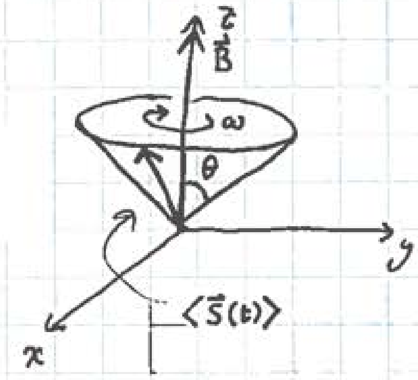


This is called Larmor ~~Freq~~ Precession

& ω is the Larmor Frequency.

introducing a time-dep. component to $\vec B$ can cause well-defined behavior & absorption or emission of a photon w/ L.F. to "flip spin" ↳ underlies NMR!

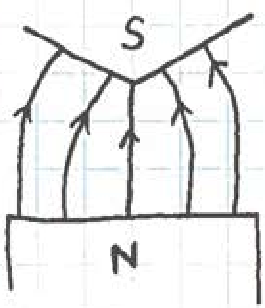
↳ resonance: match $\gamma\omega$ w/ Larmor ω.

Stern-Gerlach

Other effects of a dipole in a magnetic field? Classically:

Force $\vec F = \vec\nabla(\vec\mu\cdot\vec B)$ (force in a non-uniform B-field. Look! $F = -\nabla U$, as expected)

e.g.



$$\vec B = -\alpha x \check{x} + (B_0 + \alpha z)\check{z}$$

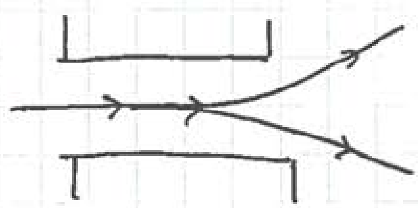
$$\Downarrow$$

$$\vec\mu\cdot\vec B = -\alpha\mu_x x + B_0\mu_z + \alpha\mu_z z$$

$$\Rightarrow \vec F = -\alpha(\mu_x \check{x} - \mu_z \check{z}) =$$

$$\text{“}\check{\vec F}\text{”} = \gamma\alpha(-\hat S_x \check{x} + \hat S_z \check{z}) \Rightarrow \text{“}\hat F_z\text{”} = \gamma\alpha S_z$$

↳ Larmor makes this not relevant



Look! A device that actually "makes a measurement"!

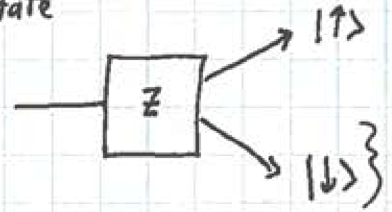
More precisely, what happens is the

spin-up part becomes a wave-fn w/ $p_z > 0$
spin-down part becomes a wave-fn w/ $p_z < 0$ ⇒ wave-packet splits in two & spreads

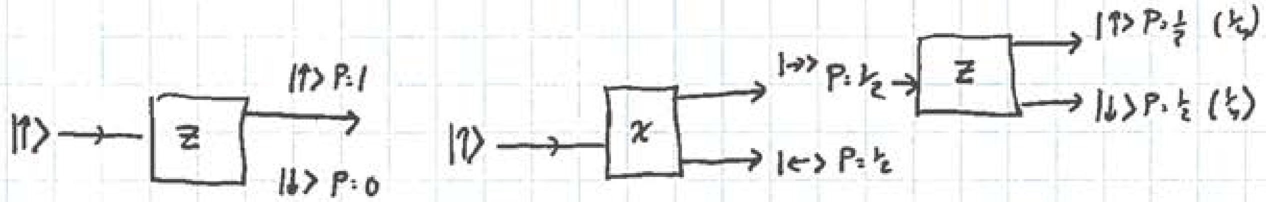
Measurement entangles space & spin d.o.f.

These now let us really explore : test quantum ideas!

- Preparing a state



SG filters act like polarizers in light!



