Aerodynamic Effects in a Dropped Ping-Pong Ball Experiment*

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This paper addresses aerodynamic modeling issues related to a simple experiment in which a ping-pong ball is dropped from rest onto a table surface. From the times between the ball-table impacts, the initial drop height and the coefficient of restitution can be determined using a model that neglects aerodynamic drag. The experiment prompts questions about modeling the dynamics of a simple impact problem, including the importance of accounting for aerodynamic effects. Two nonlinear aerodynamic models are discussed in the context of experimental results.

INTRODUCTION

ALTHOUGH OFTEN INCIDENTAL, sound signatures that occur in dynamic events can serve as useful clues in understanding the dynamic behavior of a physical system. This paper describes a simple experiment in which physical information can be extracted from the time intervals (of silence) between bounce sounds, after a ping-pong ball is dropped onto a hard table surface. (Musician Arthur Shnabel once said, 'The notes I handle no better than many pianists. But the pauses between notes—ah, that is where the art resides.') In particular, the time lapses in the audio signal resulting from ball-table collisions can be exploited to determine the height of the initial drop and the coefficient of restitution at the impacts. The experiment is simple to conduct, and affords students the opportunity to compare their results with established theoretical concepts and increasingly complex models.

The experiment, shown in the photograph of Fig. 1, involves dropping a ping-pong ball from rest and recording the acoustic signals generated by the ball-table impacts using a microphone (with an integrated pre-amplifier) attached to the sound card of a PC. Available software (shareware package, GoldWave [4]) can be configured to display the temporal history of the bounce sounds of successive impacts, from which the times between bounces, or 'flight times,' can be determined. These flight times are used to calculate the height of the initial drop and the coefficient of restitution of the ball-table impacts, under several assumptions, including negligible aerodynamic drag.

The bouncing ping-pong ball in the experiment can be modeled classically using the theory of direct, central impact of particles, which follows from Newton's laws of motion. This theory is described in traditional textbooks [5, 6] that present the fundamentals of dynamics. More advanced treatments of planar and three-dimensional impact for both particles and rigid bodies are developed in specialized textbooks [2, 7, 9]. A method for the measurement of the coefficient of restitution for collisions between a bouncing ball and a horizontal surface is given by Bernstein [1], and has been adopted as the basis for this experiment, as described below, and reported previously by Nagurka [8]. This paper extends the development and considers the applicability of aerodynamic models in light of the experimental results.

BACKGROUND: TABLE TENNIS

The official rules of table tennis specify the characteristics and properties of a ping-pong ball:

'The ball shall be spherical, with a diameter of 38 mm. The ball shall weigh 2.5 g. The standard bounce required shall be not less than 23.5 cm nor more than 25.5 cm when dropped from a height of 30.5 cm on a specially designed steel block. The standard bounce required shall not be less than 22 cm nor more than 25 cm when dropped from a height of 30.5 cm on an approved table.'

The specifications for the rebound height can be used to determine the coefficient of restitution, which reflects properties of the ball and the surface of impact.

In the interest of slowing down the game and making it more 'viewer friendly' the official rules were changed by the International Table Tennis Federation in September 2000 and now mandate a larger (40 mm) and heavier (2.7 g) ball. This paper follows the earlier rules (presented above). More information about table tennis can be obtained from the ITTF web site, http://www.ittf.org.

Fig. 1. Photograph of experimental setup (ball to be dropped in front of microphone).

THEORETICAL FOUNDATION

The development assumes that a ball is dropped from height h_0 onto a tabletop, taken as a massive (i.e., immobile), smooth, horizontal surface. The ball is modeled as a particle, and hence rotation is neglected. Vertical motion only is considered, as depicted schematically in Fig. 2, which shows the ball height from the surface as a function of time for the first few collisions. Due to the inelastic nature of the ball-table collisions, the maximum height of the ball decreases successively with each impact.

The free-body diagram of a falling ball, drawn in Fig. 3, accounts for three external forces:

- the body force $F_{body} = mg$ (the weight)
- the buoyancy force $F_{buoy} = \rho g V$
- the aerodynamic drag force $F_{aero} = \frac{1}{2} \rho A C_D v^2$

where m, v, $A = \frac{1}{4} \pi D^2$, and $V = \frac{1}{6} \pi D^3$ are the ball mass, velocity, cross-sectional area, and volume, respectively, for a ball of diameter D, density of air ρ, drag coefficient C_D, and gravity acceleration g. (For a rising ball, the aerodynamic force in Fig. 3 acts in the opposite direction.)

