Rapid Acceleration in Quadrupeds Using an Actuated Spine

Research Proposal

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Field of research: Quadruped robots; Feedback control; Mathematical modelling

This research proposal seminar, for the degree of Doctor of Philosophy in Electrical Engineering, was presented to the Head of Department and thesis Supervisor on the Wednesday 20$^{\text{th}}$ April, 2016.

Signature *Date*
Callen Fisher
Student, Department of Electrical Engineering

I confirm that the above candidate presented a seminar on Wednesday 20$^{\text{th}}$ April, 2016 in the Department on the subject of this research proposal and recommend that the proposal be approved.

Signature *Date*
Dr. Amir Patel
Supervisor, Department of Electrical Engineering

I hereby confirm that as Head of Department, I am of the view that the person nominated as the Supervisor is competent and has the time to supervise the PhD.

Signature *Date*
Prof. E. Boje
Head of Department, Department of Electrical Engineering

Abstract

Rapid Acceleration in Quadrupeds Using an Actuated Spine

Callen Fisher

Wednesday 20th April, 2016

Motion in nature is far superior to current robotic platforms. The cheetah (*Acinonyx jubatus*), along with the greyhound (*Canis familiaris*), exhibit incredible acceleration and turning abilities. During these manoeuvres the spine is observed to actively bend and flex with pronounced articulation [1] [2]. However flexible, compliant and active spines have only recently become a focal point in quadruped literature.

Current robotic platforms make use of passive compliant spines with open loop control and CPGs. The author hypothesises that a quadruped utilizing an actuated spine controlled through feedback control will exhibit far superior manoeuvrability compared to the current flexible and rigid spine quadruped robots even under uncertain friction conditions.

The effect of different spine morphologies will be investigated. These will be modelled (using Lagrange Dynamics and hybrid dynamics), simulated (in MATLAB and SIMULINK) and compared using trajectory optimisation. The stride average acceleration cost metric [3] will be used to determine the best spine for rapid acceleration manoeuvres.

The chosen spine will be designed, built and tested. A novel acceleration based controller will be designed for rapid manoeuvres across the full envelope of motion. This controller will employ the legs as well as the spine as control inputs for robust acceleration across variable surfaces.

Chapter 1

Introduction

This research is inspired by various videos and images of the cheetah (*Acinonyx jubatus*) in rapid pursuit of prey. While rapidly accelerating, it appears that the spine is playing a vital role as is evident in Figure 1.1. These transient motions, while paramount to the cheetah's survival, are generally poorly understood.

(a)

(b)

Figure 1.1: Notice how the spine flexes up [4] and down [5] while the cheetah is sprinting.

Cheetahs were filmed, by the author and dissertation supervisor, at Cheetah Outreach in Somerset West, seen in Figure 1.2. The footage shows a cheetah rapidly accelerating, braking and galloping with pronounced flexion and extension of the spine. The spine is shown to deflect about 15 degrees in both directions [6]. The deflection causes the effective back to lengthen and shorten.

Figure 1.2: A cheetah rapidly flicking its tail while braking during its daily exercise run. Image filmed at Cheetah Outreach in Stellenbosch, South Africa, by the author and supervisor.

Another animal that inspired this research is the greyhound (*Canis familiaris*, Figure 1.3) which is slower and less manoeuvrable than the cheetah yet its acceleration is still unmatched compared to state of the art robotic platforms. Greyhounds have been observed to flex their spine during acceleration manoeuvres [7].

Figure 1.3: Notice the arched back of the greyhound as it accelerates during a race [8].

Both the cheetah and greyhound are incredibly manoeuvrable animals and both have been observed to actively use their spines during acceleration manoeuvres. Yet their dynamics are poorly understood and is critical to the design of future, agile robots.

Chapter 2

Literature Review

2.1 Biology

Animals such as the cheetah and greyhound demonstrate a superior level of locomotion, agility and speed compared to their robotic counterparts [9] [10] (as seen in Figure 2.1). The main reason for this superiority is due to the fact that animal locomotion (running and turning) is not calculated, but a natural instinct that has been refined throughout the animals life using a process of trial and error [11].

Figure 2.1: Notice the arc of the spine during high speed running [12], this helps to increase the stride length of the cheetah.

The cheetah is by far the fastest land mammal with a recorded top speed of up to 25.9 m/s [13]. It has a maximum stride average acceleration of 3 m/s^2 and braking ability of 4 m/s^2 per stride. The cheetah has the ability to accelerate from zero to its top speed in three seconds [14], but it cannot maintain this speed for long. The stride length of the cheetah has been recorded to be 7 metres long [15].

From an evolutionary point of view, the spine has been considered as a propulsive engine for the body. Gracovetsky proposed the idea of the spinal engine, whereby locomotion was first achieved through the motion of the spine and the legs were developed as mere tools for the spine [16] [17]. The interaction between the spine and the legs is a highly complex interaction that has not been properly analysed in terms of nature and robotics.

There are two common gaits that are used for high speed quadruped locomotion in nature, namely the bound and gallop gait. There are two types of gallop gait, the transverse and rotary gallop as seen in Figure 2.2. The gallop gait is used for obtaining the top speed while the bound gait is used for slightly slower speeds [18]. One factor (amongst many) that increases the efficiency of a gait is the ability of the robot or animal to store energy and release it at a later stage [19]. The energy storage element can either be part of the legs, the spine or both [20].