In HW, we see that rotations are generated by ang. mom.

$$\hat{U}(\check{n},\theta) = e^{-i\theta\check{n}\cdot\hat{\vec{L}}/\hbar} \qquad \text{e.g.} \quad \hat{U}(\check{n},\theta)\,\Psi(\vec{x}) = \Psi(R\vec{x})$$

rotated $\vec{x}$ about $\check{n}$ by θ

$$\left(\hat{T}(a)\,\psi(x) = \psi(x-a)\right)$$

Interesting fact about rotating spins:

$$\hat{U}(\check{n},2\pi) = (-1)^{2s}\,\mathbb{1} \quad \text{on a spin-}s\text{ spinor} \Rightarrow \hat{U}(\check{n},2\pi)|\chi\rangle = (-1)^{2s}|\chi\rangle$$

just a phase

⇒ physically measurable things are invariant under rotation by 2π but the state isn't! mean...

- integer spins do, actually, But half-integer spins require a $\boxed{4\pi}$ (720°) rotation

(fact relating SU(2) to SO(3))

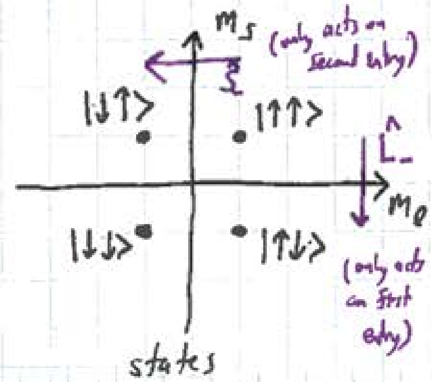
SU(2): spin rotations; SO(3): space rotations

- Dirac cup trick

Look @ spin-$\frac{1}{2}$ + spin-$\frac{1}{2}$

$|l\ s\ m_l\ m_s\rangle$ basis : $\{\underbrace{|\uparrow\uparrow\rangle}_{m_j=+1}, \underbrace{|\uparrow\downarrow\rangle}_{m_j=0}, \underbrace{|\downarrow\uparrow\rangle}_{m_j=0}, \underbrace{|\downarrow\downarrow\rangle}_{m_j=-1}\}$

$l=\frac{1}{2}$, $m_l=+\frac{1}{2}$; $s=\frac{1}{2}$, $m_s=+\frac{1}{2}$



How to create ~~ofl~~ J^2, J_z basis?

- Given (l, s) find possible values of j, m_j

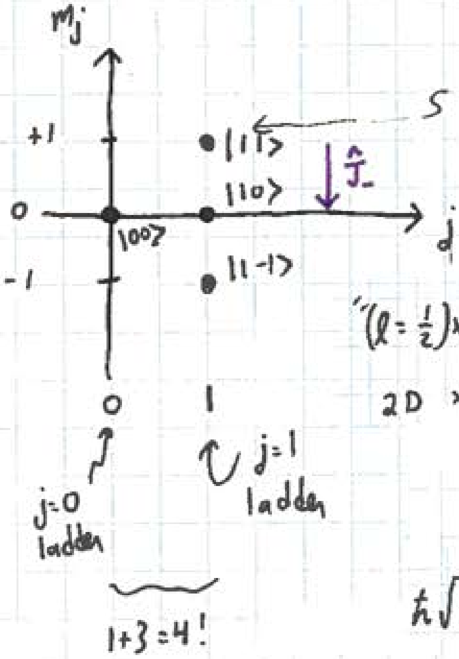
$\frac{1}{2}, \frac{1}{2} \Rightarrow \frac{1}{2}-\frac{1}{2}=0,\ \frac{1}{2}+\frac{1}{2}=1 \Rightarrow j=0,1$

$j=0$: $m_j=0$; $j=1$: $m_j=-1,0,1$

we write this as

$\underset{l}{\frac{1}{2}} \otimes \underset{s}{\frac{1}{2}} = \underbrace{0 \oplus 1}_{j\text{-values}}$

dimensions: $2 \times 2 = 1+3$ ✓



top-most rung of highest-weight ladder $|1\,1\rangle$ = "stretch state"

<u>only</u> $m_l\,m_s$ state that can go into this is $|\uparrow\uparrow\rangle$

$\Rightarrow$ Declare $\boxed{|1\,1\rangle = |\uparrow\uparrow\rangle}$

"$(l=\frac{1}{2})\times(s=\frac{1}{2}) = (j=0)+(j=1)$"