For a ping-pong ball in air at room temperature, it can be shown (see Results section) that the buoyancy force is approximately one percent of the weight. Hence, buoyancy is presumed to have a negligible effect and not treated in the analysis below.

Neglecting aerodynamic drag

For the simplest model, aerodynamic resistance is neglected. The acceleration of the ball during flight is then constant, due to gravity only. The equations of motion are $g = dv/dt$ for a falling ball and $-g = dv/dt$ for a rising ball, and are readily integrated.

Under the assumption of no air resistance, the flight time between bounces is comprised of equal and symmetrical time segments for the rise and fall
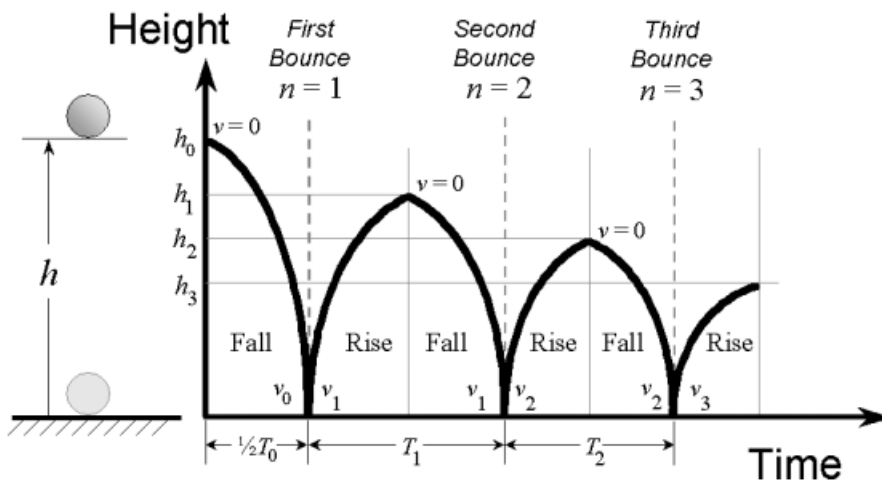

Fig. 2. Ball height vs. time for first few bounces (assuming no aerodynamic drag).
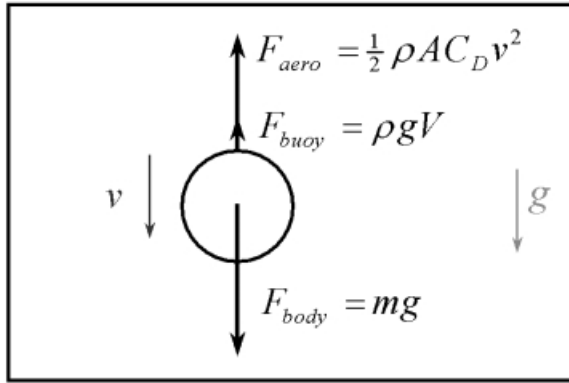
Fig. 3. Free-body diagram of falling ball showing body, buoyancy, and aerodynamic forces.

phases, i.e., the time from the surface to the top of the trajectory is the same as the time from the top back to the surface, each being half of the total flight time, T_n, between the nth and $(n+1)$th bounce. The post-impact or 'takeoff' velocity, v_n, which is the speed of the ball associated with its nth bounce is then:

$$v_n = g\left(\frac{T_n}{2}\right) \qquad (n = 1, 2, 3, \ldots) \qquad (1)$$

and is equal to the velocity prior to impact at the $(n+1)$th bounce, as shown in Fig. 2.

The coefficient of restitution is a composite index that accounts for impacting body geometries, material properties, and approach velocities. Assuming a constant coefficient of restitution, e, for all impacts,

$$v_n = e v_{n-1} = e^n v_0 \qquad (n = 1, 2, 3, \ldots) \qquad (2)$$

where v_0 is the velocity prior to the first bounce.

