# STAT 991: Topics in Modern Statistical Learning
# Lecture 1

Edgar Dobriban

January 18, 2022

# Overview

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## Introduction to Topics

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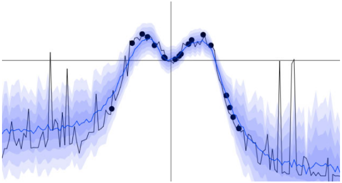

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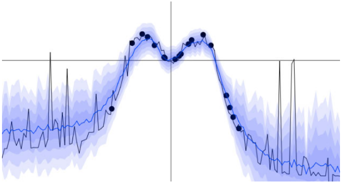

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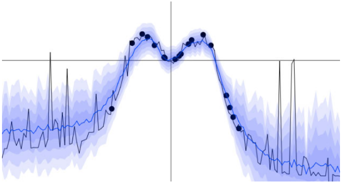

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- Some materials will be available only on Canvas (at least temporarily)

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- See syllabus for: Prerequsites, Feedback, Grading Policy

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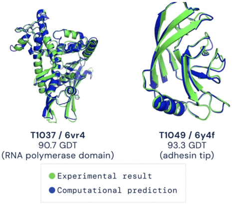
# Perspective

- Machine learning (ML): one of the most salient intellectual developments in today's world (part of broader "Data Sciences")

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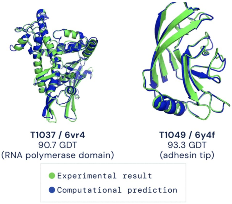


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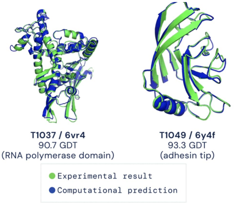


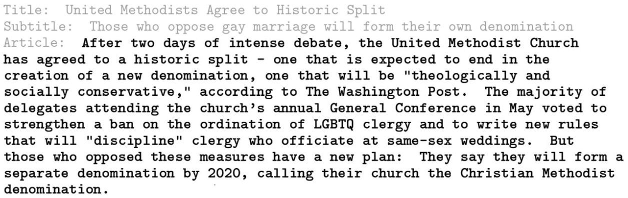
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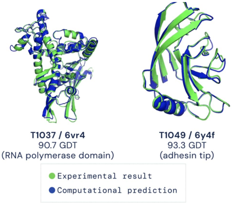


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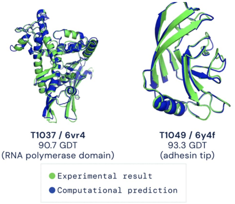


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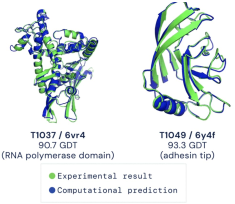


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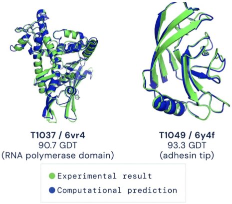


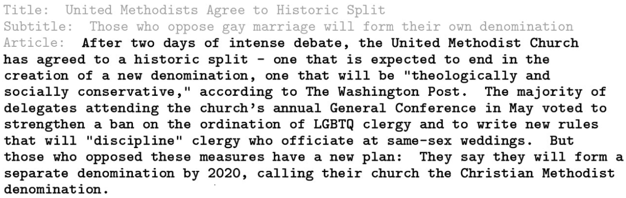

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  - Causality and learning from natural experiments (a bit broader; Econ Nobel 2021)

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- Less glitzy, but very important examples: predicting phenotype from genetic variables, Predicting response to medical treatment

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- Standard ML pipeline (see next) does not provide a solution
  - Currently no "standard" method

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  - Prediction Set: for some mapping $C$ of inputs to subsets of $\mathcal{Y}$: $P(y \in C(x)) \geqslant 1-\alpha$, for some $\alpha \in (0,1)$. Or, conditional guarantee

$$P_Z(P_{(x,y)}(y \in C(x;Z)) \geqslant 1-\alpha) \geqslant 1-\delta$$

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$$P(f(x) = y | p(x) = t) \approx t$$

[2] what is the meaning of "probability", or the source of randomness? discuss later

# Uncertainty quantification

- Uncertainty = probability[2]
- Goal: Make probabilistic statements about predictions
- Examples:
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  - Or... you name it: conditional on $x$, or on $y$, or only partial, or under dependence, or structured settings, or...
- Is uncertainty ever good?

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# Uncertainty quantification: probability

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- The language of probability is already at the core of theoretical ML: generalization bounds, probabilistic ML (graphical models), Bayesian methods, etc. — so it is already widely accepted

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  5. relatively simple, isolated problem

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  2. In the sciences, a dominating paradigm is to collect a small amount of data in a controlled way, and make inferences based on that

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    - Meteorology: calibration (aka validity, Miller, '63)
- There is no universally accepted approach. Some are mainly theoretically justified, some mainly empirically. Big open problem: can we have both?

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  - One of the most exciting areas at the interface of statistics and ML. A lot of great work being done right now, by great research groups. You can be part of it!

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