The Well-Calibrated Bayesian

Ryan Brill

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- Even if a forecaster bases their predictions on a frequentist statistical model, a Subjectivist still believes that the forecaster *believes* in their own model
- A Bayesian has a joint probability distribution over all conceivably observable quantities.
- Forecasting is then a matter of summarizing the conditional distribution of quantities still unobserved, given current information

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- This is not about the “true” probability of rain on a given day, or about any underlying “objective” probability; this refers only to observed long run proportions
- A weather forecaster is *coherent* if their set of beliefs is internally consistent, or obeys the laws of probability theory

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- Paper: *The Well-Calibrated Bayesian* by A. P. Dawid

1. Dawid's Abstract

The Well-Calibrated Bayesian

A. P. DAWID*

Suppose that a forecaster sequentially assigns probabilities to events. He is *well calibrated* if, for example, of those events to which he assigns a probability 30 percent, the long-run proportion that actually occurs turns out to be 30 percent. We prove a theorem to the effect that a coherent Bayesian expects to be well calibrated, and consider its destructive implications for the theory of coherence.

KEY WORDS: Calibration; Coherence; Martingale; Probability forecasting; Subjectivism; Weather forecasting.

2a. Patrick the Forecaster

- Meet Patrick

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- Patrick likes to predict the future
- Patrick uses his subjective probability distribution Π to predict the future

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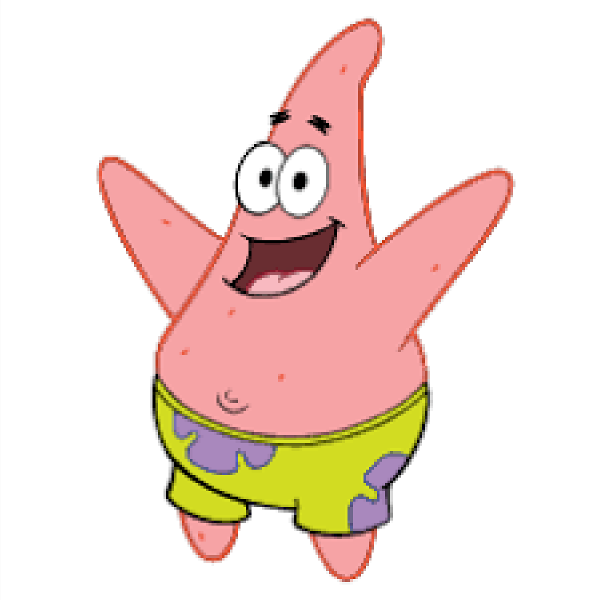

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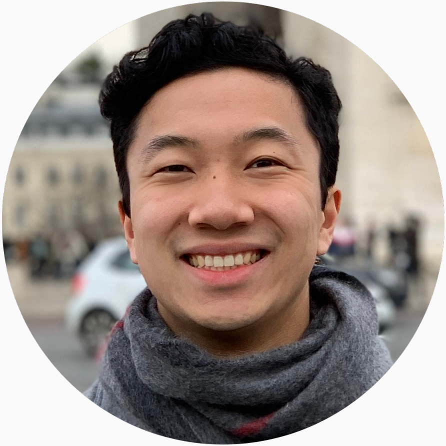

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- It's natural for Patrick to regard these quantities as probabilistically independent
- Patrick predicts a probability distribution over the outcomes, and then computes the median

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- Under Π, this event has subjective probability Binomial$(N, \frac{1}{2})$, and so is vanishingly small
- Yet it occurred!

