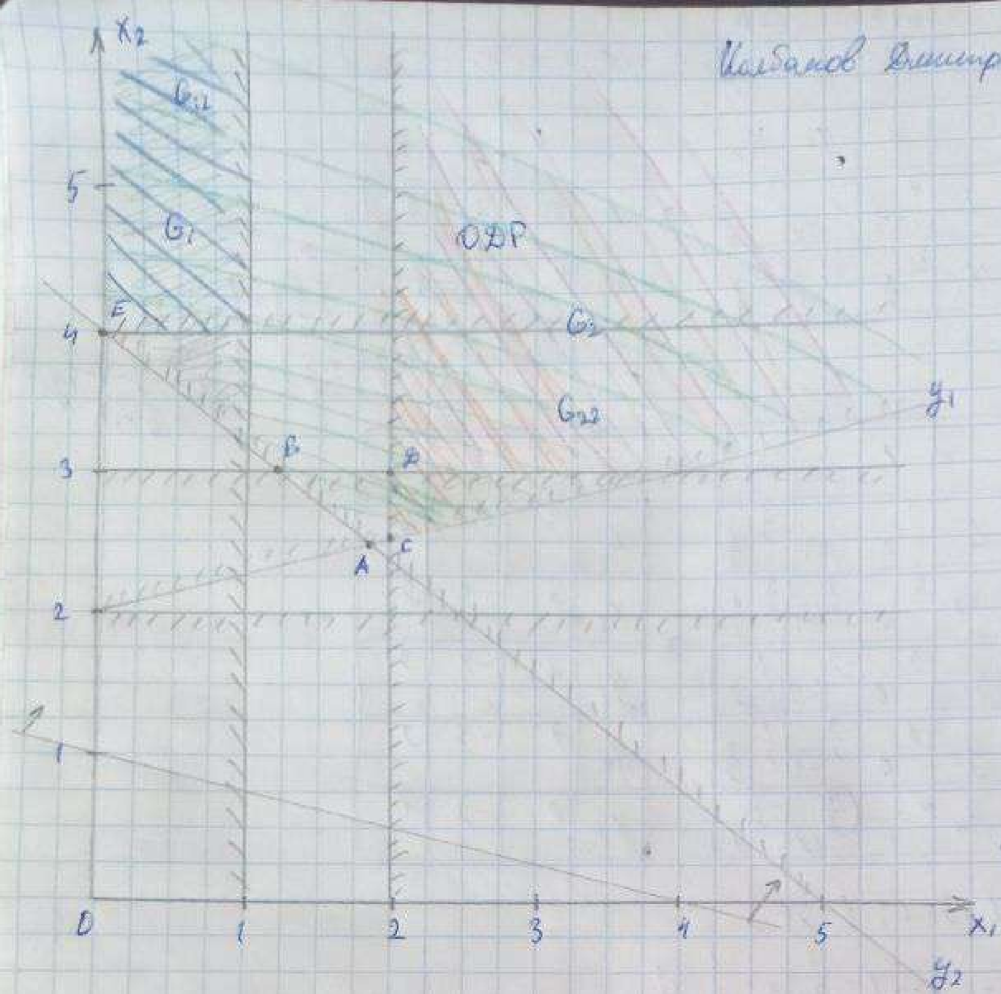
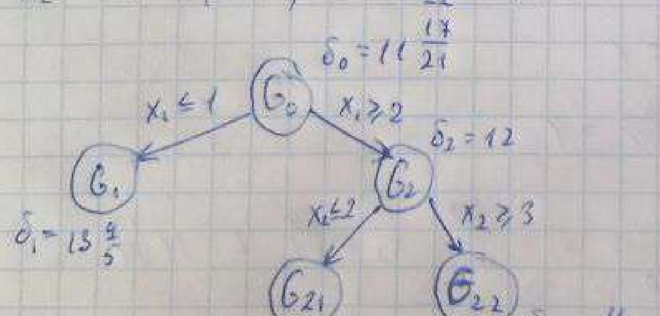


$G_2 \nearrow G_{21}: (x_2 \le 2)$ и ... $L = б_{21} = +\infty$

$G_2 \searrow G_{22}: (x_2 \ge 3)$ и ... $D(2;3)$ $б_{22} = 14$

$x_2 = 2$: нет решений $G_{21} \in \varnothing$, $L = б_{21} = +\infty$

$x_2 = 3$: $x_1 = 4$, $L = б_{22} = 14$



$L(2,3) = б_{22} = 14 \ne \min\{13\frac{4}{5}, +\infty, 14\} \Rightarrow$ решение не оптимальное

$G_1 \nearrow G_{11}: (x_2 \le 3)$ и ... $б_{11} = +\infty$

$G_1 \searrow G_{12}: (x_2 \ge 4)$ и ... $б_{12} = 16$, $E(0;4)$

$x_2 = 3$: нет решений $G_{11} \in \varnothing$, $L = б_{11} = +\infty$

$x_2 = 4$: $x_1 = 0$, $L = б_{12} = 16$

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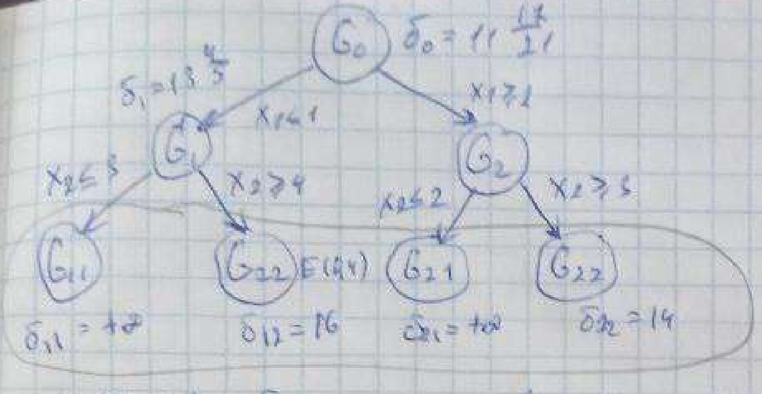

$L(2;3) = \delta_{22} = 14 = \min\{+\infty; \cancel{17}\ 16, +\infty; 14\}$ => оптим. решени

Ответ: $L = 14$; $x_1 = 2$; $x_2 = 3$