An expression for the flight time, T_n, can be obtained by equating Equations (1) and (2) giving:

$$T_n = e^n \left(\frac{2v_0}{g}\right) \qquad (n = 1, 2, 3, \ldots) \qquad (3)$$

which, after taking logarithms of both sides, can be written as

$$\log(T_n) = n \log e + \log\left(\frac{2v_0}{g}\right) \qquad (n = 1, 2, 3, \ldots)$$
$$(4)$$

Plotting Equation (4) in a graph of $\log(T_n)$ vs. n yields a straight line with slope $a = \log e$ and ordinate intercept $b = \log(2v_0/g)$. The slope can be used to determine e, and the intercept can be used to determine v_0 from which the initial height h_0 can be found. In general, the height h_n reached after the nth bounce can be written as:

$$h_n = \frac{v_n^2}{2g} \qquad (n = 0, 1, 2, \ldots) \qquad (5)$$

from conservation of energy. The coefficient of restitution and the drop height can then be expressed from the slope and intercept, respectively, as:

$$e = 10^a, \qquad h_0 = \frac{1}{8}g(10^b) \qquad (6)$$

These simple relations provide an easy means to predict the coefficient of restitution and the initial drop height solely from knowledge of the times between bounces. They are predicated on an analysis that neglects aerodynamic drag, an assumption that is addressed later in the paper.

Total time and distance

It can be shown (Nagurka, 2002) that under the assumption of no air resistance the total time required for the ball to come to rest, T_{total}, is:

$$T_{total} = \left(\frac{1+e}{1-e}\right)\left(\sqrt{\frac{2h_0}{g}}\right) \qquad (7)$$

and the total distance traveled along the path, S_{total}, is:

$$S_{total} = \left[1 + 2\sum_{n=1}^{\infty} e^{2n}\right] h_0, \qquad (8)$$

both of which are finite, although the number of bounces in theory is infinite.

Accounting for aerodynamic drag

Models that consider the retarding effect of aerodynamic drag during the flight phases can be developed. With aerodynamic resistance included, the flight time, T_n, between the nth and $(n+1)$th bounce is comprised of unequal and asymmetrical time segments for rise and fall. Since aerodynamic losses occur in both the rise and fall portions, the time from the surface to the top of the trajectory is not equal to the time from the top back to the surface, and the take-off speed v_n of the ball associated with its nth bounce is not equal to the speed prior to impact at the $(n+1)$th bounce.

The equation of motion for a falling ball can be written in a form that shows the aerodynamic force normalized with respect to the body force, i.e.

$$g\left(1 - \frac{F_{aero}}{F_{body}}\right) = \frac{dv}{dt}$$

Similarly, the equation of motion for a rising ball can be written as:

$$-g\left(1 + \frac{F_{aero}}{F_{body}}\right) = \frac{dv}{dt}$$

The magnitude of the normalized force term determines the importance of aerodynamic effects in the model.

In this paper two aerodynamic models, both of the form:

$$F_{aero} = \frac{1}{2}\rho A C_D v^2$$

are considered. One model assumes a constant drag coefficient C_D, whereas the alternative model assumes a velocity-dependent coefficient based on an empirically determined relation. The two models are developed below, and then compared in the Results section.

Constant drag coefficient

For the case in which the drag coefficient, C_D, is constant, the normalized aerodynamic force can be written as a quadratic function of velocity, namely:

$$\frac{F_{aero}}{F_{body}} = \beta v^2 \text{ where } \beta = \left(\frac{\rho A C_D}{2mg}\right)$$

The equations of motion can then be expressed as:

$$g(1 - \beta v^2) = \frac{dv}{dt} \tag{9}$$

for the ball falling, and:

$$-g(1 + \beta v^2) = \frac{dv}{dt} \tag{10}$$

for the ball rising. These equations can be solved analytically, and give:

$$v = v_t \tanh\left(\frac{gt}{v_t}\right) \tag{11}$$

for the ball falling from rest, and:

$$v = v_t \tan\left[\tan^{-1}\left(\frac{v_{off}}{v_t}\right) - \frac{gt}{v_t}\right] \tag{12}$$

for the ball rising with initial take-off velocity, v_{off}, at time $t = 0$, where $v_t = 1/\sqrt{\beta}$ is the terminal velocity, or the fastest speed the ball can attain. The expression for v_t follows from setting the derivative in Equation (9) to zero. Equation (12) is valid only as long as v is positive.

