

## Lab 2

COMP9021, Session 1, 2015

### 1 Perfect numbers

A number is perfect if it is equal to the sum of its divisors, itself excluded. For instance, the divisors of 28 distinct from 28 are 1, 2, 4, 7 and 14, and  $1 + 2 + 4 + 7 + 14 = 28$ , hence 28 is perfect.

Write a program that outputs all 3-digit perfect numbers.

You should find out that there is only one solution, equal to 496.

### 2 Finding special triples of the form $(n, n + 1, n + 2)$

Write a program that finds all triples of consecutive positive three-digit integers each of which is the sum of two squares, that is, all triples of the form  $(n, n + 1, n + 2)$  such that:

- $n$ ,  $n + 1$  and  $n + 2$  are integers at least equal to 100 and at most equal to 999;
- each of  $n$ ,  $n + 1$  and  $n + 2$  is of the form  $a^2 + b^2$ .

Hint: As we are not constrained by memory space for this problem, we might use a list that stores an integer for all indexes  $n$  in  $[100, 999]$ , equal to 1 in case  $n$  is the sum of two squares, and to 0 otherwise. Then it is just a matter of finding three consecutive 1's in the list. This idea can be refined (by not storing 1s, but suitable nonzero values) to not only know that some number is of the form  $a^2 + b^2$ , but also know such a pair  $(a, b)$ ...

The output of the program could be:

```
(144, 145, 146) (equal to (0^2+12^2, 8^2+9^2, 5^2+11^2)) is a solution.
(232, 233, 234) (equal to (6^2+14^2, 8^2+13^2, 3^2+15^2)) is a solution.
(288, 289, 290) (equal to (12^2+12^2, 8^2+15^2, 11^2+13^2)) is a solution.
(360, 361, 362) (equal to (6^2+18^2, 0^2+19^2, 1^2+19^2)) is a solution.
(520, 521, 522) (equal to (14^2+18^2, 11^2+20^2, 9^2+21^2)) is a solution.
(576, 577, 578) (equal to (0^2+24^2, 1^2+24^2, 17^2+17^2)) is a solution.
(584, 585, 586) (equal to (10^2+22^2, 12^2+21^2, 15^2+19^2)) is a solution.
(800, 801, 802) (equal to (20^2+20^2, 15^2+24^2, 19^2+21^2)) is a solution.
(808, 809, 810) (equal to (18^2+22^2, 5^2+28^2, 9^2+27^2)) is a solution.
```

### 3 Estimating the probabilities of hypotheses in the light of more and more evidence

Write a program that simulates the cast of an unknown die, chosen from a set of 5 dice with 4, 6, 8, 12, and 20 faces. To start with, every die has a probability of 0.2 to be the chosen die. At every cast, the probability of each die being the chosen die is updated using Bayes' rule (find out about it if you do not know it, it is simple...). The probabilities are displayed for at most 5 casts. If more than 5 casts have been requested, the final probabilities obtained for the chosen number of casts are eventually displayed.

Here is a possible output:

```
Enter the desired number of times a randomly chosen die will be cast: 2
```

```
This is a secret, but the chosen die is the one with 4 faces
```

```
The die that has been cast yields: 4
```

```
The updated dice probabilities are:
```

```
4: 37.04%  
6: 24.69%  
8: 18.52%  
12: 12.35%  
20: 7.41%
```

```
The die that has been cast yields: 1
```

```
The updated dice probabilities are:
```

```
4: 54.18%  
6: 24.08%  
8: 13.55%  
12: 6.02%  
20: 2.17%
```

Here is another possible output:

Enter the desired number of times a randomly chosen die will be cast: 5

This is a secret, but the chosen die is the one with 8 faces

The die that has been cast yields: 7

The updated dice probabilities are:

4: 0.00%  
6: 0.00%  
8: 48.39%  
12: 32.26%  
20: 19.35%

The die that has been cast yields: 1

The updated dice probabilities are:

4: 0.00%  
6: 0.00%  
8: 62.33%  
12: 27.70%  
20: 9.97%

The die that has been cast yields: 1

The updated dice probabilities are:

4: 0.00%  
6: 0.00%  
8: 73.51%  
12: 21.78%  
20: 4.70%

The die that has been cast yields: 3

The updated dice probabilities are:

4: 0.00%  
6: 0.00%  
8: 81.76%  
12: 16.15%  
20: 2.09%

The die that has been cast yields: 7

The updated dice probabilities are:

4: 0.00%  
6: 0.00%  
8: 87.57%  
12: 11.53%  
20: 0.90%

Here is still another possible output:

Enter the desired number of times a randomly chosen die will be cast: 20

This is a secret, but the chosen die is the one with 6 faces

The die that has been cast yields: 1

The updated dice probabilities are:

4: 37.04%  
6: 24.69%  
8: 18.52%  
12: 12.35%  
20: 7.41%

The die that has been cast yields: 4

The updated dice probabilities are:

4: 54.18%  
6: 24.08%  
8: 13.55%  
12: 6.02%  
20: 2.17%

The die that has been cast yields: 6

The updated dice probabilities are:

4: 0.00%  
6: 63.54%  
8: 26.80%  
12: 7.94%  
20: 1.72%

The die that has been cast yields: 6

The updated dice probabilities are:

4: 0.00%  
6: 72.10%  
8: 22.81%  
12: 4.51%  
20: 0.58%

The die that has been cast yields: 5

The updated dice probabilities are:

4: 0.00%  
6: 78.68%  
8: 18.67%  
12: 2.46%  
20: 0.19%

The final probabilities are:

4: 0.00%  
6: 99.68%  
8: 0.32%  
12: 0.00%  
20: 0.00%