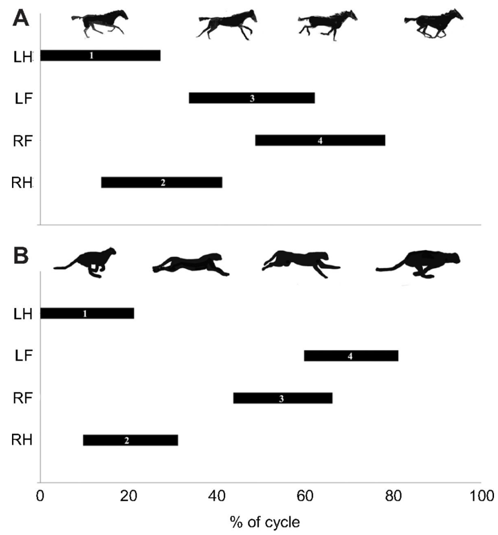


Figure 2.2: The image compares the transverse (a) and rotary (b) gallop of the horse and cheetah respectively [21].

Hildebrand argued that a flexible spine increases the maximum speed of the gait by lengthening the stride length. It was calculated that the stride length of the cheetah is lengthened by 5 percent due to its flexible spine [22] which resulted in a 10 percent speed increase [6] as seen in Figure 2.3. It was also observed that due to the spine being flexible, it allows the spine muscles, along with the leg muscles to contribute to the forward motion of the cheetah.

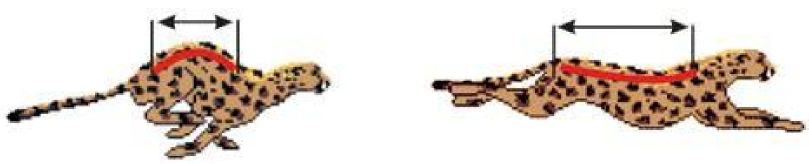


Figure 2.3: Notice how the spine lengthens and shortens [23] .

According to Hilderbrand, the flexion and extension of the spine contributes to the animals speed in the following five ways [24] [25]:

1. The distance covered during the aerial phase is increased.

2. Both the spine muscles and leg muscles contribute towards the power required for locomotion.

3. The spine contributes to the maximum forward extension of the legs, therefore increasing the maximum backward acceleration of the legs before they make contact with the ground.

4. The spine moves the body forward in an inch worm fashion.

5. The spine reduces the relative forward velocity of the girdles when the respective limbs are moving the body.

The spine does not only increase the maximum speed of the animal, it also improves the turning ability and posture of the animal [26]. It is essential to weight bearing along with protecting the neural structure and vital organs [27]. Studies done on greyhounds show that their musculoskeletal system aids in the extension and contraction of the spine during the galloping gait [28]. This clearly shows the importance of the spine and motivates further research in terms of biology and robotics.

Rapid acceleration has not had the attention that steady state motion has [29]. However, in quadrupeds, the factors limiting acceleration have been studied. These factors include the over pitch problem [3], ground traction and muscle force/power [28]. This results in a maximum average stride acceleration to avoid these conditions [3].

Investigation in rapid acceleration (non steady state gait) compared to high speed (steady state) galloping in dogs was performed by [30]. It was concluded that during acceleration the spine is flexed more than during steady state galloping and it was calculated that 43% of the body mass was supported by the forelimbs during acceleration compared to 56 to 64% during high speed galloping [30].

As is evident, the effect of spine flexion on rapid acceleration has not been investigated.

2.2 Robotics

It has been shown that robotics can be used to better understand and study nature [31]. This can be done through simulations and physical platforms. In order to investigate different quadruped morphologies (different spine and leg designs) and their respective controllers [32], a number of purely simulation based quadruped models have been developed [9] [33].

These simulated models have been used to study the effect of flexible torsos on bounding [34] [35] [36] often using reduced order models such as SLIP (spring loaded inverted pendulum). Another useful advantage of these simulations is that the stability of the platform can be analysed before it is designed and manufactured [37].

Investigation into gait transition controllers [38] and controllers that can handle and stabilize the platform after disturbances (such as foot slipping or external contacts/forces) have been designed based on simulations and implemented on physical platforms [39].

These (reduced order) simulated models often make certain assumptions such as massless legs, and therefore these models (or templates) cannot be directly applied to develop control laws for the physical robotic platform [40]. This has lead to the extension of these reduced order models to contain parameters such as leg mass to help in the design of controllers for physical robotic platforms [41]. These modified templates are still simpler then the whole body dynamics of the quadruped robot.

There are a number of quadruped platforms that have been developed by research groups (and some companies) that are pushing the boundaries of robotics in terms of performance, control and mechanical design.

The robot that inspired further quadruped robotic research and interest was the MIT Planar robot shown in Figure 2.4. This robot represented a two dimensional planar robot that was developed to analyse the effect of spine motion during high speed locomotion [42]. The robot was hydraulically powered and was capable of running at 5.9 m/s. It was invented in 1995 and set the record for the fasted legged robot (at the time).