Additional expressions can be derived from the equations of motion, Equations (9) and (10). For example, if the ball falls from rest, the displacement, y (measured vertically down from the height at release), can be written as a function of velocity as:

$$y = \left(\frac{v_t^2}{2g}\right) \ln\left(\frac{v_t^2}{v_t^2 - v^2}\right) \tag{13}$$

or, as a function of time, as:

$$y = \left(\frac{v_t^2}{g}\right) \ln \cosh\left(\frac{gt}{v_t}\right) \tag{14}$$

Rearranging Equation (13):

$$v = (\sqrt{1 - e^{-\zeta}})v_t \text{ where } \zeta = \left(\frac{2gy}{v_t^2}\right) \tag{15}$$

A ball imparted with initial take-off velocity, v_{off}, will rise to height, h, given by:

$$h = \left(\frac{v_t^2}{2g}\right) \ln\left(1 + \frac{v_{off}^2}{v_t^2}\right) \tag{16}$$

in time, t,

$$t = \left(\frac{v_t}{g}\right) \tan^{-1}\left(\frac{v_{off}}{v_t}\right) \tag{17}$$

The equations derived here rely on an aerodynamic force expression involving the velocity squared and a constant drag coefficient. The problem of selecting a proper value for this constant is addressed in the Results section, as is the validity of the underlying model.

Velocity-dependent drag coefficient

Classic fluid dynamics experiments show that the drag coefficient of flow over a smooth sphere is not a constant, but a function of the velocity. (The drag coefficient is usually plotted as a function of the Reynolds number, which itself depends on velocity.) For the case of a drag coefficient, C_D, that is velocity-dependent, the normalized aerodynamic force can be written as a nonlinear function of velocity, $F_{aero}/F_{body} = \eta(v)$ where $\eta(v) = (\rho A/2mg)C_D(v)v^2$. The equations of motion can then be expressed as:

$$g(1 - \eta(v)) = \frac{dv}{dt} \tag{18}$$

for the ball falling, and

$$-g(1 + \eta(v)) = \frac{dv}{dt} \tag{19}$$

for the ball rising. These equations can not be solved analytically, but can be integrated numerically given $C_D(v)$.

Graphs of the normalized aerodynamic force as a function of velocity for this model as well as the constant drag coefficient model are presented in the Results section.

RESULTS

A ping-pong ball (Harvard, one-star) was dropped from rest from a measured initial height of 30.5 cm onto a butcher-block-top lab bench such that it landed and bounced near the microphone. (See Fig. 1.) From the bounce sounds, indicated by spike amplitudes in the software display window of the audio signal (Fig. 4), the times between the bounces (the flight times) were found.

The buoyancy force for the ball in air at room temperature and pressure was calculated as: $F_{buoy} = 3.46 \times 10^{-4}$ N, or 1.41% of the weight ($F_{body} = 2.45 \times 10^{-2}$ N), and its effect was neglected. The coefficient of restitution, e, and the height of the initial drop, h_0, were determined from the linear regression curve fit of the $\log(T_n)$ vs. n data presented in Fig. 5 for the first ten bounces.