$2D \times 2D = 1D + 3D$ ✓

Climb down the rungs w/ $\hat{J}_-$

$\hat{J}_- = \hat{L}_- + \hat{S}_-$

$\Downarrow$

- $\hat{J}_-|11\rangle = (\hat{L}_- + \hat{S}_-)|\uparrow\uparrow\rangle$

$\hbar\sqrt{1(1+1)-1(1-1)}\,|1,0\rangle = (\hat{L}_-|\uparrow\rangle)|\uparrow\rangle + |\uparrow\rangle(\hat{S}_-|\uparrow\rangle)$

$= \hbar\sqrt{\frac{1}{2}(\frac{1}{2}+1)-\frac{1}{2}(\frac{1}{2}-1)}\,|\downarrow\uparrow\rangle + \hbar\sqrt{\frac{1}{2}(\frac{1}{2}+1)-\frac{1}{2}(\frac{1}{2}-1)}\,|\uparrow\downarrow\rangle$

$\Downarrow$

$\hbar\sqrt{2}\,|1,0\rangle = \hbar(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle) \Rightarrow \boxed{|1\,0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)}$

- $\hat{J}_-|10\rangle = (\hat{L}_- + \hat{S}_-)(\frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

$\hbar\sqrt{1(1+1)-0(0-1)}\,|1\,-1\rangle = \frac{1}{\sqrt{2}}\left[(\hat{L}_-|\uparrow\rangle)|\downarrow\rangle + \cancel{|\uparrow\rangle(\hat{S}_-|\downarrow\rangle)} + \cancel{(\hat{L}_-|\downarrow\rangle)}|\uparrow\rangle + |\downarrow\rangle\hat{S}_-|\uparrow\rangle\right]$

$\hbar\sqrt{2}\,|1\,-1\rangle = \frac{\hbar}{\sqrt{2}}\left[|\downarrow\downarrow\rangle + |\downarrow\downarrow\rangle\right]$

$\Rightarrow \boxed{|1\,-1\rangle = |\downarrow\downarrow\rangle}$

(know $|1\,-1\rangle = e^{i\varphi}|\downarrow\downarrow\rangle$ but this shows we can take phase to be 1 consistently!)

But what about the lonely $|00\rangle$ state?

Well, we know that $|00\rangle$ & $|10\rangle$ can only contain $|\uparrow\downarrow\rangle$ & $|\downarrow\uparrow\rangle$
(others don't have $m_j = m_1 + m_2 = 0$)

and they are orthogonal, $\langle 10|00\rangle = 0$

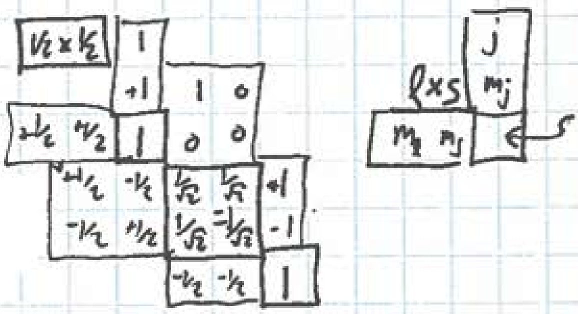
$\cdot\ |10\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle \qquad \cdot\ |00\rangle = a|\uparrow\downarrow\rangle + b|\downarrow\uparrow\rangle$

$\cdot$ orthogonal $\Rightarrow \frac{1}{\sqrt{2}}a + \frac{1}{\sqrt{2}}b = 0 \Rightarrow b = -a$

$\cdot$ normal $\Rightarrow |a|^2 + |b|^2 = 2|a|^2 = 1 \Rightarrow |a| = \frac{1}{\sqrt{2}}$

Spin Phase Convention: All coefficients real, coefficient for term w/ greatest m_ℓ is positive

$$\Rightarrow \boxed{|0\,0\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right]}$$



