An introduction to the cosmic topological defects

Li Ximeng

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1 First section

This short article is arranged as follow. In Sec 2, I will discuss the history of the defects. In Sec 3, I will present some basic phase transition ideas in the early universe, following Goldstone and Higgs ideas, and finally to the non-Abelian group Your text goes here. Without special mention, the introduction should be organized with the common QFT practice, with the **Lorentz metric convention** $\{+,-,-,-\}$ and the natural unit system $c=\hbar=k_B=1$

2 The history of the topological defects

People are familiar with the nonlinear field theory since 19 century. It is Skyrme in 1961 first noted that spontaneous symmetry break (SSB) could result in a set of degenerate vacuum manifolds, which additionally acts as non-trivial topology. Later on in 1966, Nambu take the idea of SSB into the quantum field theory. Nambu has noticed that

If my view is correct, the universe may have a kind of domain structure. In one part of the universe you may have one preferred direction of the axis; in another part, the direction of the axis may be different.

Lacking of a complete theory of the phase transition, Nambu is not able to have a more solid understanding on this prototype of the *cosmic domain wall* idea. However not long after Landau putting out the order parameter, as well as the birth of the relativistic renormalizable Yang-Mills theory, people could have various pictures on the topological defects. Although Dirac has included the idea of a magnetic monopole connected with a half-infinite *thin* string, t'Hooft and Polyakov built a more elegant monopole model with no tails in 1974. Mean while, the theory of Nielsen-Olesen vortex lines was established in 1973. The very early ideas for the topological defects in the QFT has attracted much attention.

Apart from the very microscopic QFT, the ideas of topological defects gradually tempted people to look up to the sky. The success of the big bang theory, especially the discovery of the cosmic microwave background radiation (CMBR), promoted people to think about the thermal history of our universe. In 1972, even 6 yeaes before the winning of the Nobel prize of the CMBR discovery, Kirzhnits and Linde pointed out the broken symmetry should be restored at high temperatures. Weinberg soon realized in 1974 that the domain walls, just like what have been carefully studied in the ferromagnetic phase transitions, would appear in the early universe. In the same year, Everett and Zel'dovich studied the implication for the domain wall, such as the interaction between the domain walls and the matter, as well as gravitational effects. The most important contribution came from **Kibble** in 1976, who discussed the different types of the topological defects: domain walls, strings, monopoles. Kibble also connected those defects with homotopy theory.

Dramatically, when more quantitative work is done, some contradictory problems also raised. Zel'dovich and Khlopov [1978] and Preskill[1979] found that during the early stage of the universe, the monopoles would reach an *unacceptably high density* (but the domain walls and the cosmic strings were free from this puzzle). Since no monopoles were discovered even with huge efforts, people became suspicious about the classical cosmology theories. This monopole puzzle to some extent fueled the conjecture of inflation by Guth in 1981. Of cause, the ideas of inflation is out of the contents of this short introduction, yet from the historical point of view, the development of the topological defects is of much fascination.

3 Symmetry breaking in QFT

Let's start with the very basic idea of the symmetry. For a simplest real Klein-Gordon field (the massless scalar field) ϕ, the Lagrangian should be given by

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi - V(\phi) \text{ for } V(\phi) = m^2 \phi^2. \tag{1}$$

Clearly, the least action principle requires system to be stable at the minimal of the potential. Say, at the infinity, the vacuum expectation value (VEV) of the scalar field should be ϕ_0. Another aspect of the KG field is that the system does not change under the $\mathbb{Z}_2$ symmetry for $\phi \to -\phi$.