The results, calculated from Equation (6), are: $e = 0.9375$ and $h_0 = 26.7$ cm, representing a 12.3% error in drop height. The predicted coefficient of
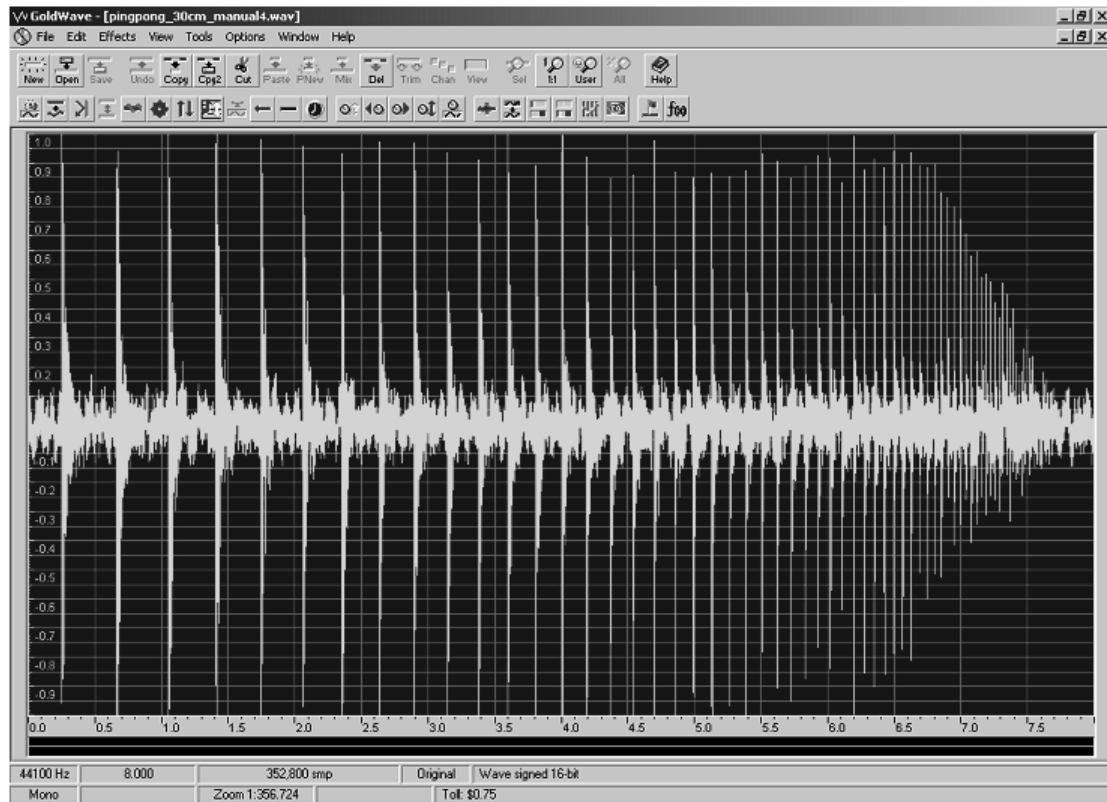
Fig. 4. Bounce history audio signal with ball-table impacts indicated by spikes. (For ball dropped from 30.5 cm, bounce sounds end in 7.5 s.)

restitution is higher than the ranges based on the rules of table tennis: $0.878 \leq e \leq 0.914$ for a 'steel block' and $0.849 \leq e \leq 0.905$ for an 'approved table'. The differences in the predicted vs. actual initial heights and in the coefficients of restitution may be attributable to several factors, including neglecting aerodynamic drag in the analysis.

Under the assumption of no drag, the total time required for the ball to come to rest is a function of the coefficient of restitution, as indicated in Equation (7). In the experiment, the total time before the ball comes to rest is 7.50 s corresponding to a coefficient of restitution of 0.938 assuming an initial drop height of 30.5 cm. Similarly, the total distance traveled by the ball before it comes to rest is a function of the coefficient of restitution for a given initial height, as indicated in Equation (8). For a drop height of 30.5 cm and a coefficient of restitution of 0.94 the total distance traversed is 4.94 m, or over 16 times the initial height.