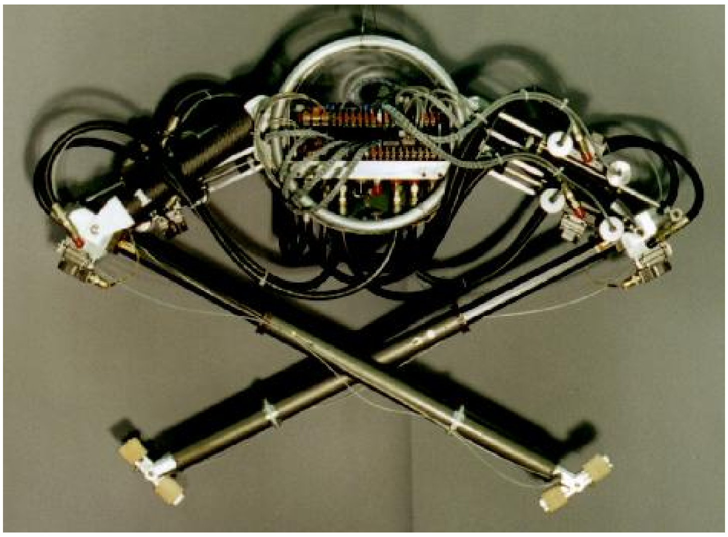


Figure 2.4: The MIT planar robot. Notice the hydraulic legs and the flexible two link spine [42].

Professor Zhao developed a quadruped robot to test the spinal engine hypothesis, and proved that locomotion is possible with un-actuated legs [16]. The platform was capable of moving at 14.5 cm/s without the use of actuated legs. The robot also showed the ability to turn at 5 degrees per second. The spine was a cable actuated spine that allowed yawing and pitching using four motors. The motors could control the stiffness of the spine and its motion. The controller consisted of four sine waves with tunable parameters to control the four motors and to generate the spine motion.

Fanari [43] was one of the first quadruped robots to utilize the **galloping** gait and was a completely passive robot with no actuators. The platform was capable of galloping down a slope utilizing only gravity and storing the energy in the compliant legs and spine. The robot is shown in Figure 2.5, and shows the springs that make the legs and spine compliant. The spine was only capable of pitching and was a flexible (multi-link) spine. The spine and gait behaviour was altered by mechanically changing the spring constants.

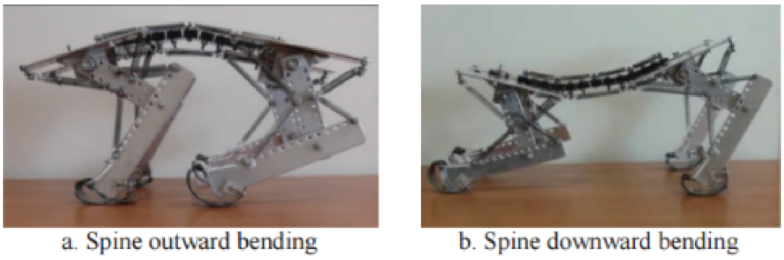
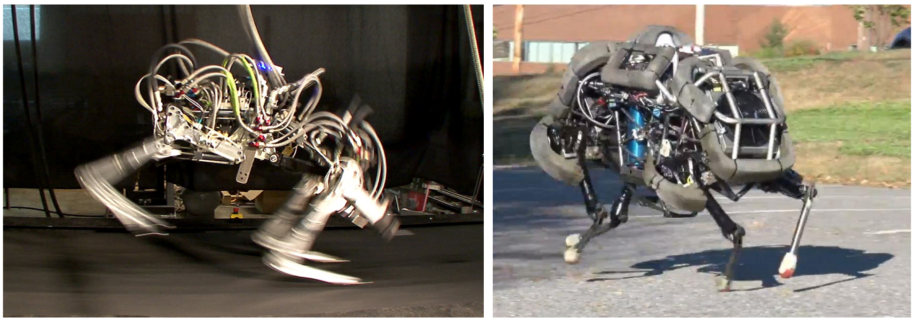


Figure 2.5: Notice the springs on the spine and the legs to add compliance. There are no actuators on this robot [43].

In 2012 MIT developed their cheetah robot [44] [45] [46] called MIT Cheetah 2, Cheetah 1 was developed in 2008 and had a flexible spine that was soon abandoned. The spine was a cable driven spine, with the cables connected to the rear legs through a differential drive mechanism resulting in the controller having no direct control over the spine motion. The flexible joints of the spine are dependent on the hip angles of the hind legs [47].

The MIT Cheetah 2 quadruped was capable of bounding (with different duty cycles) at 4.5 m/s with a rigid spine and no power or communication tether. The bounding controller used vertical impulse scaling along with a layered control scheme using MPC. In 2015 the platform was modified to jump over obstacles over half the height of the platform [48].

Boston Dynamics developed the Cheetah [49] robot and the WildCat [12] robot. Their Cheetah robot was capable of **galloping** at 13 m/s while the WildCat could only **gallop** at 7.2 m/s. The reason for this is that the WildCat is the tether free version of the Cheetah robot. The Cheetah robot required a hydraulic tether and makes use of an articulated spine. The WildCat platform makes use of a rigid spine and both these robots can be seen in Figure 2.6.



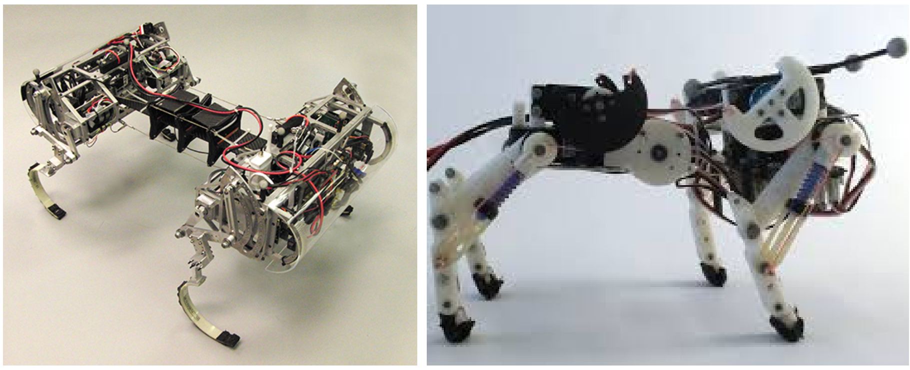
(a) (b)

Figure 2.6: The Boston Dynamics Cheetah robot is shown on the left and is one of the fastest quadruped robots. Notice the hydraulic tether [49]. The Boston Dynamics WildCat robot is shown on the right, notice the galloping gait [12].

