Three Magnetic Dipole Problem

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1 Introduction

Magnetic systems often exhibit nonlinear responses to external magnetic fields, which means that the correlation between an applied magnetic field and the resulting magnetization is not proportional, which can induce a variety of interesting phenomena, such as chaotic behavior, autoressonance, and turbulance in earth's magnetotail [1–6].

This work focuses on investigating the dynamics of a system composed of three equidistantly spaced bar magnets affixed to a table, each free to rotate about its center. Starting from an initial equilibrium state, the system exhibits unexpected and chaotic behavior when a small increase in kinetic energy is applied to a single magnet, resulting in a small initial spin. This behavior depends on specific values of the physical parameters, such as the dipole moments and moments of inertia of the dipoles. The mechanism by which kinetic energy transfers among the magnets in the system is not fully understood. Understanding this phenomenon is crucial for gaining insights into the dynamics of magnet systems and their potential applications.

In order to better analyze this system, we consider the length of the magnets to be small compared to the distance between them, allowing us to approximate the bar magnets for magnetic dipoles.

A system composed of two magnetic dipoles exhibits a nonlinear coupling, and presents two differents time scales depending on the magnitudes of the magnetic interactions, because they are of different nature [7].

Previous works shown that the influence of small fluctuations in a system composed of two magnetic dipoles can lead to two different behaviors: the dipoles fluctuate around stable fixed points (with low amplitude fluctuations), or stochastic reversals occours between stable fixed points (with strong fluctuations). Low energy fluctuations lead to disjoint basins of attraction near stable fixed points, while higher fluctuations connect basins (including an unstable fixed point) with stochastic reversals and Poisson-distributed waiting time [8].

1.1 The forced dipole

The magnetic field generated by a single dipole is given by the formula:

$$\boldsymbol{B} = \frac{3\mu_0}{4\pi r^3}\left[(\boldsymbol{m}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \frac{1}{3}\boldsymbol{m}\right], \tag{1}$$

where $\boldsymbol{m}$ is the magnetic moment of the dipole and $\boldsymbol{r}$ is the distance to the dipole.

$$\boldsymbol{m} = m(\cos(\theta), \sin(\theta), 0), \tag{2}$$

When a fixed dipole, with magnetic moment given by the equation 2, is free to spin around it's center, and it's is placed on a surfice and subjected to a homogeneous magnetic field parallel to the surfice, represented by the equation:

$$\boldsymbol{B} = B(\cos(\theta_1), \sin(\theta_1), 0), \tag{3}$$

the dipoles experiences a torque, given by the Newton's second law of motion for spinning objects:

$$\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B}_{ext} \tag{4}$$

Note that $\boldsymbol{B}_{ext}$ and $\boldsymbol{m}$ are vectors in the same plane, and therefore the resulting torque τ is perpendicular to both of them and has components only in the $\hat{\boldsymbol{z}}$ direction:

$$\tau = mB_{ext}\sin(\theta - \theta_1), \tag{5}$$

The equation of motion for the dipole is then:

$$\begin{gathered} I\ddot{\theta} - mB\sin(\theta - \theta_1) = 0 \\ \ddot{\theta} - \omega^2\sin(\theta - \theta_1) = 0 \end{gathered} \tag{6}$$

where I is the moment of inertia of the dipole, and ω its natural frequency. If we make the direction of the external magnetic field vary in time, in such a way that:

$$\theta_1 = \varepsilon\sin(\Omega t), \tag{7}$$

then equation 6 becomes:

$$\ddot{\theta} - \omega^2\sin(\theta - \varepsilon\sin(\Omega t)) = 0, \tag{8}$$

Now, the equation 6 still resembles the pendulum equation 9, but the dynamics of the system has change significantly, specially for some values of Ω, and ω.

$$\ddot{\theta} - \omega^2\sin(\theta) \tag{9}$$

Our work will try to understand how those values affect the dynamics of this simple system.

List of Figures

References

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