The first aerodynamic model, for which the drag force is a quadratic function of velocity, requires a
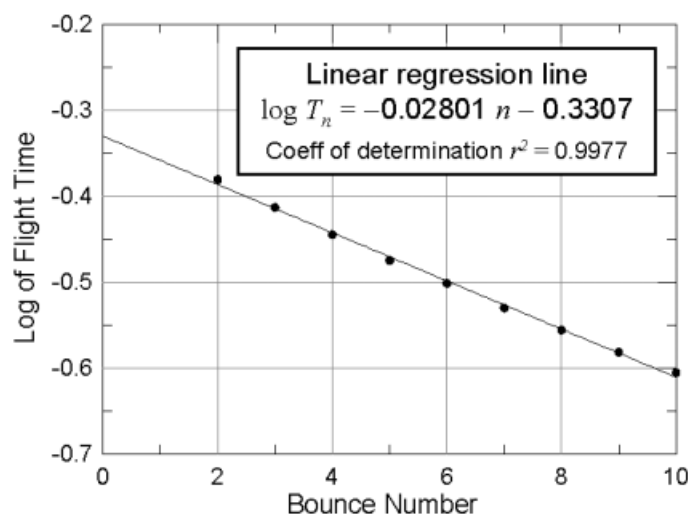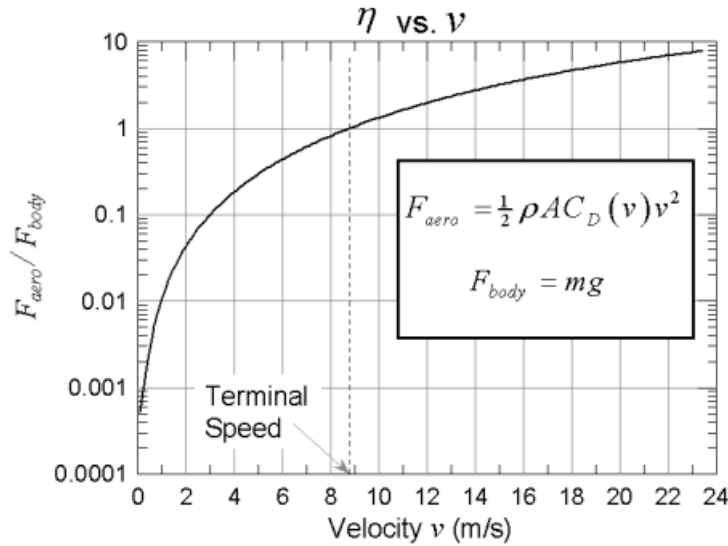

Fig. 5. Log of flight time vs. bounce number.

Fig. 6. Normalized aerodynamic force vs. velocity for nonlinear drag coefficient model.

value of the constant drag coefficient, C_D. One approach is to select a value of C_D corresponding to the maximum Reynolds number in the flow. This occurs at the maximum velocity, v_0, which arises prior to the first impact. As a first approximation, the equation, $v_0 = \sqrt{2gh_0}$, from Equation (5), which assumes no aerodynamic drag, can be used to predict a maximum approach speed of 2.45 m/s for a 30.5 cm drop. For a ping-pong ball traveling in air at room temperature at this velocity, the Reynolds number is $\mathrm{Re} = 6200$, indicating turbulent flow. Experiments in fluid mechanics show that at this value and more generally in the range $10^3 < \mathrm{Re} < 3 \times 10^5$ the drag coefficient for a smooth sphere is relatively constant at $C_D = 0.4$. (See, for example, Fig. 9.11 of [3].) Separation of the boundary layer occurs behind the sphere, and the pressure is essentially constant and lower than pressure at the forward portion of the sphere. This pressure difference is the main contributor to the drag.

A constant value of $C_D = 0.4$ corresponds to a ball terminal speed of $v_t = 9.38$ m/s. At the terminal speed, which is not attained in the experiment (since the initial drop height is too small), the aerodynamic force is in equilibrium with the weight. Accounting for aerodynamics (with the constant value of C_D), the velocity prior to the first impact can be calculated from Equation (15) as 2.405 m/s, or approximately 2% less than the non-aerodynamic prediction.

The more advanced aerodynamic model considers the fact that the drag coefficient corresponding to flow over a smooth sphere is a nonlinear function of the Reynolds number, and hence is velocity dependent (Fig. 9.11 of [3]). Using this (experimentally determined) function, the normalized aerodynamic force:

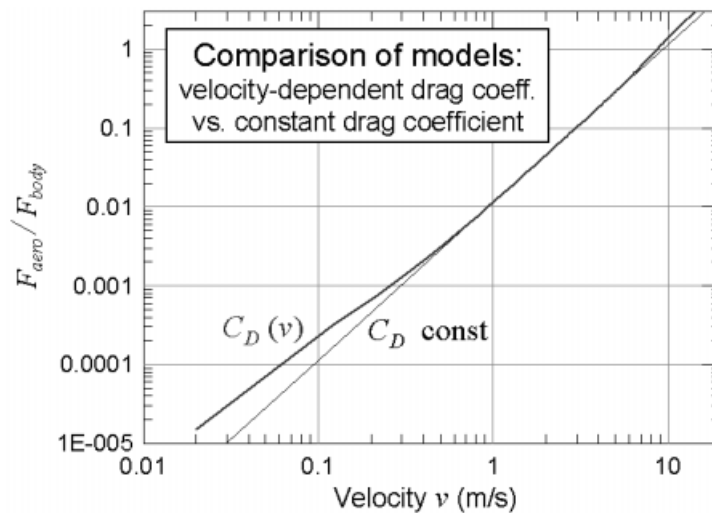$$\eta(v) = \left(\frac{\rho A}{2mg}\right) C_D(v) v^2$$