The Locomorph platform made use of a two link, torque controlled spine that was capable of deflecting 30 degrees, which is typical of the cheetah spine [6]. The compliance of the spine was also varied using springs. The main focus of the platform was energy efficient locomotion using the CoT metric. The spine was compliant and the compliance was varied through clamping springs and using the motor dynamics. Only steady state locomotion was analysed.

Canid [2] makes use of an actuated leaf spring spine to improve the bounding gait. With open loop control (a simple leg recirculation controller in the form of a PID controller) it can run at 2 body lengths per second. Body energy (sum of mechanical and potential energy) was the performance metric. Canid used springs to capture the effect of torso compliance on motion generation.

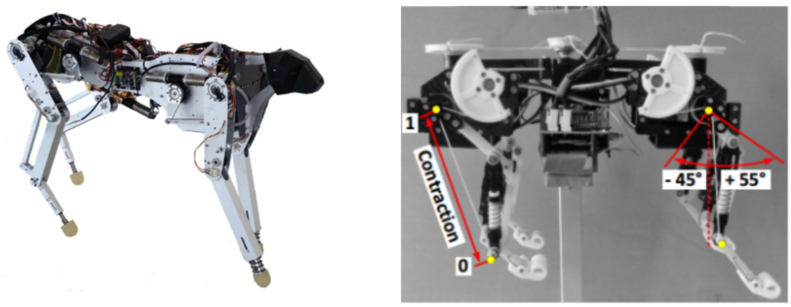
BobCat [50] [51] was used to investigate the bounding gait and was controlled via CPGs and open loop control. It was shown that the horizontal impulse for an active spine was less as the spine converted the impulse to forward motion. The speed increased from 3.7 body lengths per second to 4.7 body lengths per second with a 10 % larger stride length. These two platforms, Canid and BobCat, can be seen in Figure 2.7.



(a) (b)

Figure 2.7: The Canid robot is shown on the left. It is a modified version of the RHex platform. Notice the cable driven spine [2]. The BobCat robot is shown on the right. Notice the bio-inspired cable driven leg design and actuated spine [50].

Cheetaroid [52] [53] made use of a custom built direct drive linear actuator that was used to control the legs and the spine. Robust position tracking of the legs was performed with ground contact prediction without the use of force sensors. Cheetah-Cub-S [54] is the flexible spine version of the Cheetah Cub. Turning was investigated in this platform using open loop control via CPGs and it was found that it can turn twice as fast and maintain the bound gait compared to the rigid spine version. The spine could yaw 10 degrees. These two platforms can be seen in Figure 2.8



(a) (b)

Figure 2.8: The Cheetaroid robot is shown on the left. Notice the linear actuators powering the legs and the spine [53]. The Cheetah Cub S robot is shown on the right [54].

As can be seen in the last few years quadruped research has increased resulting in a number of new platforms. Many platforms still rely on CPGs (open loop control) and are not focused on the high speed transient motions. Further, little research has been done on the gallop gait and the effects of an active spine for acceleration and turning. It must be noted that none of the above platforms had spines that could roll, they could only pitch and/or yaw. Additionally, the aforementioned quadruped studies(simulated and physical platforms) have yet to examine how an active spine can be utilized with active feedback control to perform high speed transient motions and to improve the manoeuvrability of robotic platforms.

Chapter 3

Research Question and Hypotheses

High speed manoeuvrability along with transient motions in quadrupeds (animals and robotic platforms) has not been fully investigated. Research has shown that rapid acceleration in quadrupeds are subject to theoretical limits. These limits have been shown to be exceeded by the use of actuated tails [55]. Thus, with insight gained from the literature, the key research question is as follows:

Question. ***Can a quadruped robot utilizing an active spine controlled through feedback exhibit superior acceleration compared to the current flexible and rigid spine quadruped robots?***

This has resulted in three hypotheses that will be investigated in this dissertation:

1. An active spine improves the manoeuvrability of a quadruped (animal and robotic platform).
2. An optimal spine morphology/design exists to maximise manoeuvrability.
3. A feedback controller, using acceleration based control, will achieve a higher level of manoeuvrability even under uncertain friction conditions compared to CPG and open loop control.

Chapter 4

Methodology

Due to the multi-disciplinary nature of this research, the hypotheses will be tested via the following three methods:

1. Mathematical modelling, simulation and trajectory optimisation.
2. Feedback controller design.
3. Experimental validation on a high speed quadruped robotic platform.