Goldstone present the first idea of how a higher order term entering the Lagrangian modifies the fields. We consider the complex KG field with $\lambda - \phi^4$ potential, say

$$\mathcal{L} = \left(\partial_\mu \bar{\phi}\right)\left(\partial^\mu \phi\right) - V(\phi\bar{\phi}) \text{ for } V(\phi\bar{\phi}) = \frac{\lambda}{4}\left(\phi\bar{\phi} - \eta^2\right)^2. \tag{2}$$

The system remains unchanged under the global $U(1)$ transformation $\phi \to e^{i\alpha}\phi$, and the vacuum expectation value should be given by $\langle 0 |\phi| 0\rangle = \eta e^{i\theta}$. We immediately find the vacuum state should also be changed under the $U(1)$ transformation since θ and α are both arbitrary.

$$\langle 0 |\phi| 0\rangle \to \eta e^{i(\theta+\alpha)}. \tag{3}$$

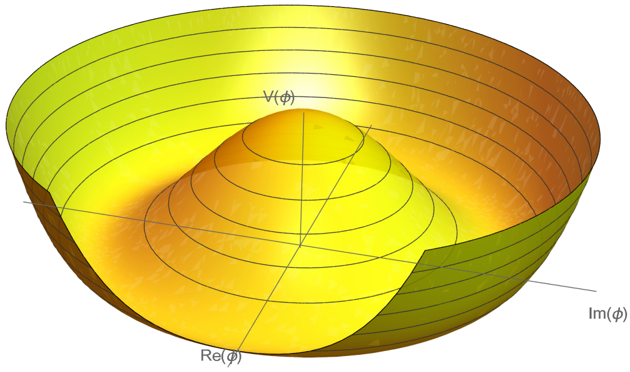

Figure 1: The $\lambda - \phi^4$ potential

There could be problems if the dynamical field ϕ stays at the origin. Imagine that $|\phi| \gtrsim 0$, the potential changing should be

$$\Delta V = \text{const.} - \frac{1}{2}\lambda\eta^2\phi\bar{\phi}, \tag{4}$$

and the minus sign implies the instability around the origin: any perturbation around $|\phi| = 0$ quickly move the system to a state with lower energy.

From the calculation before, the randomicity of θ allows us to fix the vacuum at $\theta = 0$. Further more, we can perturbatively expand the complex scalar field around $\phi = \eta$, with

$$\phi = \eta + \frac{1}{\sqrt{2}}\left(\phi_1 + i\phi_2\right), \quad \text{with} \quad \eta, \phi_1, \phi_2 \in \mathbb{R}. \tag{5}$$

Under small disturbance, the Lagrangian could be written with the form

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi_1\partial^\mu\phi_1 + \partial_\mu\phi_2\partial^\mu\phi_2\right) - \frac{1}{2}\lambda\eta^2\phi_1^2 + \mathcal{L}_{\text{int}}, \tag{6}$$

where we have absorb the cross and higher order terms into the interaction Lagrangian $\mathcal{L}_{\text{int}}$. From the re-arranged Lagrangian, one could find two new scalars appear: the one is massive, with $\mu_1 \equiv \sqrt{\lambda}\eta$. The other is massless, with $\mu_2 = 0$. The massive scalar could be understood as an oscillator along the radial direction (and since θ is fixed at 0, the oscillation should along the *Re* axis). Meanwhile the massless scalar could be regarded as the motion at a fixed radius along the cycle.

3.1 For the local $U(1)$ symmetry

Following the work of Higgs in 1964, let us consider a complex scalar field with local $U(1)$ symmetry:

$$\mathcal{L} = \bar{D}_\mu\bar{\phi}D^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V\left(\phi\right), \tag{7}$$

where $D_\mu \equiv \partial_\mu - ieA_\mu$, $F_{\mu\nu} = \frac{1}{2}\partial_{[\mu} A_{\nu]}$, and the potential V is defined before. The local $U(1)$ transformation guarantees

$$\phi(x) \to e^{i\alpha(x)}\phi(x) \text{ and } A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu\alpha(x). \tag{8}$$

Similarly, after shift the scalar field around its VEV, as $\phi \equiv \eta + \frac{1}{\sqrt{2}}\phi_1$, the Lagrangian after the redefinition becomes:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi_1)^2 - \frac{1}{2}\mu^2\phi_1^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M^2 A_\mu A^\mu + \mathcal{L}_{\text{int}}, \tag{9}$$

where the masses of the real scalar and the vector fields are $\mu \equiv \sqrt{\lambda}\eta$ and $M \equiv \sqrt{2}e\eta$.