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    - Patrick is a *potentially miscalibrated individual* - he is not sure whether his future subjective probability assignments will agree with observed frequency

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- each day, he forecasts for tomorrow, drawing on his accumulated experience all that has passed up to today

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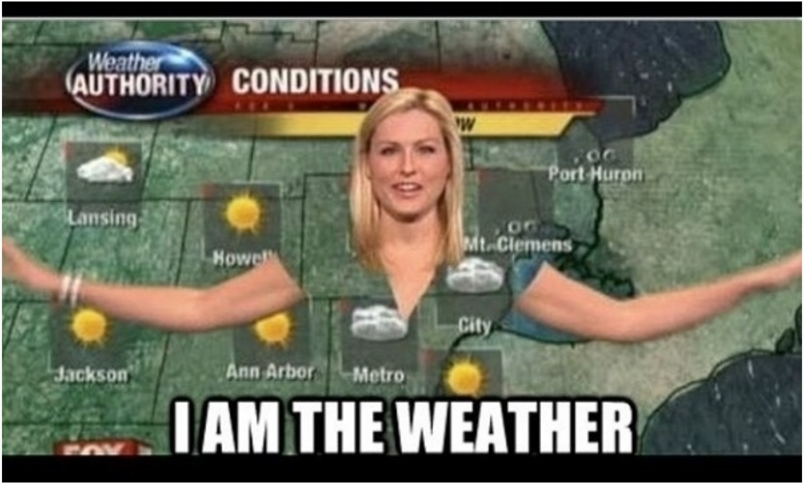

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- Patrick's arbitrary <u>*subjective*</u> probability distribution Π defined over $\cup_{i=0}^{\infty} \mathcal{B}_i$
- Patrick's forecasts made on day i are for events in $\mathcal{B}_{i+1}$, and are calculated from his current subjective conditional distribution $\Pi(\cdot | \mathcal{B}_i)$

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- By def., $\Pi(S_i|\mathcal{B}_{i-1}) = \frac{1}{2}$

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- Possibility 2: If Π is coherent, then Patrick is miscalibrated

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- *admissible*: determine the test set of days (ξ_i) sequentially, so $\xi_i \in \mathcal{B}_{i-1}$

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- Proportion of test days in which Patrick predicts S_i to occur, or the average forecast probability

$$\pi_k = \frac{1}{\nu_k} \sum_{i=1}^{k} \xi_i \tilde{Y}_i$$

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- *A coherent Bayesian feels almost certain he is well calibrated*

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- By Kronecker's Lemma, Π a.s. we have

$$p_k - \pi_k = \frac{1}{\nu_k}\sum_{i=1}^k \xi_i(Y_i - \tilde{Y}_i) \overset{\Pi \text{ a.s.}}{\to} 0$$

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- Here, $\frac{1}{\nu_k} \leq \frac{1}{\nu_i}$

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- In other words, recalibrate ω to $p = \gamma(\omega)$
- Can Patrick learn from his inadequacies as a forecaster, and use this knowledge to improve future assessments?

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- Assume Patrick is coherent. Let A_1, A_2 be disjoint events having Π probability ω_1, ω_2. Since γ is a probability,

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- Details? I can see how γ can't be $c\omega^k$, $k \neq 1$

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- Details? I can see how γ can't be $c\omega^k$, $k \neq 1$
- $c = 1$ because $\gamma(\Omega; 1) = 1 = c \cdot 1$, so

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5. How to Acheive Calibration? Recalibration?

- Paradox: How can Patrick simultaneously assign 2 different probabilities ω and $p = \gamma(\omega)$ to one event?
- He can't...
- Theorem *If ω and γ are both coherent, then they are identical*
- Assume Patrick is coherent. Let A_1, A_2 be disjoint events having Π probability ω_1, ω_2. Since γ is a probability,

$$\gamma(A_1 \cup A_2; \omega_1 + \omega_2) = \gamma(A_1; \omega_1) + \gamma(A_2; \omega_2)$$

- Then by continuity of γ, $\exists c$, $\forall 0 \leq \omega \leq 1$,

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- Corollary: *Patrick can't coherently recalibrate his forecasts*!

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- Indeed, we know no valid model for how people should change their opinions when assessments they believe are coherent systematically deviate from the observed frequencies of events
- We speculate that the fault lies in the idea that γ must depend on ω alone
- Rather, what people should learn from outcome feedback is something about properties of the world

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- Under this view, the innocent-sounding assumptions of the General Theorem - that you know the outcomes of all previous events before you state your probability of the next event - is a *psychologically strong requirement*
- Not only must you have perfect memory for past outcomes, but you must also be able to use outcome information to develop a better understanding of the world