The performance metric used to validate the hypotheses and research question is the stride average acceleration which has been used to analyse the acceleration abilities of quadruped animals [3]. The main focus of the research will be to investigate the spine as the legs are being optimised, designed and built by another PhD student. The legs will be modelled as a spring damper system with a point mass at the estimated CoM of each leg. The above methods are described in more detail below:

4.1 Mathematical Modelling, Simulation and Trajectory Optimisation

The dynamics of the quadruped robot will be modelled using Lagrange dynamics [56]. The robot can undergo a number of phases (aerial phase, rear leg in contact with ground *etc.* as seen in Figure 4.1) along with exhibiting continuous (the motion of the system) and discrete dynamics (ground contact events), therefore hybrid dynamics [57] was used to simulate the full quadruped dynamics. The equations of motion of the quadruped will be simulated in Matlab and Simulink.

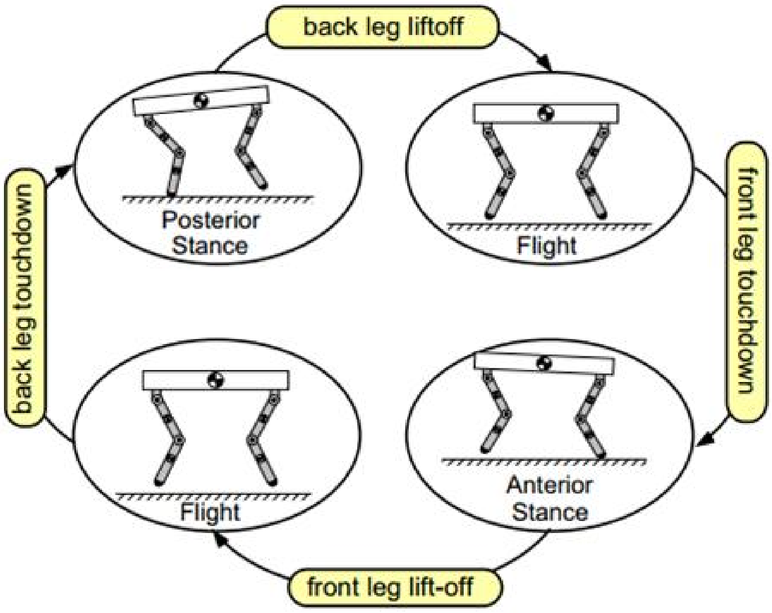


Figure 4.1: Image [58] showing the different phases for a two dimensional quadruped robot. Each phase needs to be modelled and stitched together for the hybrid dynamics system.

Video analysis of cheetahs and greyhounds accelerating gave insight into the spine design and the choice of spines. Four spines will be modelled and compared. The different spines include:

1. A rigid link spine.

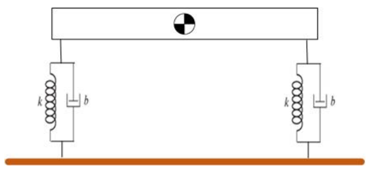


Figure 4.2: A rigid link spine model.

2. A single joint, two link rigid spine.

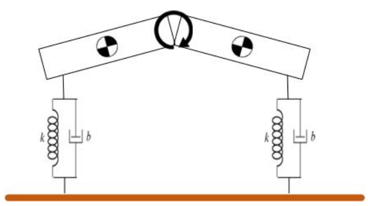


Figure 4.3: A two link rigid spine model.

3. A multiple joint, multi rigid link spine (each link represents a vertebrae), which can be viewed as a flexible spine.

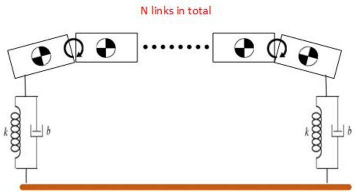


Figure 4.4: A multiple joint rigid link model.

4. A prismatic spine.

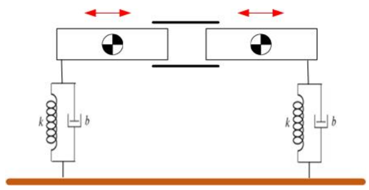
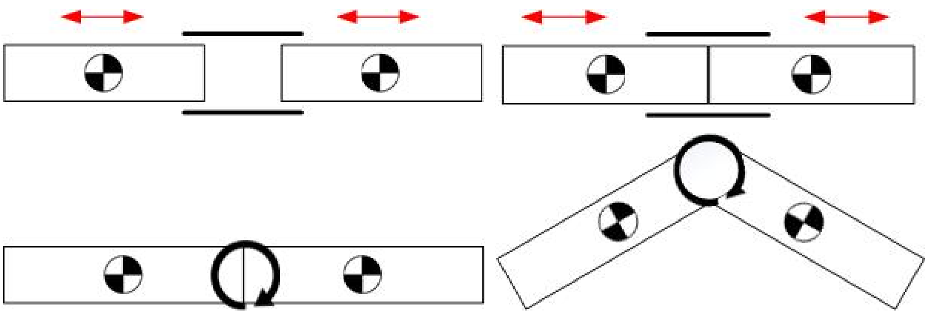


Figure 4.5: A prismatic link spine model.

The performance of the spines will be benchmarked against the rigid link spine. To mimic a flexible link, compliance can be added into the spines through the motor dynamics [6] and will be modelled. The quadruped robot will first be modelled in two dimensions (planar robot with one rear leg, one front leg and the spine). It will then be extended to three dimensions (two rear legs, the spine and two front legs). The planar robot will only have a spine capable of pitching, while the full three dimensional robot will be able to roll and pitch its spine.

The prismatic spine will not be able to pitch but will have the same effect. The prismatic spine can lengthen and shorten the spine, while a rotary joint spine lengthens and shortens its spine by pitching the links, as seen in Figure 4.6. The prismatic spine will include a joint to achieve the roll motion. The motion of the CoM for the prismatic and rotary joint spines will be investigated and it will be determined how much of a role it plays.