One could naturally raise the equation: where is the massless Goldstone? Actually, it is absorbed into the vector field, and act as a third polarization. The further discussions could be found in any QFT textbook.

3.2 A more concrete example: non-Abelian SU(2) group

From the real scalar example before, we have already found that the vacuum is not invariant under the global $U(1)$ transformation. Then how can we have a more clear view on the topological structure of the vacuum states? Similarly, let us begin with a global $SU(2)$ gauge transformation: a two-component Higgs spinor,

$$\sigma \equiv \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} \to \begin{pmatrix} \sigma_1^{'} \\ \sigma_2^{'} \end{pmatrix} = U \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}. \tag{10}$$

We also take the quartic potential $V(\sigma) \propto (\sigma\sigma^\dagger - k^2\eta^2)^2$ where k is an arbitrary positive number. Use the same tricks before, the minimum of the potential should be given at $\sigma\sigma^\dagger = k^2\eta^2$. Without loss of generality, we can take $\sigma_0^\dagger = (0, \eta)$ as one of the vacuum state, and is of the group H. Clearly, no generators could annihilate σ_0, thus the little group H is trivial. Assuming the original $SU(2)$ group is G, one immediately knows the manifold of the vacuum as:

$$G/H = SU(2) \cong S^3 \tag{11}$$

The vacuum is isomorphic to a three-sphere in 4 dimension Euclidean spaces. One could couple σ to a three gauge field W_a^μ and find the three massive gauge fields and the remaining massive Higgs boson.

Similarly, imagine a Higgs fields transform as a real scalar triplet. Then a SSB with

$$SU(2) \to U(1) \to \mathbb{Z}_2 \tag{12}$$

could happen. We also consider

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} \to \varphi_0 = \begin{pmatrix} 0 \\ 0 \\ \eta_\varphi \end{pmatrix}. \tag{13}$$

However, a rotation along the z axis could annihilate φ_0 as

$$T_3\varphi_0 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \eta_\varphi \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{14}$$

Then an $SO(2)$ rotation $R \equiv \exp(i\theta T_3)$ would leave φ_0 invariant. This implies that the unbroken group must be a $U(1)$ subgroup. Then the vacuum manifold is then given by

$$G/H = SU(2)/U(1) \cong S^2 \tag{15}$$

4 Quantum field theory in the finite temperature

When the temperature is none zero, an additional thermal contribution should be considered. Generally, we could write down the system free energy

$$F = E - TS. \tag{16}$$

In the low temperature limit, only the first term contributes to the free energy. However at high temperature, the entropy term becomes unignorable. It was pointed out that the free energy per volume is of the same diagrammatic expansion as the effective potential, just replace all Green's functions to the finite temperature Green's functions, that is

$$V_{\text{eff}}(\phi, T) = \frac{F(\phi, T)}{Vo}, \tag{17}$$

where Vo is the system volume.

Consider the thermal contribution on the leading order, that is to say, the non-interaction cases. The effective potential should be

$$V_{\text{eff}} = V_0(\phi) + V^T(\phi), \tag{18}$$

containing both zero temperature and thermal contribution of the Higgs fields.

$$V^T = \sum_n \pm T \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \ln\left(1 \mp \exp\left(-\frac{\sqrt{m_n^2 + k^2}}{T}\right)\right). \tag{19}$$

Clearly, the system could be analytically studied under two limits. When $m_N \gg T$, the Boltzmann factor could exponentially suppress the thermal term, and thus returned to the zero temperature limit. **We should notice that in the stable neutron stars, the temperature should not be higher than 1 MeV, or 2m_e, since the higher temperature would trigger the production of the electron-positron pair and instability forms.** Another limit is given with $T \gg m_n$, then the Fermionic and Bosonic thermal potential are given by polynomials:

$$\begin{aligned} F_{n,\mathrm{F}} &= -\frac{7\pi^2}{720}T^4 + \frac{m_n^2 T^2}{48} + \mathcal{O}\left(m_n^4\right), \\ F_{,n,\mathrm{B}} &= -\frac{\pi^2}{90}T^4 + \frac{m_n^2 T^2}{24} + \mathcal{O}\left(m_n^4\right). \end{aligned} \tag{20}$$