$\langle \ell, s, m_\ell, m_s | j\, m_j\rangle$

$C^{\ell s j}_{m_\ell m_s m_j}$

Uncoupled Basis

$|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$

$$\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Coupled Basis

$\underbrace{|0\,0\rangle}_{\text{"singlet"}} \quad \underbrace{|1\,1\rangle\ |1\,0\rangle\ |1\,{-1}\rangle}_{\text{"triplet"}}$

$$\begin{pmatrix}0\\1/\sqrt{2}\\-1/\sqrt{2}\\0\end{pmatrix} \quad \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \begin{pmatrix}0\\1/\sqrt{2}\\1/\sqrt{2}\\0\end{pmatrix} \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Total rotations leave singlet state alone & just mix up the three triplet states

in uncoupled basis, rot. operators are a mess. In coupled basis, much cleaner!

$$\begin{pmatrix} 1 & 0 \\ 0 & \begin{matrix}\text{usual spin-1} \\ \text{rotation} \\ \text{matrices}\end{matrix} \end{pmatrix}$$

This simple case shows up in lots of places!

e.g. "21-cm line of Hydrogen"

Some good Qs are to write rue basic operators like L_z, J_z, J^2, $L\cdot S$ in either basis!

- In Hydrogen, spin of nucleus (proton) ~~cr~~ interacts w/ spin of electron!

Most visible in Hydrogen g.s. ~~(...)~~ - only

Energy: $\hat{H}_{hf} = A\,\hat{\vec{\mu}}_e \cdot \hat{\vec{\mu}}_p = A\gamma_e\gamma_p\,\hat{\vec{S}}_e \cdot \hat{\vec{S}}_p$

electron spin, proton spin

use e,p instead of l s subscripts

- In $m_e\, m_p$ basis

$$\hat{\vec{S}}_e\cdot\hat{\vec{S}}_p|\uparrow\uparrow\rangle = \left(\hat{\vec{S}}_e|\uparrow\rangle\right)\cdot\left(\hat{\vec{S}}_p|\uparrow\rangle\right)$$

$$\hat{\vec{S}}_e|\uparrow\rangle \Rightarrow \frac{\hbar}{2}\left[\sigma_x\begin{pmatrix}1\\0\end{pmatrix}\check{x} + \sigma_y\begin{pmatrix}1\\0\end{pmatrix}\check{y} + \sigma_z\begin{pmatrix}1\\0\end{pmatrix}\check{z}\right] = \frac{\hbar}{2}\left[\begin{pmatrix}0\\1\end{pmatrix}\check{x} + \begin{pmatrix}0\\i\end{pmatrix}\check{y} + \begin{pmatrix}1\\0\end{pmatrix}\check{z}\right]$$

$$\hat{\vec{S}}_e|\downarrow\rangle = \frac{\hbar}{2}\left[\begin{pmatrix}1\\0\end{pmatrix}\check{x} + \begin{pmatrix}-i\\0\end{pmatrix}\check{y} + \begin{pmatrix}0\\-1\end{pmatrix}\check{z}\right]$$

$$\frac{\hbar^2}{4}\left[|\downarrow\rangle\check{x} + i|\downarrow\rangle\check{y} + |\uparrow\rangle\check{z}\right]\cdot\left[|\downarrow\rangle\check{x} + i|\downarrow\rangle\check{y} + |\uparrow\rangle\check{z}\right] = \frac{\hbar^2}{4}\left[\cancel{|\downarrow\downarrow\rangle} - \cancel{|\downarrow\downarrow\rangle} + |\uparrow\uparrow\rangle\right] = \frac{\hbar^2}{4}|\uparrow\uparrow\rangle$$

$$\hat{\vec{S}}_e\cdot\hat{\vec{S}}_p|\uparrow\downarrow\rangle = \frac{\hbar^2}{4}\left[|\downarrow\rangle\check{x} + i|\downarrow\rangle\hat{y} + |\uparrow\rangle\check{z}\right]\cdot\left[|\uparrow\rangle\check{x} - i|\uparrow\rangle\check{y} - |\downarrow\rangle\check{z}\right]$$

$$= \frac{\hbar^2}{4}\left[|\downarrow\uparrow\rangle + |\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle\right] = \frac{\hbar^2}{4}\left[2|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle\right]$$

$$\Downarrow$$

$$\mathbb{H}_{hf} = A\frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

uncoupled don't form good interaction basis!