(a) (b)

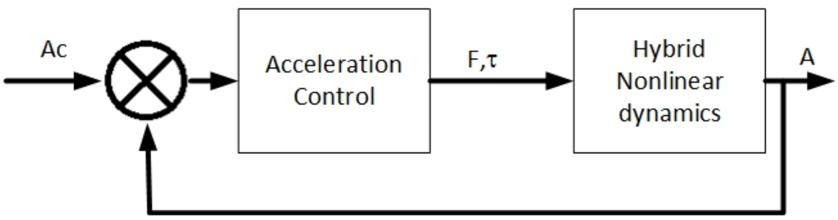
Figure 4.6: Image (a) showing the prismatic and two link spine in the unbent formation. Image (b) showing how the two link spine achieves spine shortening.

The four hybrid dynamic models of the quadruped (a model for each of the three spines of interest and the rigid spine) will be optimized for a locally optimal trajectory where the cost metric is the stride average acceleration over a set distance. The stride average acceleration cost metric was chosen as it is the metric used when analysing the acceleration limitations due to pitching and traction in quadrupeds [3]. SNOPT (sparse non-linear optimizer), along with Matlab, will be used to optimize the trajectory. The spine morphology with the largest stride average acceleration will be chosen as the spine to be designed and built.

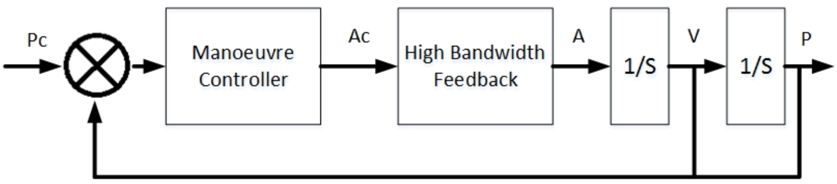
4.2 Feedback Controller Design

A partial feedback linearisation controller will be designed on the simulated model using the optimal trajectory based on the maximum stride average acceleration for the chosen spine morphology. The feedback will consist of motor encoders, IMU data and force sensors to determine the ground contact forces. The ground contact will cause the hybrid dynamic model to switch to the relevant phase.

The robotic platform will be commanded with an acceleration vector as seen in [59]. Acceleration control allows for high bandwidth mitigation of the uncertainties in the model such as friction, Figure 4.7. A desired high level acceleration vector for the entire quadruped robot will be commanded. The controller will consist of multiple layers, the layers below the acceleration control will be in charge of controlling the motors to achieve stable locomotion and the desired acceleration.



(a) Acceleration control using the hybrid dynamics of the system.



(b) The acceleration controller is seen as a high bandwidth feedback block.

Figure 4.7: Images showing the general layout of the proposed controller.

4.3 Experimental Validation

The selected spine will be designed in SolidWorks and manufactured via 3D printing. Stress analysis will be performed to ensure the 3D printed structures are strong enough to withstand impacts and will not deform under high torques. Motion analysis will be performed to ensure the spine design functions as required. The spine will first be designed for a planar (pitch only spine) quadruped and will then be further developed for the full quadruped robot (roll and pitch spine).

The quadruped robot will be made to accelerate from rest on a treadmill system that will keep the platform in the required plane of motion while still allowing rotation about the CoM and translation in the required plane, Figure 4.8. The stride average acceleration of the platform will be measured and benchmarked against the stride average acceleration of the same platform with a rigid spine. The gait of the quadruped robot will be analysed and compared to footage of cheetah's and greyhound's running.

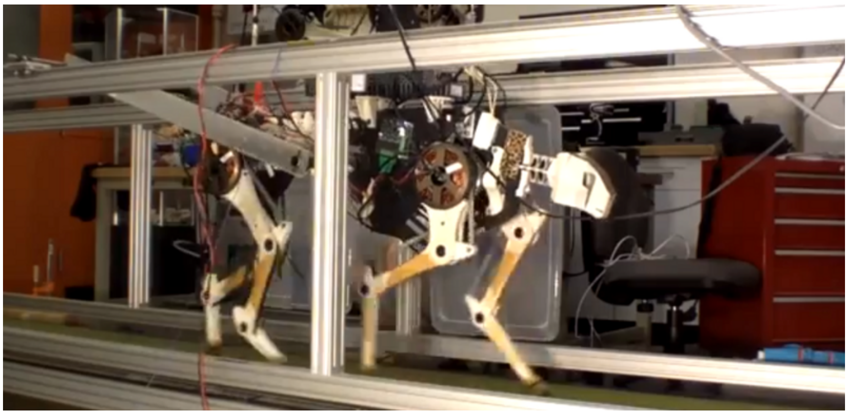


Figure 4.8: Image of a quadruped robot walking on a treadmill [60]. Note the mechanism keeping the robot in the vertical plane while still allowing translational and rotational motion.

The motion of the quadruped robot will be compared to the simulation, if they differ then system identification will be conducted until the physical platform matches the simulation. The simulation will help with more advanced controller design and analysis of the spine and leg interaction. The effects of the active spine can then be properly analysed using the simulation instead of the platform.