Clearly, in the high temperature limit, the thermal potential has mass contributions. Still take the Abelian Higgs Lagrangian (7) and the $\lambda - \phi^4$ potential (2) and assume that $\lambda \gg e^4$ to ignore the Higgs radiative correction. We have already know the mass of the gauge boson as $M = \sqrt{2}e|\phi|$, thus, the effective potential with one loop contribution should be

$$V_{\text{eff}}(\phi, T) = V_0(\phi) + \frac{\lambda + 3e^2}{12} T^2 |\phi|^2 - \frac{2\pi^2}{45} T^4. \tag{21}$$

The coefficient 4 comes from the Bosonic degree of freedom $\mathcal{N}_F$. Now we can read the thermal mass:

$$m^2(T) = m^2_{\phi,0} - \frac{\lambda + 3e^2}{12} T^2 \tag{22}$$

A very basic idea is that when T is high, the thermal mass square could becomes negative, which could *flip* the top of the Mexico hat around $\phi = 0$ to the negative part. Then a second order phase transition (SOPT) could happen: the lowest point of the potential is smoothly transfer from a non-zero value to zero.

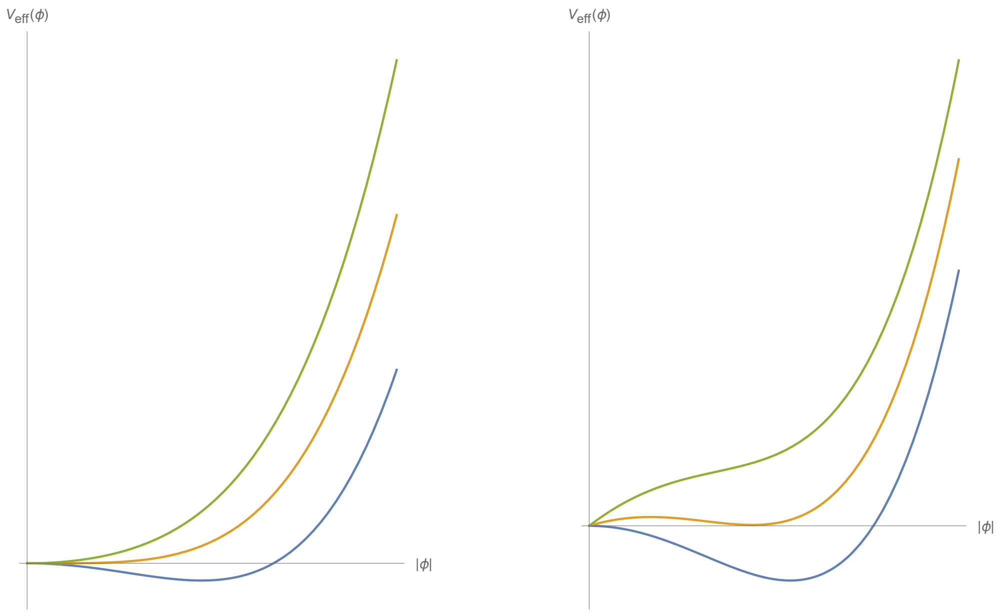

Figure 2: The effective potential as a function of the temperature and the field. Left: SOPT at $e^4 \gg \lambda$. Right: FOPT at $\lambda \gg e^4$. In the both figures, the green lines represent the $T \gg T_c$ cases, the orange lines represent the $T = T_c$ cases, and the blue lines represent the $T = 0$ cases. The second minimum point transfer smoothly from none-zero to zero in the SOPT scenario, while jumping rapidly from non-zero to zero in the FOPT scenario.