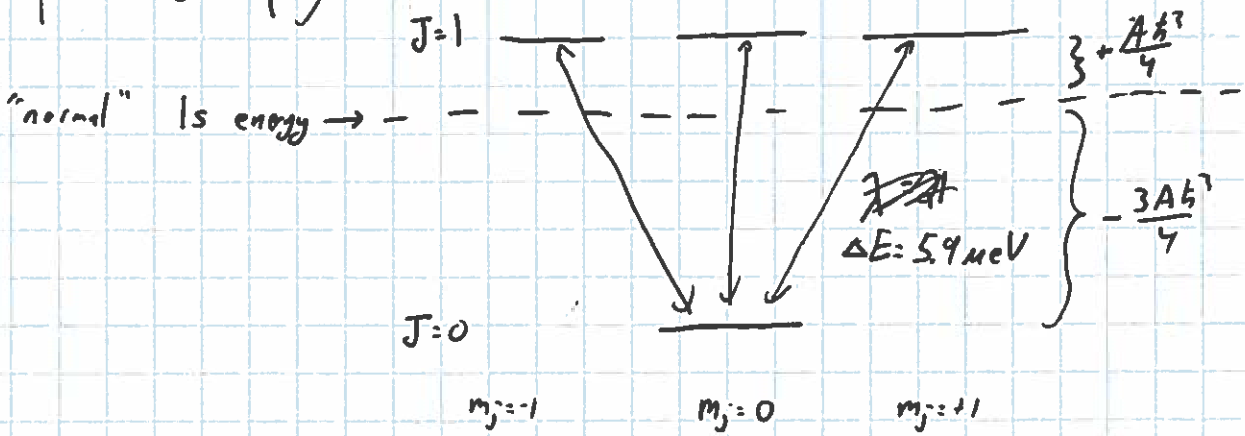
But in Coupled basis, $\hat{\vec{S}}_e \cdot \hat{\vec{S}}_p = \frac{1}{2}(\hat{J}^2 - \hat{S}_e^2 - \hat{S}_p^2)$

$$J^2 |j\, m_j\rangle = \hbar^2 j(j+1) |j\, m_j\rangle \qquad S_{e,p}^2 |j\, m_j\rangle = \hbar^2 \tfrac{3}{4} |j\, m_j\rangle$$

$$\Rightarrow \hat{\vec{S}}_e \cdot \hat{\vec{S}}_p |0\,0\rangle = \frac{\hbar^2}{2}\left[0 - \tfrac{3}{4} - \tfrac{3}{4}\right] |0\,0\rangle = -\tfrac{3}{4}\hbar^2 |0\,0\rangle$$

$$\hat{\vec{S}}_e \cdot \hat{\vec{S}}_p |1\, m_j\rangle = \frac{\hbar^2}{2}\left[2 - \tfrac{3}{4} - \tfrac{3}{4}\right] |j\, m_j\rangle = +\frac{\hbar^2}{4} |j\, m_j\rangle$$

$$A\frac{\hbar^2}{4}\left(\begin{array}{c|ccc} -3 & & 0 & \\ \hline & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & 0 & 0 & 1 \end{array}\right) \Rightarrow |j\, m_j\rangle \text{ are interaction/energy eigenstates!}$$



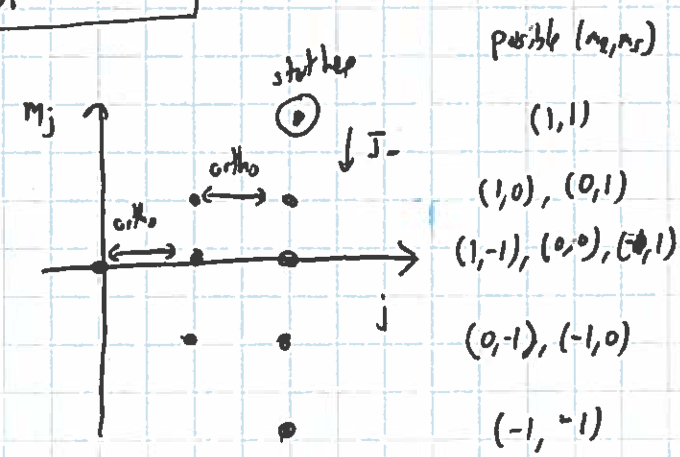
$\boxed{\lambda = 21 \text{ cm!}}$

Another important addition situation: $\ell = 1 \quad s = 1$

$$1 \otimes 1 = 0 \oplus 1 \oplus 2$$

$$3 \times 3 \qquad 1 + 3 + 5$$

singlet, triplet, quintuplet



This pops up classically. A "1" or triplet transforms under rotations like a vector