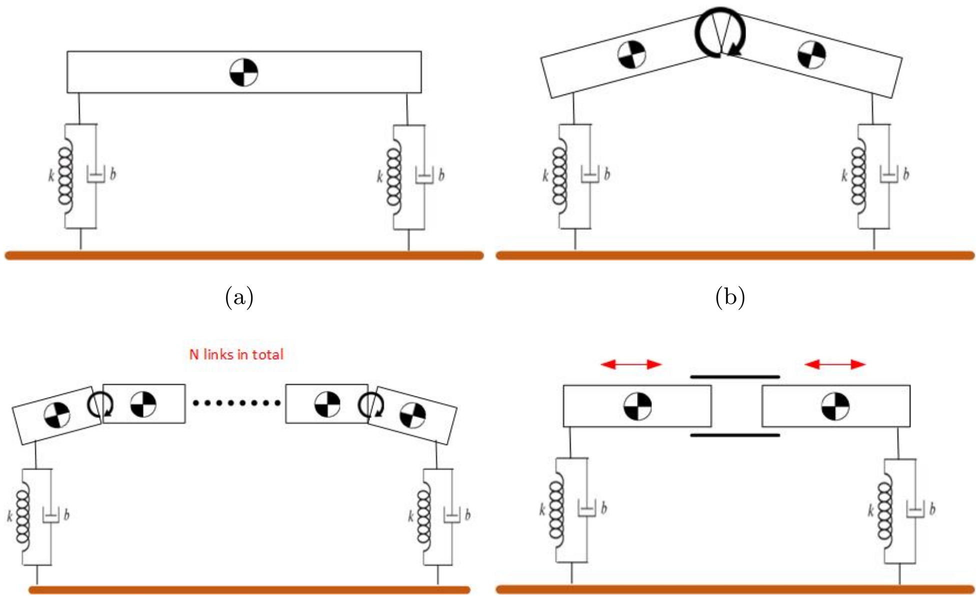
Chapter 5

Preliminary Results

Research was started in January 2016. Cheetahs at Cheetah Outreach in Stellenbosch were filmed and their motion has been analysed. Figure 5.1 shows one of the tail flicks that was filmed at the Cheetah Outreach.

Figure 5.1: A cheetah rapidly flicking its tail while braking during its daily exercise run. Image filmed at Cheetah Outreach in Stellenbosch, South Africa, by the author and supervisor.

Currently the four spines along with the quadruped robot has been modelled using Lagrange dynamics as seen in Figure 5.2. The quadruped was modelled as a number of rigid links with point masses, spring and damping coefficients. A generic algorithm was developed that generates the EoM of the system. By specifying the number of legs, number of links per leg, number of spine links and the degrees of freedom of each link, the EoM will be generated for all the relevant phases. The phases include all the possible stances (one rear foot in contact with the ground, flight phase with no ground contact points, one front foot in contact with the ground *etc.*).



(a) (b) (c) (d)

Figure 5.2: Image showing the four spine models. (a) is the rigid body spine. (b) is the two link spine. (c) is the multi link spine and (d) is the prismatic spine.

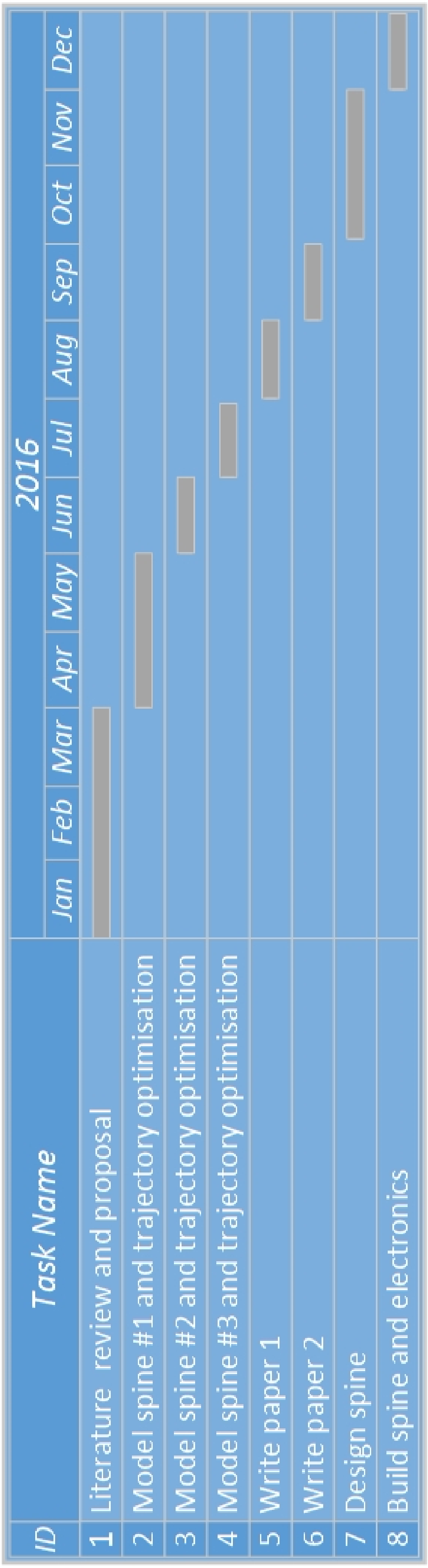
In total there are sixteen possible phases that potentially need to be modelled. So far four of these phases have been completed (enough for the planar model of the quadruped). The model includes the spring and damper terms, prismatic links and the ground contact forces.