The critical temperature T_c is estimated with the zero point of the effective thermal mass square, or $T_c^2 = \dfrac{12m^2_{\phi,0}}{\lambda + 3e^2} \sim \dfrac{12m^2_{\phi,0}}{\lambda}$.

Out of SOPT, some models allow the first order phase transition (FOPT). We look ahead to the vector boson cases, and some difficulties happen. First of all, when the condition $\lambda \gg e^4$ is not strictly obey, it should be introduced with the radiation correction at the zero temperature:

$$V = \frac{\lambda}{4}(\phi\bar{\phi} - \eta^2)^2 + B\phi^4 \left(\ln \frac{\phi^2}{M^2}\right) \tag{23}$$

Where M is a renormalization scale, and $B = \frac{1}{64\pi^2}\left(\frac{5}{8}\lambda^2 + 3e^4\right) \sim \frac{3e^4}{64\pi^2}$. However, the radiation correction makes things to be difficult, since the high-T expansion is no longer valid (this could be found in any TQFT textbook). The problem is that with out the high-T limit, the effective potential is had to be studied analytically. The numerical results show that the FOPT potential has a $false vacuum$ state at $T = T_c$, and is metastable. The transition from the false vacuum to the true vacuum should be accomplished by energy release, thus is widely linked with the early universe.

5 Topological defects

We have already shown how the phase transition in the TQFT could happen: the alignment of the symmetry broken expectation value could be different in adjacent causal domains. Now a simple $\lambda - \phi^4$ model will be adapted to describe the defects analytically.

5.1 Domain walls

Still use the Lagrangian (2). For simplicity, only consider the real scalar field, and consider the field to be time independent (finite energy). The EOM (modified K-G equation) should be given with

$$\frac{\partial^2\phi}{\partial x^2} = V'(\phi) \Longrightarrow x = \pm \int \frac{\mathrm{d}\phi}{\sqrt{2V}}, \tag{24}$$

or

$$\phi\,(x) = \eta \tanh\left(\sqrt{\frac{\lambda}{2}}\eta(x - x_0)\right) \tag{25}$$

We find the solution behaves as some *localized* behavior shows. This is remarkable, since the dissipative solitons could be Lorentz boosted to arbitrary velocities. Further more, when analyze the stability of the solution, we find that any small departure from the $\pm\phi_0$ results in returning to the original fields, thus the kink soliton is stable. Then a question naturally raises: how do we understand the stability of the kink (as well as anti kink) solitons?

The result has already mentioned before: the disconnected vacuum manifold $\mathcal{M}$. As the scalar field transforms from $-\phi_0$ in the $-\infty$ to ϕ_0 at the ∞, or from one potential minimum (vacuum state) to another different one. The scalar field could then be regarded as a *mapping* from the spatial infinity in the physical space into the vacuum manifold M. One might ask, what would happen when we remove the kink soliton? Although on the both sides of the soliton, the energy remains the ground state, one should left the all the fields on the one side over the Mexico hat top – this could explain the classical stability.