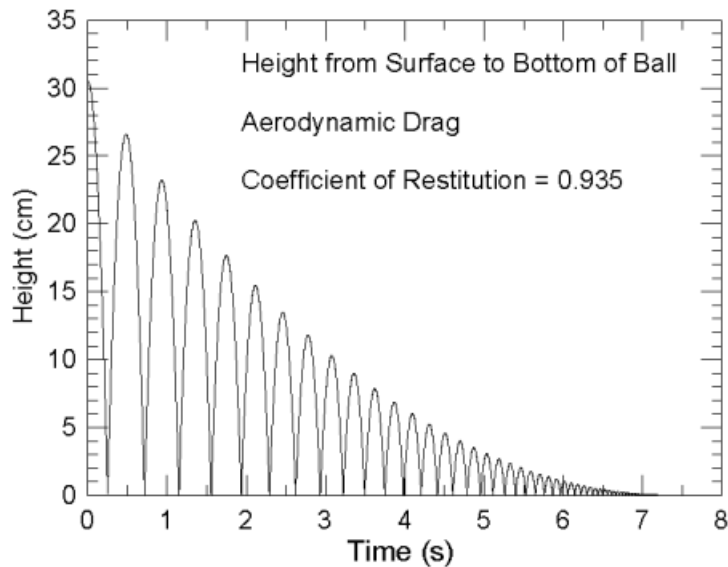Fig. 7. Comparison of normalized aerodynamic force vs. velocity for two models.

Fig. 8. Ball height vs. time for entire bounce history accounting for aerodynamic effects.

can be constructed, as shown in Fig. 6. At the maximum approach speed of 2.4 m/s for a 30.5 cm drop, the aerodynamic force is 6.4% of the body force. Although not achieved in the experiment, a ball terminal speed of 8.8 m/s is predicted using the nonlinear drag coefficient model.

A comparison of the two models, i.e., the constant vs. nonlinear drag coefficient cases, shows that the differences are minor. When plotted in the log-log graph of Fig. 7, the nonlinear drag coefficient model can be seen to closely match the straight line representing the constant coefficient model (with slope 2) except for slight deviations. The differences occur at low velocities where the aerodynamic force levels are insignificantly small, and at higher velocities that are not achieved by the ball in the experiment. The figure implies that the two models are virtually equivalent (for speeds below 0.5 m/s the force differences are inconsequential), and suggests indistinguishable effects on the predicted behavior of the ball. This has been confirmed by simulation studies, which show no differences when comparing the height histories of the two models. Figure 8 gives the height vs. time predicted by both models for the case of a coefficient of restitution of 0.935. There appears to be no additional predictive value using the more advanced model that accounts for the velocity dependent drag coefficient.

CONCLUSION

This paper has addressed aerodynamic modeling issues related to a simple experiment that engages students in creative thinking about the dynamics of a simple impact problem. Several findings emerge:

1. By adopting a model which neglects aerodynamic resistance, the time intervals between bounces can be used to determine with reasonable accuracy the initial height of a dropped ping-pong ball and the coefficient of restitution (assumed constant).

2. Although aerodynamic effects do not dominate the trajectory dynamics, they influence the bounce history and are needed for more accurate predictions.

3. The constant drag coefficient model is a very reasonable approximation of the nonlinear coefficient model, and can be used to determine the aerodynamic force and the predicted height history of the ball.

This experiment has been incorporated successfully into a junior-level mechanical engineering course, 'MEEN 120: Mechanical Measurements and Instrumentation' at Marquette University. The experiment has minimal requirements for hardware (PC with sound card, microphone, ball), software (GoldWave shareware), and time (taking approximately 5–10 minutes to conduct). Since it does not require any special laboratory facilities, the experiment can be demonstrated in a classroom using a notebook computer with a sound card and microphone.

The 'open-ended' nature of the experiment, as well as its amazing simplicity, has been attractive to students. Many questions beyond those addressed here can be posed to trigger discussion and prompt student thinking about the physics of impact and assumptions of appropriate models.

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