Chapter 6

Timeline

Main research goals:

1. August 2016: Trajectory optimize the four planar spines and write a paper for ICRA detailing the results and the choice of optimal spine morphology.
2. March 2017: Develop a planar robot capable of acceleration on a treadmill system. A planar robot will consist of two legs (one front and one rear leg) and the 1 DoF active spine. Write a paper for IROS detailing the results of the stride averaged acceleration of the active spine robot compared to the rigid spine robot.
3. September 2017: Trajectory optimize the full robot (four legs and 2 DoF active spine), design and build it. Write a paper for ICRA detailing the results obtained focusing on the mechanical design and novelty of the spine.
4. March 2018: Controller designed for entire system.
5. June 2018: Test the controller on the full platform.
6. August 2018: Write up and submit the dissertation. Afterwards write a journal paper focusing on the effects of an active spine on the stride average acceleration of quadrupeds.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Task Name
2016
Jan
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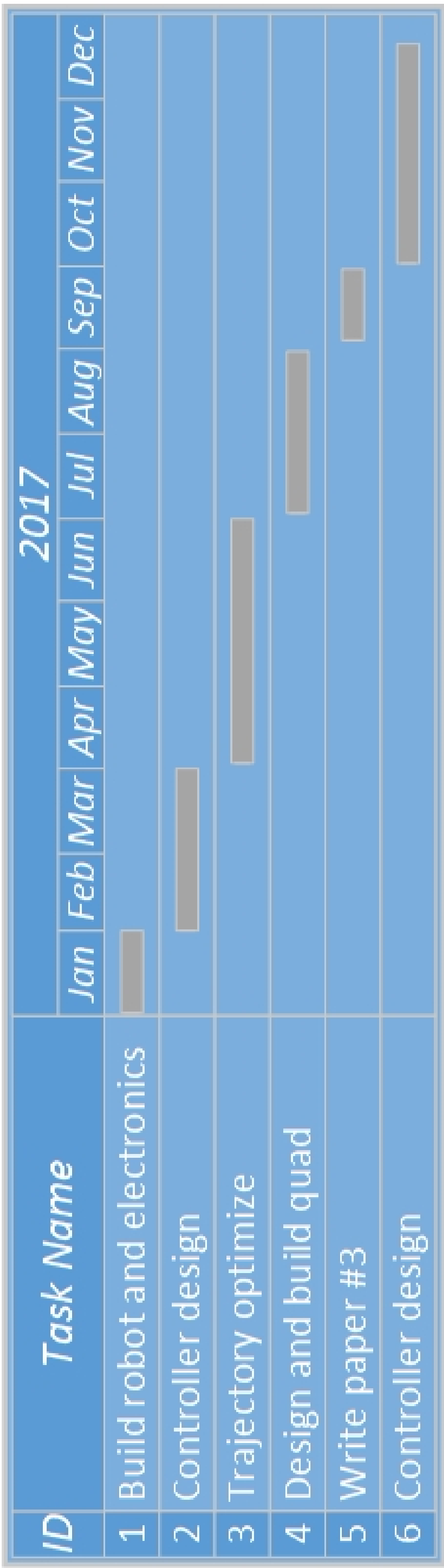
1 Literature review and proposal
2 Model spine #1 and trajectory optimisation
3 Model spine #2 and trajectory optimisation
4 Model spine #3 and trajectory optimisation
5 Write paper 1
6 Write paper 2
7 Design spine
8 Build spine and electronics

(a)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2017
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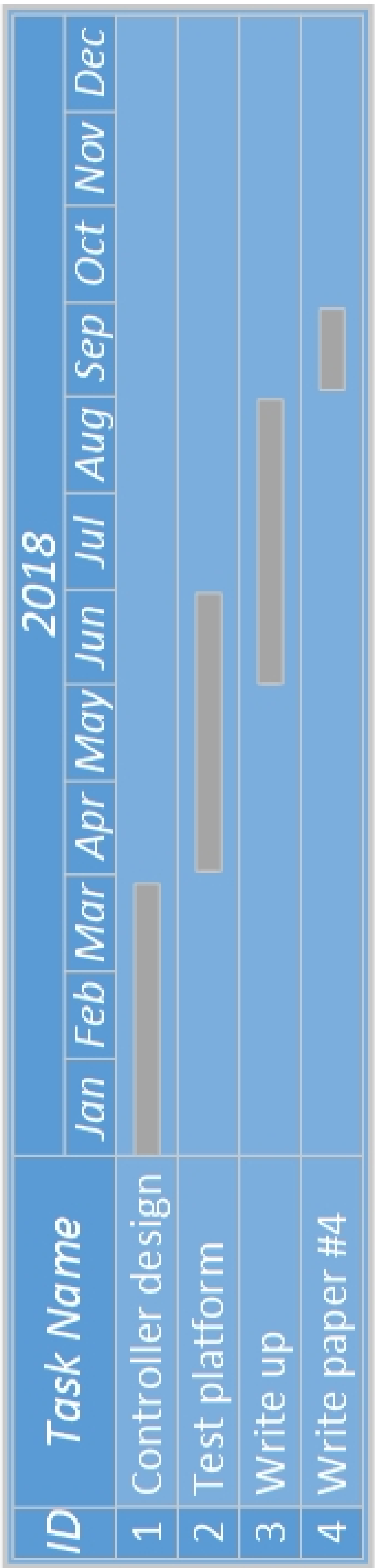
Nov
Dec
1 Build robot and electronics
2 Controller design
3 Trajectory optimize
4 Design and build quad
5 Write paper #3
6 Controller design

(b)


ID
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Apr
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1
Controller design
2
Test platform
3
Write up
4
Write paper #4

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