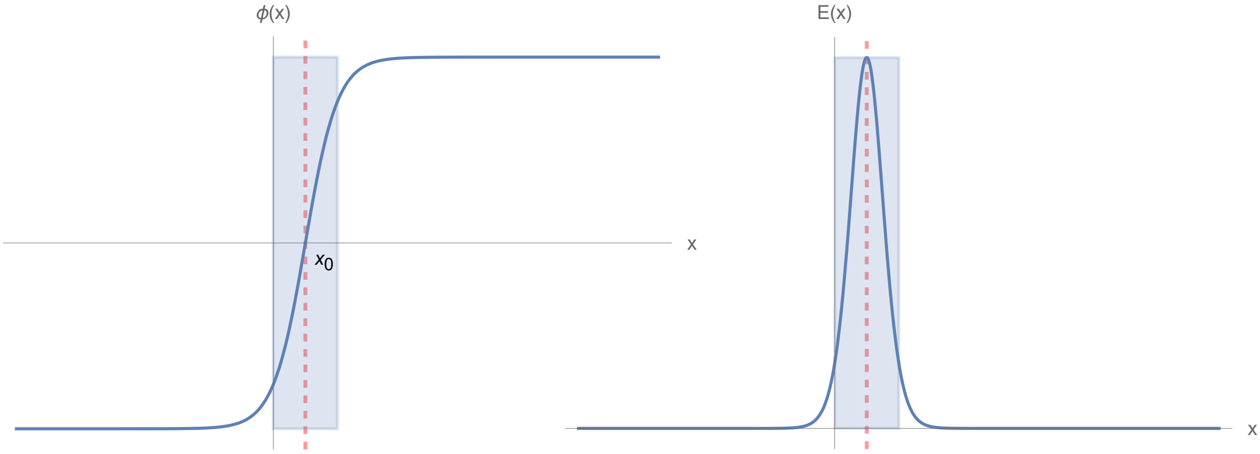

Figure 3: Left: the potential of the scalar field as a function of position x, which is a kink soliton solution. Right: the energy distribution of the scalar field. The shadowed area is the domain wall. The thickness of the wall is estimated with $\delta \sim (\sqrt{\lambda}\eta)^{-1}$, and the energy density inside the wall should be $e = \frac{2}{3}\sqrt{2}\lambda\eta^3$.

Another interesting aspect is to sum over a pair of kink and anti kink solutions, and only trivial solution is left. We see a kink soliton could be annihilated by an anti kink soliton. This is some kind of topological *charge* conservation. Now the stability of the kink soliton becomes a requirement of a topological charge conservation law. Skyrme [1961] has pointed out that the conserved topological current could be written as

$$j^{\mu} \equiv \epsilon^{\mu\nu}\partial_{\nu}\phi, \tag{26}$$

as well as the topological charge

$$Q = \int \mathrm{d}x j^0 = \phi \Big|_{-\infty}^{+\infty} \tag{27}$$

Clearly, this is promised by the Noether's theorem of the (continuous) symmetry of the vacuum. It is found that the topological charge is only depend on the two endpoint of the scalar field, and one need to know nothing about the details of the field distribution.

However, another series problem shows that the energy density of the domain wall scales as $R(t)^{-1}$ (where $R(t)$ is the scale factor of the universe. The relation could be calculated with the thickness δ and the energy density e). Comparing with the radiation ($\propto R^{-4}$) and matter ($\propto R^{-3}$), it is clear that domain wall could become a dominated component of the universe. The solution is that we don't prefer a Higgs field with disconnected vacuum states. Besides, there should be an inflationary stage to dilute the domain wall energy density.

5.1.1 Time-dependent solitons: Q-ball

Domain wall solutions are get without time varying. Rosen [1] and T. D. Lee[2] have pointed out a kind of non-topological defects defining with time varying scalar fields:

$$\phi(\boldsymbol{x}, t) = \phi(\boldsymbol{x}) e^{-i\omega t} \tag{28}$$

The charge Q of the scalar field is given with

$$Q = i \int \mathrm{d}^3 x \left(\bar{\phi}\dot{\phi} - \dot{\bar{\phi}}\phi \right) \sim \omega \left|\phi\right|^2 Vo, \tag{29}$$