Combining two vectors gives us three special objects under rotations:

- $\vec{A}\cdot\vec{B}$, which transforms as a scalar (j=0 - singlet!)
- $\vec{A}\times\vec{B}$, which transforms as a vector (3 components - triplet - j=1!)
- $\frac{1}{2}(A_aB_b+A_bB_a)-\frac{1}{3}(\vec{A}\cdot\vec{B})\delta_{ab}$, ← 5 components which form a traceless symmetric matrix (quintuplet)

e.g. Quadrupole Moment! (j=2)

Entanglement & EPR

I can't tell you before the measurement what the values are but if I measure one aspect of the system I automatically know a result of a ~~diff~~ measurement of a different aspect.

Entangled state

or

$$\boxed{\begin{array}{c}\text{I know less about the parts than I do about the whole!}\\ \text{mathematically, state is not "simple" } |\varphi\rangle|\psi\rangle\end{array}}$$

Setup: 2 particles

Typical example: Singlet state!

$$\overset{j\ m_j}{|0\ 0\rangle} = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

e.g. I can produce this by the decay of a π^0 meson

π^0 $\overset{S\ m_S}{|1\ 0\ 0\rangle}$

$\Downarrow$

$|\tfrac{1}{2}\ \tfrac{1}{2}\ 0\ 0\rangle$

$s_1=\tfrac{1}{2}$ $s_2=\tfrac{1}{2}$ ↑ cons. of ang.

π^0

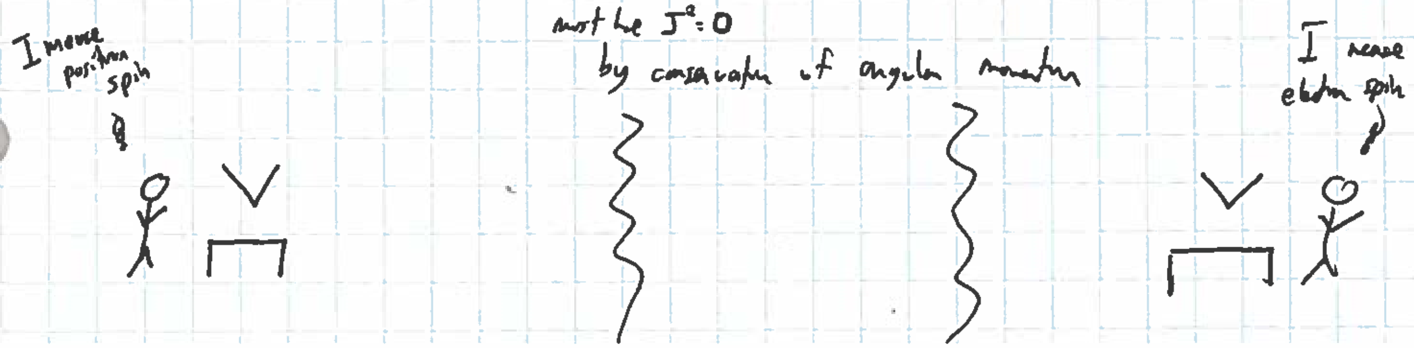
a spin-0 particle

decays to a positron & electron (each spin ½)

$\Leftarrow e^+ \qquad e^- \Rightarrow$

must be $J^2=0$

by conservation of angular momentum



EPR experiment:

π^0 ⟲ $s=0$

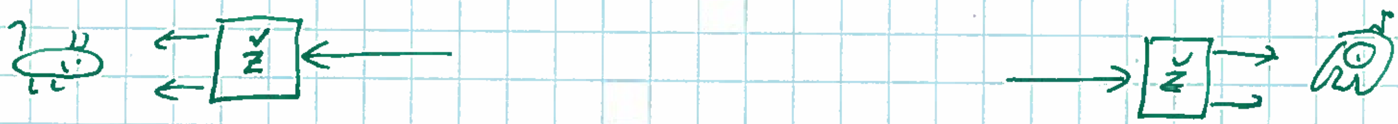
$\Rightarrow |0\ 0\rangle$

decays $\Downarrow$

$\Leftarrow e^+ \qquad e^- \Rightarrow$

Total ang is 0 $\Rightarrow$ $|s_1\, s_2; j\, m_j\rangle = |\tfrac{1}{2}\, \tfrac{1}{2}; 0\, 0\rangle$

$$\Rightarrow |00\rangle = \frac{1}{\sqrt{2}}\Big(\underbrace{|\uparrow\downarrow\rangle}_{\substack{+\text{ is up}\\ -\text{ is down}}} - \underbrace{|\downarrow\uparrow\rangle}_{\substack{-\text{ is up}\\ +\text{ is down}}}\Big) \Leftarrow \text{entangled}$$



If [figure] sees spin up $\Rightarrow$ "measure S_{1z}, get $+\hbar/2$"

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - \cancel{|\downarrow\uparrow\rangle}\right) \Rightarrow |\uparrow\downarrow\rangle \Leftarrow \text{collapsed state!}$$

$\Rightarrow$ [figure] must find spin down & vice-versa.

Paradox: How does 2nd particle "know" the collapse has happened!?
Does this mean FTL? Can we use this to communicate?

No.

It's only when the two compare their answers that the correlation is evident
(one didn't know when the other made a measurement)

Soln 1: Hidden variables (QM is "incomplete" - there's something underneath we just don't know about, like in thermo.)

<rock example>

Bell: Here's an experiment: Look at correlations in spin measurements.

$\boxed{\check{a}}$ $\boxed{\check{b}}$

Define: F = average of $\begin{pmatrix}\text{sign of}\\ A\end{pmatrix} \times \begin{pmatrix}\text{sign of}\\ B\end{pmatrix}$

Singlet: $F(\check{z},\check{z}) = -1$ (if + then − & if − then + w/ 100%)

In general, we can determine $F(\check{a},\check{b}) = -\check{a}\cdot\check{b}$

$$\langle S_{1a} S_{2b}\rangle = \langle (\check{a}\cdot\hat{\vec{S}}_1)(\check{b}\cdot\hat{\vec{S}}_2)\rangle = -\frac{\hbar^2}{4}\cos\theta$$

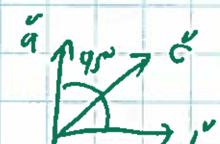
(exercise for student!)

If there were a classical hidden thing determining beforehand the outcome of the experiment (so that P is from ignorance of a knowable thing) then we must have

$$|F(\check{a},\check{b}) - F(\check{a},\check{c})| \le 1 + F(\check{b},\check{c})$$

see 12.2 in Griff.

But our QM result violates this!



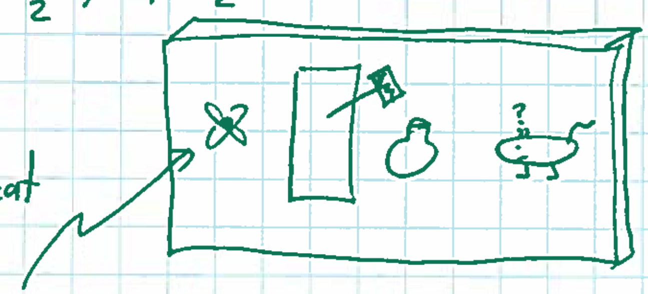
$F(a,b) = 0$

$F(b,c) = F(a,c) = -\cos(45°) = \frac{-\sqrt{2}}{2}$

$0.707 \Rightarrow \frac{\sqrt{2}}{2} \ne 1 - \frac{\sqrt{2}}{2} = 0.293$

Sch. Cat is another entanglement!

<u>If</u> nucleus decays, cyanide kills cat



after 1 τ, $|\text{nucleus state}\rangle = \frac{1}{\sqrt{2}}(|\text{undecayed}\rangle + |\text{decayed}\rangle)$

$$\boxed{\frac{1}{\sqrt{2}}(|\text{un, alive}\rangle + |\text{dec, dead}\rangle)}$$

quantum thing!

we don't see this due to <u>decoherence</u> of macroscopic objects.