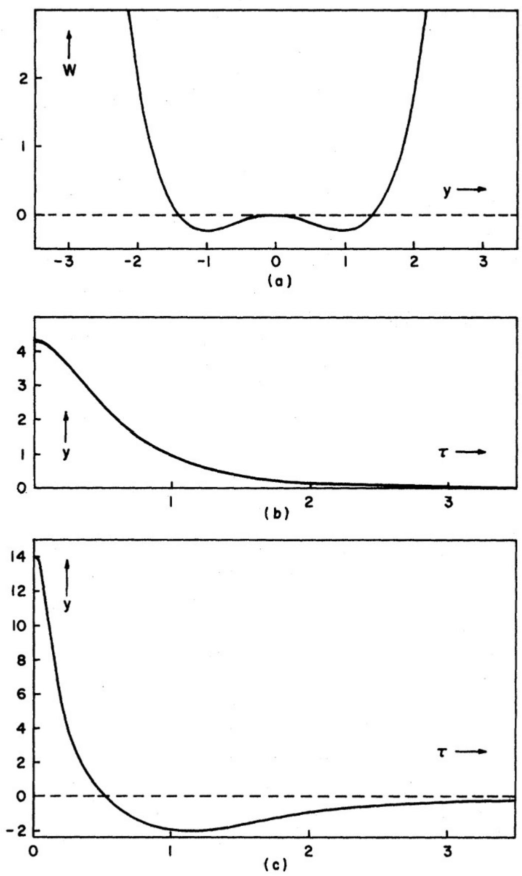

Figure 4: The potential of the time varying soliton, taken from T.D.Lee. From above to below: Higgs potential, ground state of the scalar, the first excited state.

[1]J. Math. Phys. 9, 996 (1968)
[2]PhysRevD. 13.2739

where Vo is the volume. We find that this (Noether) charge is conserved.

5.2 Cosmic strings (necklaces)

Looking back into the complex $\lambda - \phi^4$ model, one finds that the vacuum manifold is no longer a disconnected manifold. However, it still has a topological defect: there is a *hole* inside the manifold, around which a loop could be trapped. Take the Lorentz gauge $\partial_\mu A^\mu = 0$. The global $U(1)$ symmetry requires:

$$\phi \to e^{i\alpha}\phi \tag{30}$$

In order to have a minimum of the potential $\frac{\lambda}{4}(\phi\bar{\phi} - \eta^2)^2$, the complex Higgs field could be written as

$$\phi_0 = \eta e^{in\theta} \tag{31}$$

Where n is an integer, now define it as the string *winding number*. As mentioned before, the solution has broken the global $U(1)$ symmetry. This equation of motion is hard to solve, but we could write the solution under the cylindrical polar coordinates as

$$\phi = \phi_0 f(\rho) \tag{32}$$

If $\rho = 0$, the θ would be undefined. Thus a physical meaningful solution requires when $\rho \to 0$, $\phi \to 0$. The asymptotic condition requires when $\rho \to \infty$, $f(\rho) \to 1$. Re-arrange the equation of motion, there should be

$$\frac{\mathrm{d}^2 f}{\mathrm{d}\rho^2} + \frac{1}{\rho}\frac{\mathrm{d}f}{\mathrm{d}\rho} - \frac{n^2}{\rho^2}f = m_\phi^2 f\left(f^2 - 1\right) \tag{33}$$

We find that the mass square appears on the right handed side $m_\phi^2 = \lambda\eta^2$ behaves as a scale factor of the equation of motion as $\rho \to \sqrt{\lambda}\eta\rho$. The solution of the cosmic string becomes a tube with finite radius $\sqrt{\lambda}\eta$. Inside the tube, the vacuum is false. Further more, if we additionally consider the **local** $U(1)$ gauge symmetry, there would be quantized gauge flux around the string as:

$$A_\mu = \frac{1}{ie}\partial_\mu \ln\phi \;\Rightarrow\; \Phi = \int \boldsymbol{B}\cdot\mathbf{d}\boldsymbol{S} = \oint \boldsymbol{A}\cdot\mathbf{d}\boldsymbol{l} = \frac{2\pi n}{e} \tag{34}$$

This requires that the cosmic string to be either infinite length or closed loops (otherwise, the winding number or the topological charge is not conserved). Similarly, the gauge field could be written as

$$\boldsymbol{A} = \frac{n}{e\rho}a(\rho)\hat{\boldsymbol{\theta}}, \tag{35}$$

with $m_A^2 = e^2\eta^2$.

Both the Higgs field and the gauge field have a characteristic length scale: the energy density of ϕ is localized at m_ϕ^{-1}, while of $\boldsymbol{A}$ is localized at m_A^{-1}. Both fields are naturally localized without introduced a cutoff.

The energy per unit length of the cosmic string should be given by

$$\frac{E}{l} \sim \int \rho \mathrm{d}\rho \int \mathrm{d}\theta \partial_\rho \bar{\phi} \partial^\rho \phi \propto \eta^2 \tag{36}$$

Interestingly, if the symmetry breaking scale η is Planck scale (10^{19}GeV), the energy per unit could becomes as large as 10^{28}g/cm. This enormous energy density implies the possibility of primordial black hole forming and many other astrophysics aspects.

5.3 Monopoles

To be added.

5.4 Skyrmions (textures)

To be added.

6 The gravitational effects or the defects

As mentioned before, domain walls are not preferred as a cosmic structure in the universe nowadays. Some studies have discussed the axion domain walls induced CMB polarization, as the CMB polarization has already been observed by the PLANCK satellite. This section mainly focuses on the observational effect of the (gauge) cosmic strings.

Since the cosmic string should be either infinity length or closed, for the observation purpose, imagine the strings with the length sufficiently larger than the thickness m_ϕ^{-1}. Then the strings are regarded as a thin line with zero radius. A dimensionless parameter could be introduced to study the cosmic string, with

$$G\mu \sim \left(\frac{\eta}{m_{pl}}\right)^2 \tag{37}$$

When the symmetry breaking scale η is Planck scale, $G\mu \sim 1$. However for the electroweak scale $\eta \sim 100$GeV, $G\mu \sim 10^{-34}$. We point out (without the detailed steps) the metric around the string with cylindrical polar coordinates (ρ, θ, z) is

$$\mathrm{d}\tau^2 = \mathrm{d}t^2 - \mathrm{d}z^2 - \mathrm{d}\rho^2 - (1 - 4G\mu)^2 \rho^2 \mathrm{d}\theta^2. \tag{38}$$

After defining $\vartheta \equiv (1 - 4G\mu)\theta$, the spacetime return to the Minkowski spacetime

$$\mathrm{d}\tau^2 = \mathrm{d}t^2 - \mathrm{d}z^2 - \mathrm{d}\rho^2 - \rho^2 \mathrm{d}\vartheta^2 \tag{39}$$

However, a conical singularity appears as θ ranges from 0 to 2π, but $0 \le \vartheta < 2\pi(1-4G\mu)$. This is how we make up a circular cone with a circle cut off a sector. A star behind a cosmic string could therefore has double images, which is similar to the gravitational effect. The following figure [3] has shown the double images of one source.

[3] https://www.intechopen.com/chapters/69434

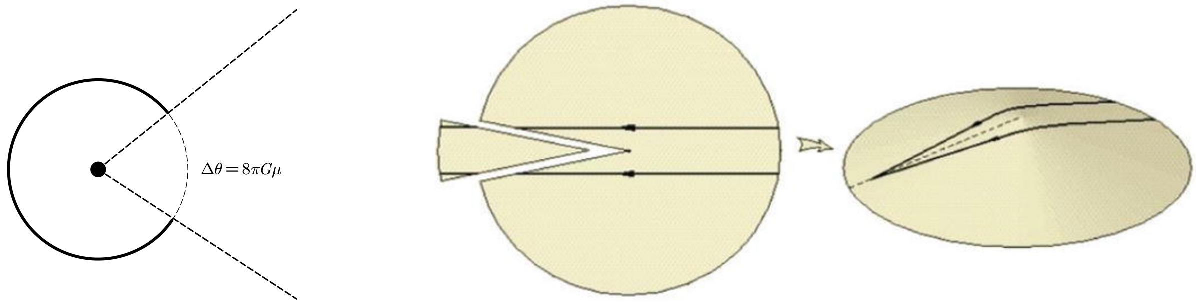

Figure 5: Left: the conical singularity, by cutting of a sector from the flat manifold. Right: The double image (lensing) of the cosmic strings.

Besides, if a cosmic string is moving perpendicularly to the light of sight (LOF) at the speed $\boldsymbol{v}$, for the two images P_1, P_2 of one source, a Doppler shift will happen, and could results in a discontinuity of the temperature at about $\delta T/T \sim 8\pi G\mu|\boldsymbol{v}|$.

6.1 The radiation of the local cosmic strings

To be added.